Modeling Broaching-to and Capsizing with Extreme Value Theory

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ABSTRACT

The paper reviews recent research on the application of extreme value theory for stability failures associated with qualitative physical change: capsizing in waves with account of stability change in waves and broaching-to. As these events are very rare, direct numerical simulation of these events with a code of reasonable fidelity is hardly practical. The assessment of probability must therefore be done without direct observation. This is done using the split-time framework, in which a metric of the likelihood of the failure is introduced. The metric is computed by perturbation of the dynamical system, in phase space, towards the failure state, therefore accounting for changing physics of extreme motions. Extreme value theory is applied to this metric to extrapolate a rate of failure.

Keywords: *Broaching-to, Capsizing in waves, Extreme values*

1. THEORY OF EXTREME VALUES

Any intact stability failure is an extreme event in the sense that its probability is very small, so the value of response associated with the failure, which might be a roll angle for capsizing or a yaw deviation for broaching-to, is quite far on the tail of its distribution. Extreme value theory is a part of mathematical statistics that studies those tails.

The essence of the extreme value theory is that the maxima of independent and identically distributed random variables have a limiting distribution, which is known as a Generalized Extreme Value (GEV) distribution. This is stated by the 1st extreme value or Fisher-Tippet-Gnedenko theorem. Another important distribution is the Generalized Pareto Distribtion (GPD), which is derived from GEV as a conditional distribution above a “large-enough” threshold. The ability of GPD to approximate any tail above a certian threshold is stated by the 2nd extreme value or Pickands-Balkema-de Haan theorem.

These theorems present a possibility of modeling the behavior of the tail without modeling the entire distribution. This is, indeed, a very attractive way to solve many safety-related engineering problems because the safety hazards are associated with large and rare excursions. Thus, the probabilistic assessment of ship stability does not require modeling of roll distribution over its full range – it is enough to know the tail. Both GEV and GPD have three parameters, counting location /threshold. It is therefore necessary only to find those parameters from simulated or measured data and the whole problem of probabilistic stability assessment is solved.

Unfortunately, the simplicity of this approach is quite superficial. Available procedures for finding those parameters simply find the values that fit the data best. However, a ship as a dynamical system is nonlinear and the nature of those nonlinearities manifest itself for the large roll angles. Both GEV and GPD are limit distributions so the applicability of extreme value theory is related to the context of the problem and specific physical mechanism of stability failure.

A review and principle logic of the derivation of both extreme value theorems is available from Coles (2001). The first application of extreme value theory to the stability problem is attributed to McTaggart (2000) and McTaggart and de Kat (2000).

2. NONLINEARITY AND STATISTICS

Peak-over-threshold (POT) is a form of application of the extreme value theory to data exceeding a certain threshold. Campbell, et al. (2016) reviewed the application of POT for roll peak data using the GPD. Smith and Zuzick (2015) described a statistical validation effort of roll data POT. The method seems to work well even for a target angle beyond the maximum of the roll restoring (GZ) curve; however, the confidence interval becomes rather large.

In principle, a decrease of the confidence interval may be achieved without increasing the sample size by introducing a deterministic relationship between the GPD parameters based on a physical consideration. If the shape parameter of GPD is negative, it has an upper limit with the probability equal to zero above that limit. Glotzer, *et al*. (2017) describe how the uncertainty of pitch extrapolation can be decreased by introducing a pitch angle limit of about 12 degrees. This limit was based on the idea that as the longitudinal GZ becomes flat, the ship can no longer receive significant energy from wave excitation.

Peaks of roll motions have a complex distribution tail structure. The possibility of capsizing implies an upper limit of roll peaks as a peak stipulates return. However, the statistics of roll peaks typically shows a positive shape parameter, suggesting that no limit exists. This problem was considered in Belenky, *et al.* (2016). It was found that the softening nonlinearity of the GZ curve around its maximum value leads to positive shape parameter through stretching in the phase plane.

Nonlinearity of the dynamical system may lead to a complex structure of the distribution tail; however, this structure can be revealed and included into the model.

