Regulatory Use of Nonlinear Dynamics: an Overview

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Abstract: The paper is focused on the physical background of the second level vulnerability criterion for surf-riding /broaching-to as a part of the second generation IMO intact stability criteria. The criterion is based on Nonlinear Dynamics, homoclinic bifurcation, in particular, and uses the Melnikov method for calculations. While, well understood in the scientific community, these concepts may present a challenge for regulatory use as most practicing Naval Architects are not familiar with these concepts. The paper presents an explanation of the criterion background using conventional Naval Architecture physical concepts, and gives an overview of the dynamical aspects of the calculation procedure.

Key words: Surf-riding, dynamical system, equilibrium attraction, t

1. Introduction

Current development of the IMO second generation intact stability criteria brought a number of new problems and solutions that are not familiar to a practicing Naval Architect [1]. The reason is not as much new physical phenomena of stability failures, but rather related to the fact that the new criteria are based on first principles. Thus, the new criteria have to rely on a mathematical model of the stability failure; the only input is hull geometry, propulsion and environment characteristics. The development experience has shown that one of the least familiar mathematical techniques is the Melnikov method [2] used in the second level vulnerability criteria for surf-riding and broaching-to [3, 4]. The objective of this paper is bring this subject to the attention of the expert community at the Workshop, as the regulatory use of this technique requires an explanation accessible for practicing Naval Architect.

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2. The Description of the Failure Mode

2.1 General

Broaching-to is a violent uncontrollable turn, occurring despite maximum steering effort in the opposite direction. As with any other sharp turn event, broaching-to is frequently accompanied with a large heel angle, which may lead to partial or total stability failure. Broaching-to occurs in following and stern-quartering seas. Broaching-to is usually preceded by surf-riding. Surf-riding occurs when a wave, approaching from the stern, captures a ship and accelerates her to the speed of the wave profile - wave celerity. While surf-riding, the wave profile does not move relative to the ship. Most ships are directionally unstable in the surf-riding situation; and this leads to the uncontrollable turn, defined as broaching-to (or often, just “broaching”). Therefore, the likelihood of surf-riding can be used to formulate vulnerability criteria for broaching-to [5].

2.1 Surf-Riding Equilibria

When a ship sails in longitudinal waves, three main forces act in the axial direction: thrust, resistance and surging wave force. Since the surf-riding occurs with the speed equal to wave celerity, it is convenient to locate the frame of reference on the wave crest. As the reference frame moves with the wave, the ship remains unmovable in this frame of reference while she surf-rides.

For most practical cases, the surf-riding phenomenon is associated with acceleration of a ship to wave celerity. Thus, the thrust is not sufficient to provide speed equal to the wave celerity in calm water. Consider the difference between the thrust and resistance in calm water within the accepted frames of references; this difference is negative, when the resistance is greater than thrust.

The value of the wave force depends on the location of the ship on the wave. The front slope of the wave pushes a ship forward; while the back slope does the opposite. Indeed, there are neutral points around the wave crest and wave trough. If the wave is sufficiently long and steep, the pushing action of the wave force is sufficient to compensate the negative balance between thrust and resistance and create two equilibria. See Fig. 1 where the wave force, taken with opposite sign, is shown for different positions of a ship on a wave.

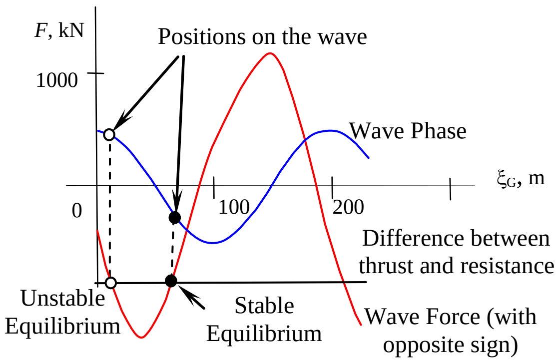

Fig. 1 – Wave forces and balance between thrust and resistance shown for different positions of ship on a wave.

