Quantitative Analysis of Parametric Roll and Operational Guidance

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ABSTRACT

In this paper, it is shown that a large number of numerical simulations are needed to get a stable long-term statistics of parametric roll due to the non-ergodicity of this physical problem. To this end, a very fast and effective tool with an acceptable accuracy is proposed. This method is based on 1 DOF roll equation accounting for GM variation and approximated GZ curve and tested in both of regular and irregular waves. Using this tool, the vulnerability to parametric roll is quantified for a post-Panamax container ship and VLCC by means of long-term prediction in North-Atlantic seas using Monte Carlo Method. For the operational guidance, state-of-art tool based on IRF method is introduced, and it is shown that the application to operational guidance is quite successful for a very large container ship. Based on this study, the parametric roll characteristic is studied in the view of ship operation.

KEYWORDS

Parametric roll, Quantitative analysis, GZ appoximation, GM RAO, Operational Guidance.

INTRODUCTION

In seakeeping problem, the assumption of linear system and Gaussian excitation can be quite valid for the wave elevation and heave and pitch motion. However, roll motion equation is not linear system due to nonlinear damping and restoring moment. If the process is not ergodic, a number of realizations are necessary to get a stable probability density function. More number of realizations will give more stable results and the proper number of realization will depends on the degree of non-ergodicity (Belenky, 2003).

Considering many sea states in wave scatter diagram, the total number of realizations to get a stable long term prediction significantly increases. Therefore very fast and effective numerical tool is needed for quantitative analysis under some simplification of real physics.

If heave and pitch are given, GM fluctuation can be calculated with consideration of ship motions and wave. The GM fluctuation can be expressed by Fourier series. The mean value and first component are very important and these values are quite linear for post-Panamax container carrier practically. Under this assumption, transfer function of mean and first component can be used as GM variation in irregular seas.

Simplification of righting arm using still water GZ curve and above GM variation can make it possible to carry out a huge number of numerical realizations by Monte Carlo Method. Using this simplified method, the vulnerability of parametric roll will be measured quantitatively for a post-Panamax container ship with several GM values considering all seas states in North Atlantic seas. This kind of quantitative results can be useful for ship design and validation of safety level to decide a proper criteria and standard.

If the ship design is not satisfied with a standard of parametric roll corresponding to certain safety level, the severe parametric roll should be avoided in operational stage by means of provision of a proper guidance. The guidance should be developed using state of art tools as far as possible to support ship crew's proper decision making in urgent situation according to 'Framework of New Generation of Intact Stability Criteria' (IMO, 2010).

The impulse-response-function (IRF) approach formulated by Cummins (1962) can be a good candidate to compromise the accuracy and efficiency of numerical computation. This approach solves the equation of ship motion by using pre-computed hydrodynamic coefficients. Using this advantage, Spanos and Papanikolaou (2007) have applied the IRF approach in the parametric roll analysis of a fishing vessel.

In this paper, SNU-PARAROLL based on IRF method is applied to develop operational guidance for a container ship and the overall characteristics of parametric roll are observed in the view of ship operation.

MATHEMATICAL MODEL

GM fluctuation

The shape of still water GZ curve is highly influenced by GM which is the slope of the curve at a zero heeling angle. The GZ curve is quite linear up to certain heeling angle before the side of a deck is immersed into water for many ship types. The linear region of GZ curve is almost up to 30 degrees for container ships and even larger in case of tankers. This is still true for a ship in waves. The Fig. 1 shows the GZ curves in every time steps when the crest of a wave moves forward along the ship length. The bold line stands for still water GZ curve. Therefore, the role of GM is very important to predict the initial occurrence of parametric roll and its growth to some extent before the roll is bounded to a steady angle due to nonlinear components of GZ curve.

Fig. 2 shows the GM variation of a post-Panamax container ship, accounting for wave, heave, pitch and geometrically nonlinear hull shape.