3. CAPSIZING IN WAVES

Qualitative change of physics

Capsizing is a transition to the motions around another stable equilibrium that is dangerous from practical point of view, i.e. “mast down”. During this transition the dynamical system passes the unstable equilibrium at the point of vanishing stability, see Figure 1. The presence of the unstable equilibrium defines the topology of the phase plane in its vicinity and serves as a “separator” between the domains of attraction to the motion around the upright and capsized equilibria. This influence in a statistical sense can be detected when the system is passing relatively close to the unstable equilibria (see considerations on “inflection point” in Belenky, et al. (2016a)). Indeed, this information is absent in the roll motion data set that does not contain a statistically significant number of capsizes or “near-misses”.

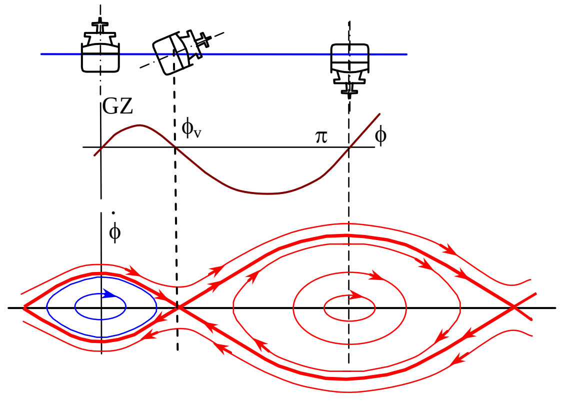

Figure 1: Phase plane of un-damped roll motion.

While the capsizing data is absent from the sample, it is still possible to compute a value reflecting how likely the capsizing is at any given instant of time using the motion perturbation method (MPM). In this method, the roll rate is perturbed until the capsizing is observed (see Figure 2) and the perturbed roll rate is recorded. The difference between the critical roll rate leading to capsizing and the observed roll rate provides a metric of the likelihood of capsizing at this instant of time.

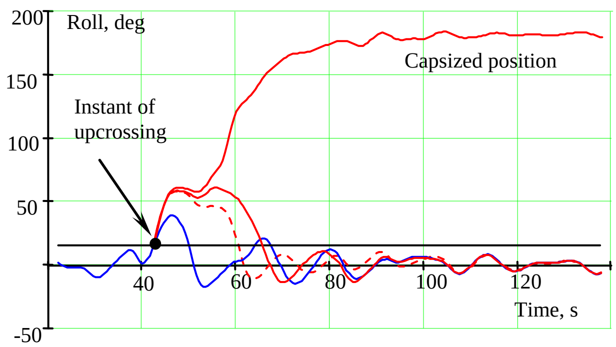

Figure 2: Calculation of critical roll rate (Belenky, et al. 2016b)

This metric is a random variable, as the phasing of the excitation and the stability in wave are random. The metric values can be considered independent if they are computed at the instances that are far enough from each other – say, beyond the de-correlation duration. The independence of the data points in the sample allows extreme value theory to be applied straightforwardly to the metric values. With the motion perturbation method, the metric sample set reflects the change of physics as all of the effects of the transition are explicitly included in the calculation of the metric. Once the GPD is fitted to the metric data, the probability of capsizing can be found as the probability of the event that the observed roll rate reaches the critical roll rate.

In order to relate the probability of capsizing with time, the calculation of the metric can be carried out at the instant of upcrossing of an intermediate level by the roll angle. Capsizing is therefore defined as an upcrossing of an intermediate level in which the metric of capsizing exceeds its critical value (i.e. distance to failure falls below zero). This is how the probability of capsizing is treated under the split-time framework, whose development is described in Belenky, *et al.* (2016b).

Properties of tail of the metric

The application of the extreme value theory through the split-time method for capsizing has been successfully tested via statistical validation carried out for 14 combinations of sea state, heading and speed combination (Weems, et al. 2016). While the performance of the method was good, it could be improved by decreasing the uncertainty of the final estimate. To do this without additional data, the structure of the distribution tail of the metric has to be studied.