Superimposed with the difference between thrust and resistance, the crossings with the wave force mark the position of two equilibria along the wave. One could note that the difference between thrust and resistance is referred to as "balance between thrust and resistance" in some literature, *e.g.* [4], however this term will not be used here.

3. Mathematical Model of Ship Motions

3.1 Mathematical Model of Resistance and Propulsion

Given wave parameters (length and height), calculating the position of these equilibria does not go beyond conventional Naval Architecture calculations. The first element needed is the approximation of the calm water resistance with a cubic polynomial:

$$R(V_S) = r_1 V_S + r_2 V_S^2 + r_3 V_S^3 \qquad (1)$$

Here V_S is ship speed in m/s, while r_1, r_2 and r_3 are curve-fitting coefficients. Curve fitting is a standard operation, available from a number of software packages, including Microsoft Excel.

The second element is thrust in calm water as a function of commanded number of revolution n and V_S is ship speed in m/s

$$T(V_S, n) = \tau_0 n^2 + \tau_1 V_S n + \tau_2 V_S^2 \qquad (2)$$

The coefficients τ_0, τ_1 , τ_2 for thrust are defined as

$$\tau_0 = c_0 (1 - t_p) \rho D^4 \qquad (3)$$

$$\tau_1 = c_1 (1 - t_p)(1 - w_p) \rho D^3 \qquad (4)$$

$$\tau_2 = c_2 (1 - t_p)(1 - w_p)^2 \rho D^2 \qquad (5)$$

Here t_p is the coefficient for thrust deduction, while w_p is the wake fraction coefficient. Both coefficients are evaluated for calm water. D is the propeller diameter and ρ is mass density of water. Coefficients c_0, c_1, c_2 came from polynomial presentation of the coefficient of thrust K_T:

$$K_T = c_0 + c_1 J + c_2 J^2 \qquad (6)$$

Where J is the advance ratio

$$J = \frac{V_S (1 - w_p)}{nD} \qquad (7)$$

Thrust and resistance are plotted in Fig 2. Indeed, the curves are crossing in self-propulsion point in calm water; Fig. 2 also shows the balance between thrust and resistance as the difference at the speed corresponding to wave celerity.

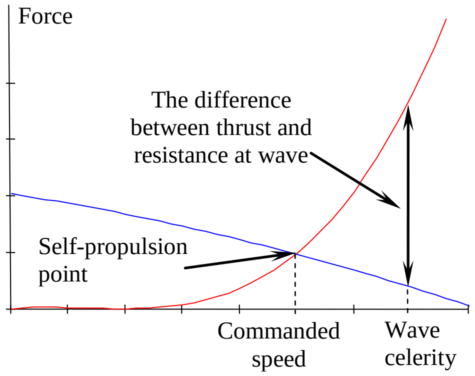

Fig. 2 – Resistance and propulsion showing self-propulsion point and thrust-resistance difference

3.2 Mathematical Model of Wave Surging Force

The surging wave force is a result of the projection of the wave pressure on the longitudinal axis. When a ship is moving in waves, the wave pressure are usually influenced by the presence of the ship. The ship generates waves because of her motions; these waves radiate from the ship and interfere with in incoming waves. Also, the waves that reach the ship, will be reflected from her as from any other obstacle (diffraction). These reflected (or diffracted) waves will also interfere with incoming waves changing the wave pressure on the hull.

However, when considering surf-riding, the ship speed is close to wave celerity. Thus, the encounter frequency is close to zero; no significant ship motions can be expected. Hence, the influence of radiated waves cannot be significant either. Also, if an obstacle moves with a wave, the reflection is going to be weak. Thus diffraction and radiation wave forces can be assumed small and excluded from the consideration.