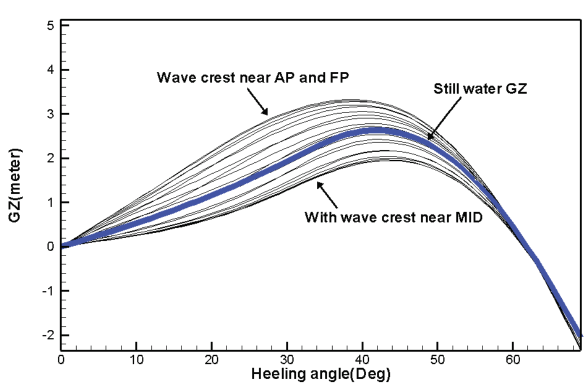

Fig.1: GZ curves in a wave

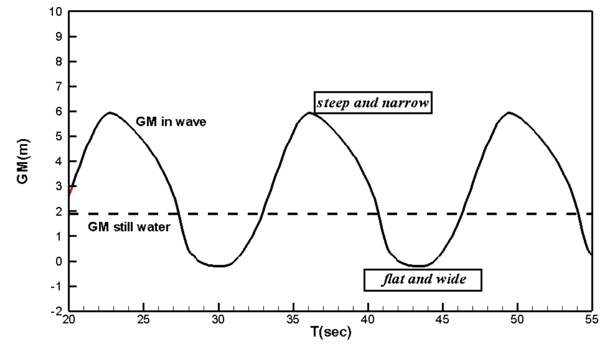

Fig.2: GM fluctuation

When the wave crest is in midship region, the parallel middle body is immersed and the water plane area is a little bit smaller than that of still water and the variation of water plane area is quite small as the crest passes along the long parallel body. On other hand, when the wave crest is near bow and stern, the water plane area is much larger and the variation of water plane area is quite big because the overhanging transom and bow flare are immersed and wetted area is rapidly changed. It is expected that decomposition of GM fluctuation $GM(t)$ into Fourier series can be useful to analyze it as follows.

$$GM(t) = GM_f(t) + GM_{still} \tag{1}$$

$$GM_f(t) = GM_0 + \sum_{n=1}^{\infty} GM_n \cos(n\omega t + \alpha_n) \tag{2}$$

Fig.3 shows mean value and first and second harmonic amplitudes with respect to various frequencies for a container ship and a tanker respectively.

It shows that GM_0 is always positive and the amplitudes of the first component is significant. The maximum amplitude occurs when the wave length is near $0.8L$. The role of GM_0 and GM_1 is obvious to check vulnerability of parametric roll by means of Mathieu equation (ABS, 2004). The GM_0 and GM_1 are induced by geometrical nonlinearity but Fig. 4 shows that these values are quite linear up to very high wave height before the deck is immersed. This kind of linearity is purely dependant on each ship type's hull shape.