Does the distribution tail of the capsizing metric have a limit? Some general argument can be made on this matter. The metric, which is formulated in Belenky, et al. (2016b), has two random components:

$$y_i = 1 - \dot{\phi}_{Ui} + \dot{\phi}_{Cri}\,; \quad i = 1,\ldots,N_U \qquad (1)$$

$\dot{\phi}_{Cri}$ is the critical roll rate calculated for the i^{th} upcrossing, and $\dot{\phi}_{Ui}$ is the roll rate observed at the i^{th} upcrossing.

Both of these random variables are, in principle, limited. The minimum roll rate at upcrossing is a small positive number; a value of zero corresponds to a "touch," so for an upcrossing event to occur, the derivative must be positive.

The critical roll rate must be limited if the capsized equilibrium is stable. Since the capsizing condition always exists in terms of roll velocity, there should be maximum roll rate leading to capsizing from the least probable initial conditions. This idea is illustrated in Figure 3 for the case of damped calm-water roll motion. As the first guess, the limit of the critical roll rate can be taken as an intersection of the separatrix with the vertical axis of the phase plane.

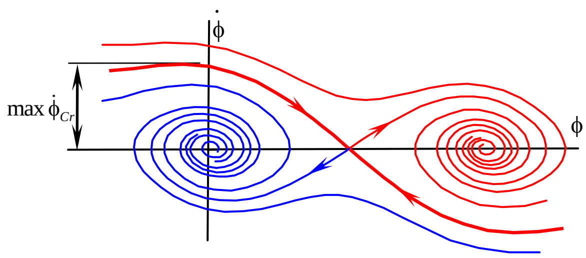

Figure 3: On the maximum critical roll rate

If this argument is correct, the fitting of the GPD is expected to yield a negative shape parameter; however, in many cases, the estimate of the shape parameter is positive (Weems, *et al* 2016).

A similar picture has been observed for roll peaks and described in Belenky, *et al.* (2016). The value of the capsizing metric (1) below 1.0 corresponds to a large roll angle, thus it describes the same random event as the distribution of roll peaks. Does this mean that the tail of the metric (1) has a similar structure as the tail of roll peaks? Can the position of inflection point estimated for the roll peaks be extended for the metric (1)? These questions remain to be answered.

4. BROACHING IN IRREGULAR WAVES

Qualitative change of physics

Broaching-to is a violent uncontrolled turn occurring in following or stern-quartering waves despite full control effort applied on the opposite side. The most frequent mechanism of broaching includes surf-riding, after which the ship becomes directionally unstable. This directional instability leads to repelling in yaw direction.

Surf-riding in regular waves is driven by a dynamic equilibrium that appears when the surging component of the incident wave (Froude-Krylov) force compensates for the difference between the available thrust and the ship's resistance at a speed equal to wave celerity. A similar force balance can occur at instantaneous wave celerity in irregular waves, but such points are not strictly equilibria. The irregularity of the waves and wave forces make both celerity and force change with time so those balance points move unsteadily in the phase plane. The "acceleration" creates additional inertial forces that prevent the ship from staying at such balance points. Thus, those points are not a solution of the equation of motion. To reflect this fact, those points are further referred to as "pseudo-equilibria."

These pseudo-equilibria define the topology of the phase space and create an attraction subset of initial conditions, known in literature as "Lagrangian Coherent Structure", see Kontolefas and Spyrou (2016) for details. The appearance of the pseudo-equilibrium near the current position of a ship (within the coherent structure containing ship position) will accelerate the ship towards the instantaneous wave celerity. If this specific coherent structure makes the ship directionally unstable and if this directional instability lasts long enough, broaching must follow.

Thus, the development of broaching-to is related with the qualitative change of physics related to the appearance of the coherent structure capable of directional instability. If a time history or set of time histories from numerical simulations does not contain attraction events, attempts to fit GPD or GEV are futile as the sample does not contain relevant information on extreme behavior.

Metric of broaching likelihood

Broaching behavior may be included in extreme value consideration within the split-time framework using the motion perturbation method. The metric of broaching likelihood described in Belenky, *et al.* (2016) is based on a concept of "dangerous points" located inside those coherent structures. Not every point inside the structure leads to broaching as the structure may quickly disappear and a significant yaw angle may not have enough time to develop from the directional instability. As a result, the yaw angle deviation has been chosen as a criterion for the selection of dangerous points.