This simplifies the problem: integrating the pressures along the hull lead to the following formula for the wave surging force:

$$F_W(\xi_G) = -\rho g k \zeta_A [A_S \sin(k\xi_G) - A_C \cos(k\xi_G)] \quad (8)$$

Where ρ is the density of water; g is gravity acceleration; ζ_A is the amplitude of the wave, ξ_G is the position of a ship on the wave; k is the wave number, also known as the spatial frequency of a wave of length λ:

$$k = \frac{2\pi}{\lambda} \quad (9)$$

A_S and A_C are sine and cosine amplitudes of the wave force, respectively:

$$A_S = \sum_i S_i \exp(-0.5kd_i) \cdot \cos(kx_i)\Delta x \quad (10)$$

$$A_C = \sum_i S_i \exp(-0.5kd_i) \cdot \sin(kx_i)\Delta x \quad (11)$$

Here x_i are the distance to station i, measured from the amidships, Δx is the distance between the stations and S_i is the submerged area of station i and d_i is the draft at the station i.

The amplitude of the surging wave forces shown in Fig. 1 is calculated as:

$$A_F = \rho g k \zeta_A \sqrt{A_S^2 + A_C^2} \quad (12)$$

Usually, the value A_S is about 10 times larger than A_C, thus the latter can be safely neglected from equations (8) and (12)

4. The Physics behind the Criterion

4.1 The Mechanics of Surging

The mechanics of surging can be illustrated using just the curves of thrust and resistance. Consider relatively small surging motion while the curves of thrust and resistant are not very different from the tangent lines plotted at the self-propulsion point in calm water, see Fig. 3.

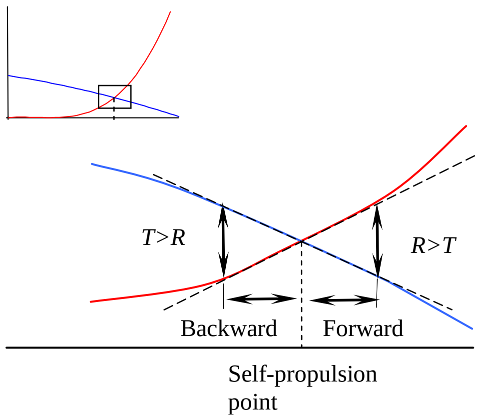

Fig. 3 – Small surging motions around self-propulsion point

Once the surging force pulls the ship backwards, the instantaneous speed decreases and the resistance becomes less than the thrust. The difference between thrust and resistance is directed forward, against the surging speed. When the wave surging force pushes the ship forward, the instantaneous speed increases and the resistance exceeds the thrust. The difference between thrust and resistance is directed against the surging speed again.

Consider the case where the wave force pushes the ship forward. The ship continues motion in the same direction even when the wave force changes sign. Now both wave force and the difference pulls the ship backward. Eventually, the ship changes the direction and surges backward. Once the self-propulsion point is passed the difference between the thrust and resistance changes sign and the surge starts to slow down. Then the surging force also changes the sign and start pushing the ship forward.

We now need to consider how the surging motion is stabilized, i.e. how the steady state amplitude is established?

One can consider the energy balance: the wave transfers to the ship some kinetic energy through the wave force. The difference between thrust and resistance disperses this energy; the balance between the work of these forces establishes the amplitude of surge.

4.2 Stability of Surf-riding Equilibrium

The surf-riding equilibria were referred to as stable and unstable in Fig.1. How it can this be shown?

Consider a ship in a surf-riding mode; midship is located around 70 m forward of the wave crest (marked as stable equilibria near wave trough in Fig. 1) and has a speed that is equal to the wave celerity.

Let the ship be perturbed from this location forward, towards the wave trough. The surge force is smaller there and the difference between thrust and resistance pulls back, since the wave celerity is larger that the commanded speed. Thus, the instantaneous speed of the ship decreases and the wave starts overtaking the ship. Once the ship slips back towards the wave crest, the wave surge force increases and pushes her back to the equilibrium.

Now, let the ship be perturbed from the equilibrium backwards, *i.e.* towards the wave crest. The wave force becomes larger than the difference between thrust and resistance. Thus, the ship will be pushed back to the surf-riding equilibrium (trough).