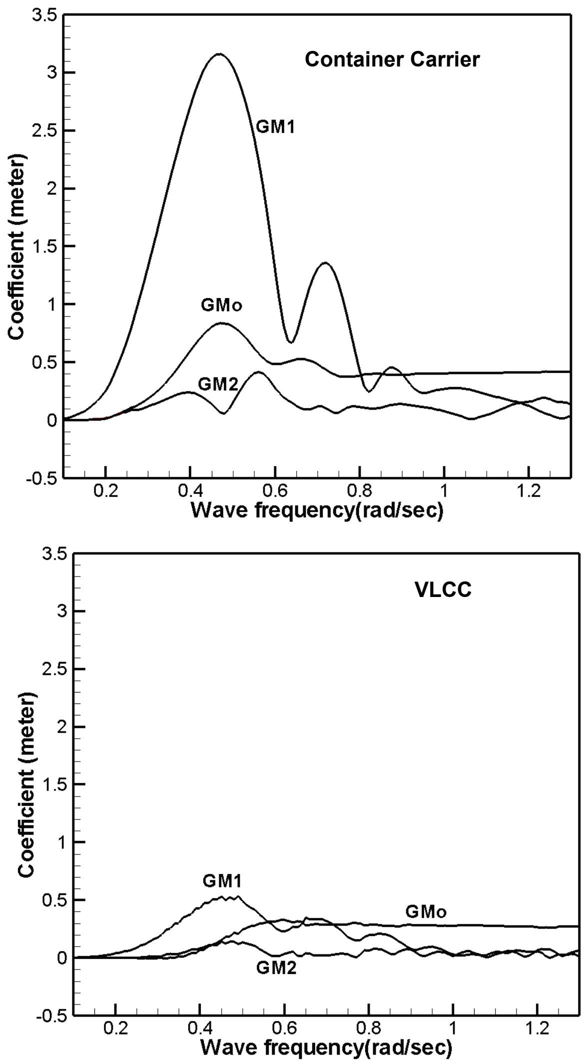

Fig.3 Fourier coefficients for a container ship and a VLCC

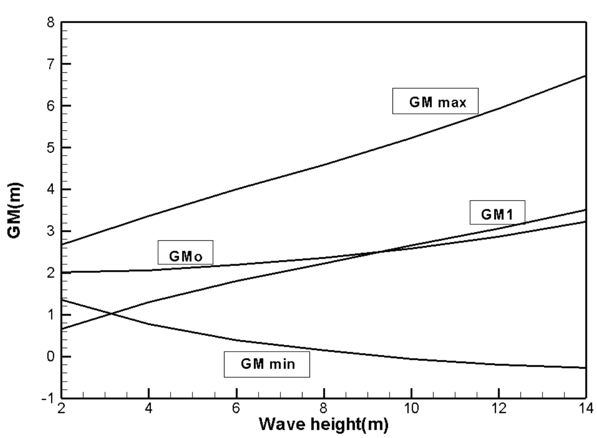

Fig. 4: GM variation for wave height increase

The assumption of linearity of GM_1 in container ship can be also validated in irregular waves. If GM_1 is linear, the response spectrum of GM_1 $S(GM_1, \omega)$ can be obtained by RAO of GM_1 $RAO_{GM0}(\omega)$ and wave spectrum $S(\omega)$.

$$S(GM_1, \omega) = RAO_{GM0}(\omega)^2 S(\omega) \quad (3)$$

Fig. 5 shows the comparison between the response spectrum GM_1 and Fourier transform results from numerical time simulations of *GM*.

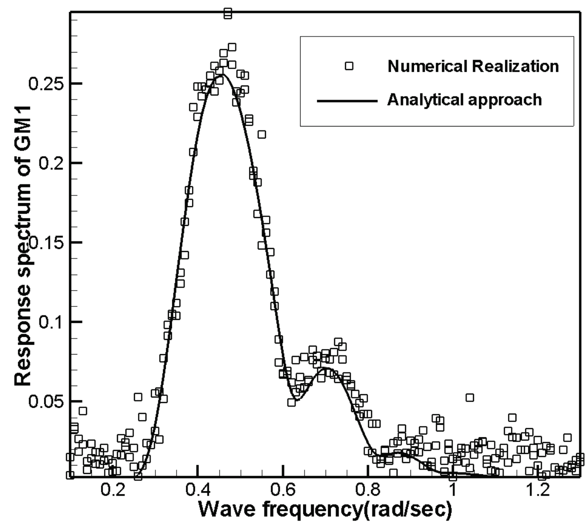

Fig. 5: Comparison of response spectrum of GM1 between numerical and analytical approach