Figure 4 shows a perturbation from an observed position of a ship towards the dangerous point in the surging phase plane (Figure 4a), while the dangerous points are defined as a set of initial conditions leading to large deviation of the yaw angle (25 degrees in Figure 4b).

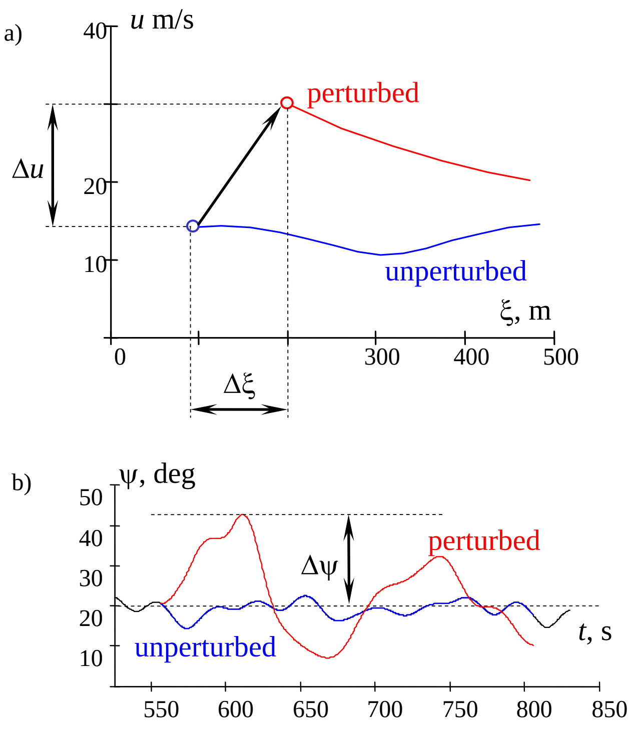

Figure 4: On the definition of the dangerous points: surging phase plane (a) and yaw time history (b)

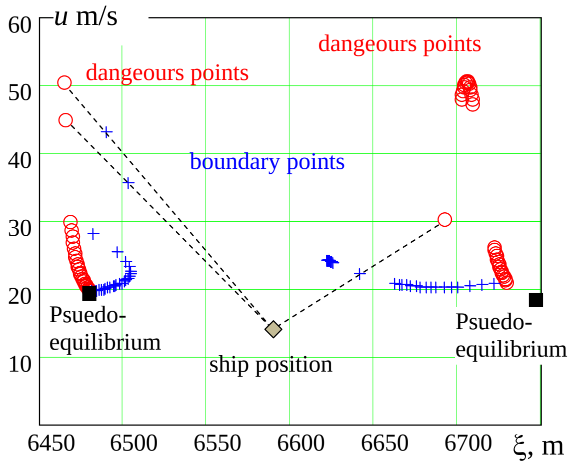

Figure 5: Dangerous and boundary points in the surging phase plane

Figure 5 shows a number of dangerous points found in the vicinity of two pseudo-equilibria closest to the ship position. The "boundary" points are defined as a set of initial conditions leading to exactly specified yaw deviation and are found along a line, in phase space, between the ship position and each dangerous point. The distance to the closest boundary point, referred further as a "critical distance", is the basis of the metric value.

Distribution of the broaching metric

Further calculation procedure includes fitting of the GPD distribution as an approximation of the right tail. To facilitate this, the metric is formulated as

$$z_i = 100 - d_i; \quad i = 1,..., N_U \qquad (2)$$

where d_i is the critical distance at the i^{th} up-crossing. When the critical distance equals zero, the yaw deviation is expected to be "dangerous" and the metric value equal to 100. Figure 6 shows the histogram of the metric before the dependent values of the metric. As the GPD requires independent points, a de-correlation time is used to eliminate dependent points prior to fitting the GPD.