These simple considerations show that if one tries to perturb the ship from the equilibrium near wave trough, a resultant force pushes it back to the equilibrium. Thus, the equilibrium near the wave trough is stable.

Consider a ship in a surf-riding mode; she is located around 30 m forward of the wave crest (marked as unstable equilibria near wave crest in Fig. 1) and has a speed equal to wave celerity.

Let the ship be perturbed from this location forward, towards the wave trough. The wave surging forces is increasing there; it will push the ship further forward, until she ends up at the stable equilibrium near the wave trough.

If the ship is perturbed from this location backward, towards the wave crest, the wave force is decreased and the instantaneous speed also starts to decrease. The difference between thrust and resistance pulls the ship back and nothing keeps the wave from overtaking the ship. There are several scenarios that consider what may happen next (to be considered in the next subsection), but one thing is clear, the ship does not return back to the equilibrium.

These considerations show that as one tries to perturb the ship from the equilibrium near wave crest, a resultant force takes it away from that equilibrium. Thus, the equilibrium near the wave crest is unstable.

4.3 Attraction to Surf-riding Equilibrium

If surf-riding equilibria do not exist, surf-riding is not possible and the ship will simply surge. That means that all the combinations of instantaneous speed and position on the wave lead to the same outcome *i.e.* it does not matter where the motion has started from.

However, once the equilibria points appear at certain positions on the wave, not all the combinations of the wave position and instantaneous speed lead to the same response.

If a ship is "placed" exactly at the location of the stable equilibrium near wave trough and accelerated to the wave celerity, indeed, she will stay there indefinitely. Any small perturbation from this position will return the ship back to equilibrium (see the discussion in the previous subsection). If a ship is placed to the unstable equilibrium near wave crest, accelerated to wave celerity and then perturbed towards the wave trough, she will end up at the stable surf-riding equilibrium as well.

Thus, there is a set of combinations of wave positions and instantaneous speeds that will lead to surf-riding. One can say that these combinations form a "domain of attraction to surf-riding equilibrium." What happens to a ship outside of this domain?

For translating ship motions in the longitudinal direction, two options are possible: surging or surf-riding. So, in principle, once outside of the attraction domain, the ship either continues to surge or is attracted to surf-riding equilibrium on some other wave. How is the choice between these options determined?

Consider again the energy/work balance of the wave surging force and the difference between thrust and resistance. As it was discussed in the subsection 4.1, the latter disperses the kinetic energy obtained from wave. Once the balance between these two works is established, the ship's response is surging. What if a wave provides the ship with more kinetic energy than the difference between thrust and resistance can disperse?

Eventually, this excessive kinetic energy leads to acceleration and to attraction to the surf-riding equilibrium. The surf-riding becomes a new energy balance between the works of wave surging force and the difference between thrust and resistance. The ship is captured by the wave. Once the surf-riding equilibria appear, is surf-riding inevitable and will occur on one of the succeeding waves?

As it was discussed in the beginning of this section not all the combinations of position on the wave and instantaneous speed lead to the same result. Indeed, the front slope of the wave provides more chances for surf-riding because the wave surging force is directed forward. If started on the back slope of the wave, the wave surging force is directed backward and the surging energy balance still may be achieved. That means: surging and surf-riding may co-exist for the same speed setting and wave parameters. How this can be explained?

If the initial energy level can be dispersed by the difference between thrust and resistance, surging will occur. If the initial energy level is too high (say, front slope of the wave and/or high instantaneous speed) to be dispersed, surf-riding will occur.

If the wave adds too much kinetic energy (say, wave is steep) to ship motions that it cannot be dispersed by the difference between thrust and resistance (say, commanded speed is too large), then surging motions are no longer possible. Even when starting with low initial energy level on the back slope of the wave and commanded speed, each sequential wave will add a bit of kinetic energy that cannot be dispersed; then sooner or later the surf-riding will occur as the ship moves towards stable equilibrium.