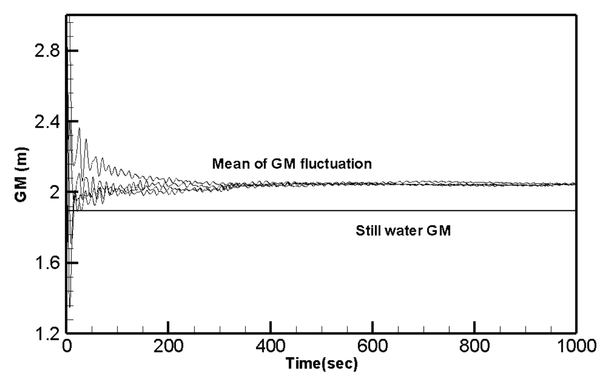

Fig. 6: Variation of mean value of GM for several numerical realizations

Fig. 6 shows also the temporal means of GM variations by several numerical tests. It shows that temporal means of GM variation is asymptotically converged to certain value and its value is positive like a regular wave case. The shift of GM_0 in irregular waves can be obtained by following equation under the assumption of linearity and Rayleigh distribution of GM_0.

$$m_0 = \int S(Hs, Tz, \omega) RAO_{GM0}(\omega)^2 d\omega \qquad (4)$$

$$GM_o(Hs, Tz) = 1.25\sqrt{m_o} \qquad (5)$$

Approximation of GZ curve

There are several approaches to calculate GZ curves (Vidic-Perunovic, 2010). However, the approximation of GZ curve is essential to reduce computation cost. The approximation of GZ curve by still water *GZ* curve and *GM* variation (IMO, 2010) is very useful to reduce computation cost with keeping an acceptable accuracy. Fig. 1 shows that the angles corresponding to zero restoring moment are not changed according to the position of the wave crest. It is easily understood that the variation of water plane area in a wave is small when a ship is inclined to severe roll angle.

The GZ curve in waves is fitted using still water GZ curve $GZ_{still}(\phi)$ and zero GZ roll angle ϕ.

$$GZ(\phi, t) = GZ_{still}(\phi) + \{GM(t) - GM_{still}\} \times \{\sin(\phi) - \sin(\phi)^3 / \sin(\varphi)^2\} \qquad (6)$$

1 DOF roll motion equation

Following 1 DOF roll motion is proposed accounting for GM and approximated GZ curve

$$\begin{aligned}(I_{44} + A_{44})\ddot{\phi} + 2\delta\dot{\phi} + \{GM_{still} + GM_0(Hs, Tz) \times \\ \sum \zeta_i RAO_{GM1} \cos(\omega_e t + \alpha_i)\} + \\ \Delta g[GZ_{still}(\phi) + \{GM(t) - GM_{still}\} \times \\ \{\sin(\phi) - \sin(\phi)^3 / \sin(\varphi)^2] = 0\end{aligned} \qquad (7)$$

Where, I_{44} and A_{44} are roll moment inertia and added mass in roll, and δ is linearized equivalent damping coefficient. The wave height can be obtained from following equation using wave spectrum

$$\zeta_i = \sqrt{2 \cdot S(Hs, Tz, \omega_i) d\omega} \qquad (8)$$

Fig. 7 shows parametric roll analysis for various wave frequencies and slopes. If the wave slope increases, the GM_0 also gets larger and it will shift the resonance frequency to higher one and increase the threshold value of GM_1. Finally the parametric roll will vanish from certain wave slope. It means the higher wave height with same frequency may mitigate the parametric roll angle.