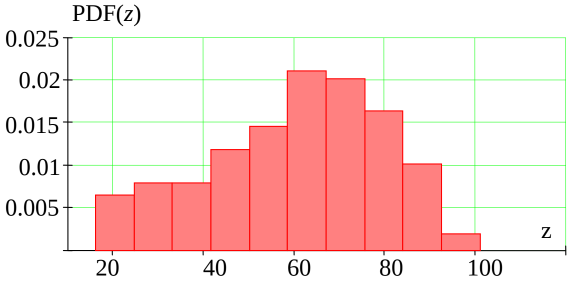

Figure 6: Histogram of the broaching metric before removing dependency

The shape of the distribution suggests a light tail; the initial fit indicates values of the shape parameters around -0.4 after the dependent points were removed.

SUMMARY AND CONCLUSIONS

Two problems were examined from the point of view of extreme value theory: the probability of capsizing and probability of broaching. Both problems are characterized by significant nonlinearity and a substantial change of physics during the transition to the state of failure.

If the information of those changes is not present in the available data, the direct application of extreme value theory will not be successful using only statistical methods. However, constructing an artificial value that does include the change of physics allows application of the extreme value theory to estimate the probability of failure. For the present problems, this is done by formulating metrics based on motion perturbation analysis.

The structure of the tail is a problem of special interest, as the appearance of the upper bound of Generalized Pareto Distribution may indicate the existence of a physical limit. Some considerations have been given to this physical limit of the metric of capsizing in waves. Initial results of the broaching metric calculation indicate the existence of a limit as well.

Further understanding of a nature of those limits and the development of techniques for their estimation may be of significant practical and theoretical interest.

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REFERENCES

Belenky, V., Glotzer, D., Pipiras, V. and T. Sapsis 2016 "On the Tail of Nonlinear Roll Motions," *Proc. 15th Intl. Ship Stability Workshop*, Stockholm, Sweden, pp. 109-114.

Belenky, V.L., Spyrou, K.J. Weems, K.M., 2016a "On Probabilistic Properties of Surf-Riding and Broaching-to in Irregular Waves," *Proc. 31st Symp. Naval Hydrodynamics,* Monterey, California, USA.

Belenky, V., Weems, K. and W.M. Lin 2016b "Split-time Method for Estimation of Probability of Capsizing Caused by Pure Loss of Stability," in *Ocean Engineering*, Vol. 122, pp.333-343.

Campbell, B., Belenky, V. and V. Pipiras 2016 "Application of the Envelope Peaks over Threshold (EPOT) Method for Probabilistic Assessment of Dynamic Stability," in *Ocean Engineering*, Vol. 120, pp. 298-304.

Coles, S., 2001, An Introduction to Statistical Modeling of Extreme Values. London: Springer-Verlag.

Glotzer, D., Pipiras, V., Belenky, V., Campbell, B., T. Smith, 2017 "Confidence Interval for Exceedance Probabilities with Application to Extreme Ship Motions," *REVSTAT Statistical J.* (Accepted)

Kontolefas, I., Spyrou, K.J. 2016 "Coherent structures in phase space, governing the nonlinear surge motions of ships in steep waves," *Ocean Engineering*, Vol 120, 339-345.

McTaggart, K. A., 2000 "Ongoing Work Examining Capsize

Risk of Intact Frigates Using Time Domain Simulation,”. *Contemporary Ideas of Ship Stability*, D. Vassalos, *et al.* (eds), Elsevier Science, pp. 587–595.

McTaggart, K. A., de Kat J.O., 2000 “Capsize Risk of Intact Frigates in Irregular Seas,” *Trans. SNAME*, Vol. 108, pp 147–177.

Smith, T. C., and Zuzick, A., 2015 “Validation of Statistical Extrapolation Methods for Large Motion Prediction,” *Proc. 12th Intl. Conf. on Stability of Ships and Ocean Vehicles (STAB 2015)*, Glasgow, UK.

Weems, K., Belenky, V. and B. Campbell 2016 “Validation of Split-time Method with Volume-Based Numerical Calculations,” *Proc. 15th Intl. Ship Stability Workshop*, Stockholm, Sweden, pp. 103-108.