4.4 Influence of the Commanded Speed

The discussion in the previous subsection led to the conclusion that if a ship cannot disperse kinetic energy by the difference between thrust and resistance, then surf-riding becomes inevitable. Thus, the commanded speed defines the surf-riding likelihood for the given wave parameters.

If the commanded speed is low, the difference between thrust and resistance (at the speed of wave celerity) is larger than the amplitude of the wave surging force, the intersection (like in Fig.1) does not exist, and the surf-riding is impossible.

Increase of commanded speed leads to appearance of surf-riding equilibria (seen as the intersection in Fig. 1). Surf-riding may be possible for some combinations of wave position and instantaneous speed. Other combinations with lower initial energy level still lead to surging as the difference between thrust and resistance still is capable of dispersing the additional energy. This is the case with the co-existence of surging and surf-riding. The minimal commanded speed corresponding to appearance of the equilibria (i.e. leading to the difference between thrust and resistance equal to the amplitude of the wave surging force) is commonly referred as “the first threshold.”

Further increase of the commanded speed will illuminate the surging mode of motions, because the difference between the thrust and resistance becomes too small to disperse additional kinetic energy obtained from the wave surging force. Surf-riding becomes inevitable. The lowest commanded speed leading to inevitable surf-riding is commonly referred as “the second threshold.”

5. The Reasoning behind the Criterion

5.1 Choice of the Criterion

Two thresholds described at the end of previous section seem to be natural candidates for the criterion. Given the wave parameters, one can find the commanded speed corresponding to one of these thresholds. If a ship cannot make this speed, there is no vulnerability for surf-riding and broaching-to. Which threshold should be used for the criterion?

Use of the first threshold seems to be more conservative as the surf-riding is impossible for the commanded speed below it. However, a simple calculation with formulae (1), (2) and (12) show that the surf-riding equilibria may exist even for ships that have never been observed to surf-ride, such as bulk-carriers. Thus, the criterion based on the first threshold would lack the discriminating power to single out the ships vulnerable for broaching. Why?

Appearance of the surf-riding equilibria makes broaching possible, but requires a ship be placed into the domain of attraction to the stable surf-riding equilibrium. This domain is defined for combinations of wave positions and instantaneous speeds. So it is not enough for the ship to be on the front slope of the wave, but also needs to obtain an instantaneous speed close to wave celerity. For example, for a ship of 180 m length and the wave of the same length, the speed close to the wave celerity will be just above 30 knots. There is no real reason for a ship with the service speed of, say 18 knots, to be spontaneously accelerated up to 30 knots.

At the same time, the second threshold guarantees surf-riding for any ship that can make the speed above the threshold for a given wave. This gives the criterion its discriminatory power and this is why the second level vulnerability criterion is based on the second threshold.

5.2 Evaluation of the Criterion

Use of the criterion requires a way to calculate the commanded speed (setting of number of revolutions or the throttle setting) that corresponds to the second threshold. In principle, it can be done by numerical simulations [4]. The Melnikov analysis gives a process to do it quickly and easily [2].

Consider two or three sequential waves. Let’s assume, one has found the boundaries of the domain of attraction to stable surf-riding equilibrium. If the commanded speed is below the second threshold and allows co-existence of surging and surf-riding, the boundary of the attraction domains of sequential waves, must have some separation between them to allow combinations of position on a wave and instantaneous speed leading to surging.

There is a class of mathematical models, known as Hamiltonians, that provide analytical solutions for these boundaries. Unfortunately, they cannot be applied directly because they do not include any energy dispersion, which is essential for the problem at hand.