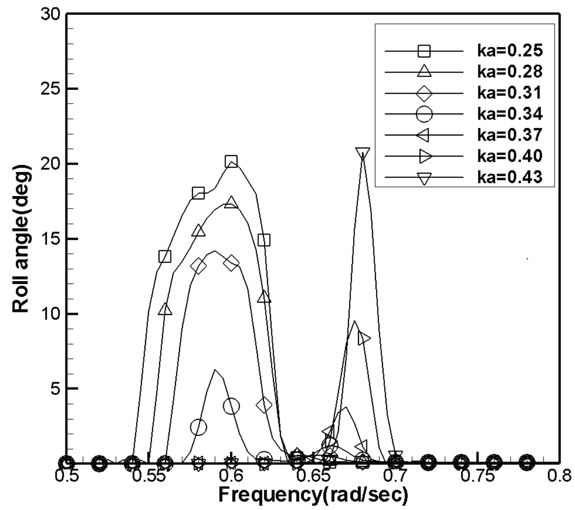

Fig. 7: Parametric roll angle w.r.t wave slope

Fig. 8 shows the number of parametric roll occurrence during 100 times of numerical realizations, for each sea states. Parametric roll never occurs in sea state A and C and always occurs in B.

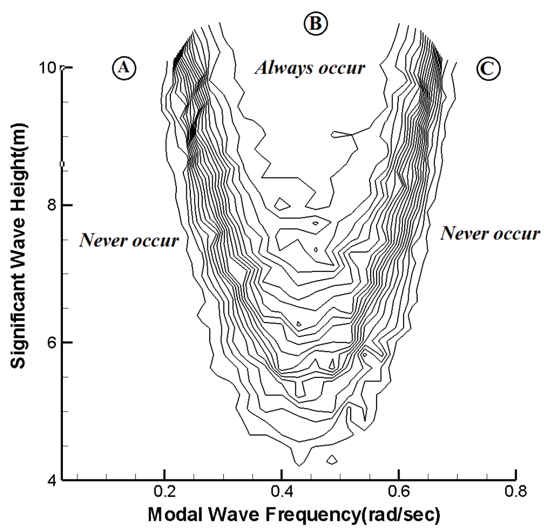

Fig. 8: Map of parametric roll occurrence for irregular waves

Following Fig. 9 shows response spectrum of GM_1 for point A, B and C. In point B, the response spectrum of GM_1 is maximized and the energy is concentrated near the resonance frequency (0.48) while the spectrum is spread and low at point B and C. It means the distance between the modal frequency of response spectrum of GM_1 and resonance frequency and the area near resonance frequency can be a good index for parametric roll occurrence in irregular waves.

COMPUTATION COST

It is necessary to carry out a large number of numerical realizations to get a stable result of parametric roll due to its non-ergodicity. Fig. 10 shows cumulative distribution functions of parametric roll for 50, 500 and 2500 times of numerical realizations for a sea state respectively. Each realization is carried out for 10 minutes as a real scale. The number of realizations depends on the degree of non-ergodicity and confidence level.

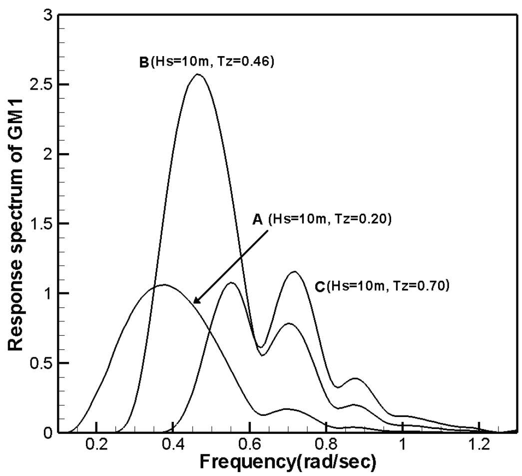

Fig. 9: Response spectrum of GM_1

Current state-of-art tools solving 6 DOF can give a very exact ship motions but it is very time consuming jobs. For example, if it is assumed that 1000 times numerical tests are carried out for each sea state and each test takes 10 minutes, several years are needed for long-term prediction based on IACS standard wave data. Considering above results, even 500 times may give quite unstable results. Therefore the calculation speed seems more important than the accuracy of the numerical code in cases of highly non-ergodic process.

QUANTITATIVE ANALYSIS

Using GM and approximated GZ method, the probability of exceeding roll angle is calculated by Monte Carlo Method considering the wave statistics of the North Atlantic.

$$P(x > \phi) = \sum\sum P(x > \phi, Hs_i, Tz_i)P(Hs_i, Tz_i) \quad (9)$$

1000 times of numerical simulation are carried out for each short-term sea state, and Fig. 11 shows long-term distribution results of roll angle for various GM value.

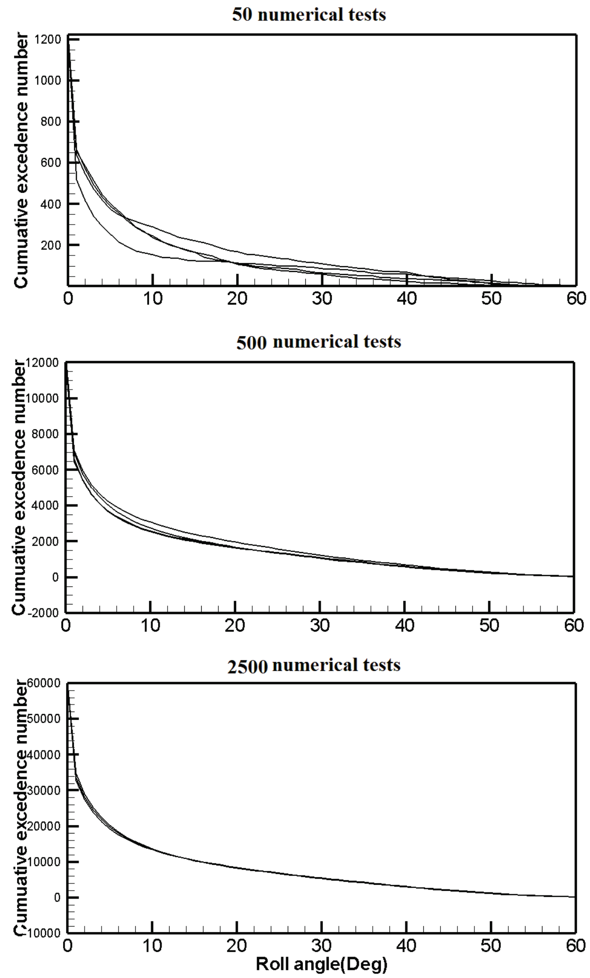

Fig. 10: Cumulative number of exceeding during 50, 500, 2500 realizations respectively

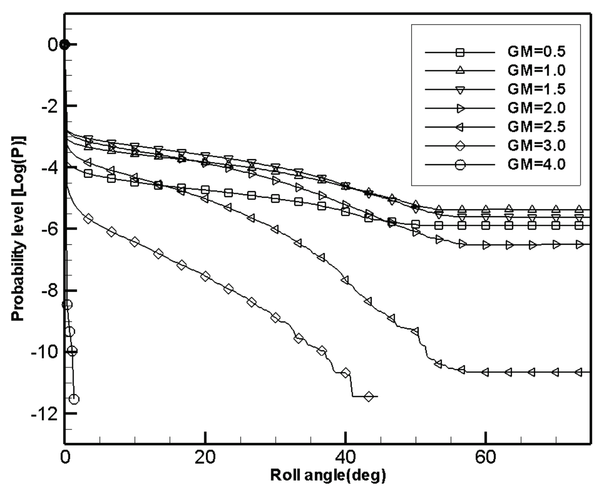

Fig. 11: Long-term prediction of parametric roll for container ship

Several long-term characteristics of parametric roll are observed in the Fig. 11.

Each graph starts from certain probability level because parametric roll are not observed in mild sea state or the wave frequency is far from the resonance frequency. Larger GM gives low exceeding probability in general. If the *GM* is large, the resonance frequency is shifted to high frequency where larger GM_1 and more self exciting energy is needed to overcome threshold value. However, the exceeding probability with *GM* 0.5 is smaller than that of 1.0 and 1.5. In this case, the resonance frequency is shifted to lower frequency where threshold value is also increased.