The Melnikov analysis is an asymptotic expansion, (similar to Taylor series) where the Hamiltonian is used as the first term. The influence of the energy dispersion terms is included in the higher order terms. This approach allows expressing the distance between the boundaries (Melnikov function, see [2] for derivation) for a given commanded number of revolutions *n*:

$$M(n) = -\frac{r(n)}{q} - \frac{4}{\pi} p_1(n) + 2p_2 - \frac{32}{3\pi} p_3 \qquad (13)$$

The terms in this equation has the following meaning:

$$r(n) = \frac{k\left(T(c,n) - R(c)\right)}{\left(m + m_x\right)} \qquad (14)$$

Here $T(c,n)$ is the thrust at the speed equal to wave celerity, k is the wave number (spatial frequency, see formula 9), $R(c)$ is the resistance at the speed equal to wave celerity, m is mass of the ship and m_x is the added mass of the ship computed for zero-frequency.

$$q = \frac{k \cdot A_F}{m + m_x} \qquad (16)$$

The amplitude of the wave surging force, A_F, is defined by formula (12).

$$p_1(n) = \frac{3r_3c^2 + 2(r_2 - \tau_2)c + r_1 - \tau_1 n}{\sqrt{kA_F\left(m + m_x\right)}} \qquad (15)$$

$$p_2 = \frac{3r_3c + 2(r_2 - \tau_2)}{k\left(m + m_x\right)} \qquad (16)$$

$$p_3 = \frac{r_3\sqrt{A_F}}{\left(\sqrt{k\left(m + m_x\right)}\right)^3} \qquad (17)$$

The coefficient r and t are defined by formulae (1) through (5). The appearance of the Melnikov Function (13) is given in Fig. 4.

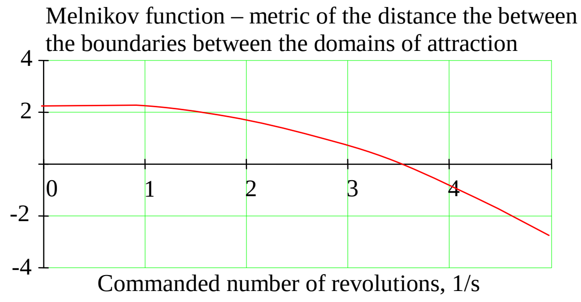

Fig. 4 – Melnikov function

The zero value of Melnikov function approximately corresponds to a zero distance between the boundaries of the domains of attraction to stable surf-riding equilibrium for the sequential waves. Indeed, the number of commanded revolutions is an approximation for the second threshold that was chosen as a criterion.

5.3 Wave parameters

The calculation described in the previous subsection is performed for a given set of wave parameters. How to choose these parameters to reflect a realistic seaway?

The idea is to approximate a realistic seaway as a series of regular waves with random lengths and heights. Then, the parameters of each wave become random numbers and can be obtained from known probability distributions, see [5] for details. In principle, the final form of the criterion is probabilistic and based on a critical wave/ wave group approach, see [6], and [7]. Discussion of the probabilistic aspects of the second level vulnerability criteria falls outside the scope of this paper.

6. Summary and Concluding Comments

The paper is focused on dynamical aspects of the second level vulnerability criterion for surf-riding / broaching-to. The criterion is based on the commanded speed corresponding to the second threshold, exceedance of which makes surf-riding inevitable on a given wave. The appearance of such a threshold is associated with a phenomenon known in nonlinear dynamics as “homoclinic bifurcation” [8]. However, its physical background can be explained without the vocabulary of Nonlinear Dynamics using physical concepts available in Naval Architecture.

The phenomenon of surf-riding is essentially the attraction to the surf-riding equilibrium created when the wave surging force is large enough to compensate for the difference of thrust at the commanded speed and resistance at the speed of the wave profile (wave celerity).

While surging, the difference between thrust and resistance disperses the additional kinetic energy obtained from the wave surging force. When the kinetic energy is too large or the difference between thrust and resistance is too small, the additional kinetic energy cannot be dispersed and the attraction to the surf-riding equilibrium becomes inevitable.

Calculation of the criterion, i.e. the commanded speed leading to inevitable surf-riding on a given wave, can be calculated using the Melnikov method, which is an asymptotic expansion of an analytical solution of this problem. These calculations involve a numerical solution of an algebraic equation, requiring approximate resistance, propulsion and hull geometry data.

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