There is a certain threshold value of GM where parametric roll never occurs. For this container ship, the threshold GM value is between 3 and 4.

The angle from which the curve is kept constant stands for total stability failure angle. If GM increases, the angle starts from low value. It is because large GM results in large roll acceleration and total stability failure starts from smaller angle due to large inertia forces.

The quantitative analysis of parametric roll is also carried out for VLCC. However it is never occurred with regardless of GM value. Considering high GM is kept for VLCC always, the possibility of parametric roll occurrence can be neglected.

Based on above results, the parametric roll seems to occur under the two conditions. The first condition is that GM should be quite low so the wave energy near resonance frequency is sufficiently large. The other condition is the variation of water plane area is large enough to overcome threshold value.

Using the results of quantitative analysis, the parametric roll criteria can be proposed with critical roll ϕ_{cr} and target exceeding probability P_{design}.

$$P(x > \phi_{cr}) < P_{design} \tag{10}$$

If ϕ_{cr} and P_{cr} are assigned as 40 and 10^{-6} respectively for instance, the criteria is not satisfied when the *GM* is 1.5. In this case, the designer can change the hull shape with small flare angle or install counter measure to increase the roll damping.

OPERATIONAL GUIDANCE

The change of ship design is unfavorable for both of ship designer and owner because it increases the shipbuilding cost and results in less ship efficiency. Another choice is to increase the GM above certain value. However, if the GM is high, the transverse acceleration induced by roll angle is very severe and it will results in loss of containers and crew.

The best way to solve this problem is to provide operational guidance to ship crew to avoid very dangerous situations. This kind of operational guidance can be made by using of state of art technology which can deal with 6 DOF freedom, arbitrary heading angle and ship speed. In this study SNU-PARAROLL which is developed based on weakly nonlinear approach and IRF are used to accelerate the calculation speed keeping a high degree of accuracy. The motion equation of SNU-PAARAROLL is as follows.

$$\begin{aligned}(M + M_\infty)\ddot{\xi} + \int_0^t R(t-\tau)d\tau = \\ (F_{F.K.})_{nonlinear} + (F_{Diff}) + F_{viscous} + F_{external}\end{aligned} \tag{11}$$

$$M_\infty = M_\infty(\omega) + \frac{1}{\omega}\int_0^\infty R(t)\sin(\omega t)dt \tag{12}$$

$$R(t) = \frac{2}{\pi}\int_0^\infty b(\omega)\cos(\omega t)dw \tag{13}$$

where the force terms consist of Froude-Krylov, restoring, diffraction, viscous and external forces. The infinite added mass M_∞ and retardation function $R(t)$ can be pre-calculated damping coefficients $b(\omega)$ from a frequency domain code as follows.

Table 1 shows the calculation conditions. The occurrence map is developed with respect to ship speeds and heading angles which are controllable under wave heights and periods and GMs which are given.

Only one realization is carried out for one sea state. Fig. 12 shows examples of operational guidance. The values stand for the average of one third maximum rolls. The figure shows very clear indication of danger zone and safe zone even if one realization is performed. Considering the purpose of operational guidance is not to give exact roll angle but to indicate the degree of danger, it can be acceptable. The most dangerous region is more than 10 knots in following sea with wave length near 200m and very low GM. In this region, the ship is very vulnerable to parametric roll because the encounter modal frequency is near resonance one in that speed and the response spectrum is very narrow-banded and wave energy is concentrated on resonance frequency. When the wave length is near the ship length, both of following sea and head sea are dangerous because GM_1 is maximized. Table 2 shows the summary of operational guidance.

Table 1: Calculation condition for operational guidance

Parameter	Value
Ship Type	10,000 TEU Container carrier
L,B,D(m)	334, 45.6, 27.3
Hs(m)	6, 7, 8
H. Ang.(deg)	0 ~ 360 (15 increment)
Tz(sec)	25, 18, 14, 10, 8
GM(m)	0.73, 1.2, 1.8
Speed(knots)	0 ~ 15(1.5 increment)

Table 2: Summary of operational guidance

GM	Guidance
$GM > 1.8$	Safe
$GM > 1.2$	Cautious(Following sea, 10sec)
	Safe otherwise
$GM < 1.0$	Safe(Hs<6, Tz<8, Tz>25)
	Danger(Head, Following sea , Tz=14)
	Very Danger(Following sea, Tz=10)

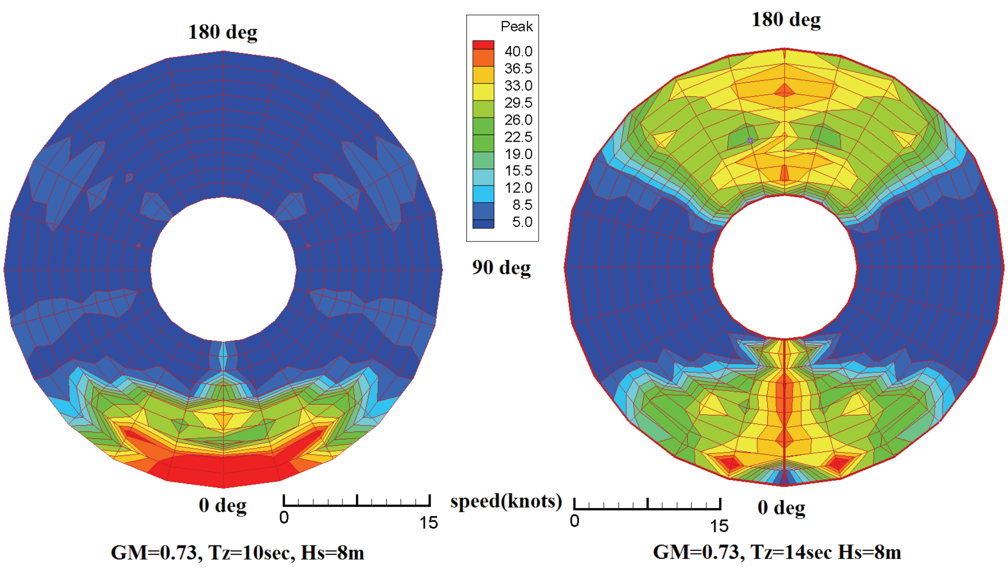

Fig. 12: Examples of operational guidance

CONCLUSIONS

The linearity of mean and first harmonic amplitudes of GM fluctuation is valid for Post-Panamax container ship. The mean and first harmonic amplitudes are dominant to the occurrence of parametric roll. The second harmonic components can result in parametric roll in a very high regular wave but it is impossible in a real sea case. The roll may decrease as wave height increase due to change of resonance frequency induced by shift of mean value.

The GZ can be approximated by mean and first harmonic amplitudes of GM fluctuation and also by still water GZ curve. It gives very fast computation keeping some degree of accuracy.

1 DOF roll equation based on above assumption and approximation is proposed and tested for both of regular and irregular waves. This method is very useful to get a stable long-term prediction accounting for a large number of numerical realizations by Monte Carlo Method.

Using the method, the vulnerabilities to parametric roll can be quantified in a practically short time. The vulnerability to parametric roll of Post-Panamax ship is governed by GM value and if the GM is over certain critical value the parametric roll never occurs.

The operational guidance can be developed successively based on IRF method, giving ship operators enough information to avoid severe roll angle even if one realization is carried out. These results can give ship operator qualitative information such as very danger zone, danger zone, cautious zone and safe zone.

The approach in this paper can be used to measure the vulnerability to parametric roll quantitatively and to check the safety level of parametric roll standard. If the ship cannot satisfy the standard, the ship designer can change the ship design or develop operational guidance using IRF method practically.

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