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Novel Statistical Prediction on Parametric Roll Resonance by Using Onboard Monitoring Data for Officers

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ABSTRACT

A novel statistical prediction method on occurrence of roll with large amplitude is proposed based on a methodology applied exponential autoregressive (ExpAR) modeling procedure, which is a kind of nonlinear time series analysis. The verification of the proposed method is implemented by using results of model experiments concerning the parametric roll resonance. It can be con-firmed that the large amplitude roll motion can be predicted based on the predictive probability distribution calculated by using the statistically optimum ExpAR model determined by Akaike Information Criterion (AIC).

Keywords: *Parametric roll resonance, ExpAR model, AIC, Predictive probability distribution*

1 INTRODUCTION

There are many studies concerning a para-metric roll resonance that have been published in the past Stability of Ships and Ocean Vehi-cles (STAB) conference and the International Ship Stability Workshop (ISSW) (e.g. Belenky & Campbell 2012; Bulian & Francescutto 2012; Cooper & McCue 2012; Katayama *et. al.* 2012; Miguez-Gonzalez *et. al.* 2012; Hashimoto & Umeda 2012; Ovegard *et. al.* 2012; and so on) from the viewpoint of naval architecture. And excellent knowledges concerning this issue has been showed. However, they are not enough from the viewpoint of ship officers, since ship motions under navigation are big different due to lording conditions of cargoes and external forces such as waves, wind, current and so on. Therefore, it is very important for officers to understand the state of roll motion in which it is the steady or the unstable.

From this background, one of authors (Ter-ada, 2014) suggested that during navigation, of-ficers should keep monitoring the roll motion. In that study, the dynamical system on roll mo-tion can be approximated by an exponential autoregressive (ExpAR) model that is a kind of a nonlinear time series model, and it showed that roots of a characteristic equation on the ExpAR model are an evaluation index useful as the method to confirm the state of roll mo-tion. In short, if all characteristic roots lie in-side of the unit circle, then the system is sta-tionary and stable. Moreover, when the real part of the characteristic root changes from pos-itive/negative to negative/positive, the dynam-ical system for the roll motion can be evaluated as nonlinear for the damping force. Moreover, when the imaginary part of the characteristic root changes from positive/negative to nega-tive/positive, the dynamical system for the roll motion can be evaluated as nonlinear for the restoring force. Therefore, since officers can un-derstand the detailed dynamics of the roll mo-tion under navigation, it is considered that the proposed method is useful for promoting safer navigation. However, we has been pointed out that it is very difficult for officers to understand the result, and we need to solve this issue.

On the other hand, it is possible to consider that the ExpAR model is one class of the ra-dial basis function (RBF) approximation model in the neural network approach. Ueno & Han (2013) attempted to predict the time series of the roll motion, they showed it's effectiveness. Note that this kind of approach cannot use in actual navigation, since officers do not steer confirming the time series of the roll motion and serious accidents occur with the failure of the prediction.

In this study, we attempt to establish a novel statistical prediction method, which uses an upper and lower endpoint of a pre-dictive probability distribution calculated from a stochastic simulation based on the ExpAR model, in order to give the significant infor-mation concerning the roll motion to officers. To confirm the effectiveness of the proposed method, we analyzed the data of the paramet-ric roll resonance. The obtained findings are reported.

2 RELATIONSHIP BETWEEN NONLIN-EAR STOCHASTIC DYNAMICAL SYS-TEM AND TIME SERIES MODEL

Firstly, we mention a relationship be-tween nonlinear stochastic dynamical system and time series model according to Terada& Matsuda (2011) and Terada (2014).

Consider the following nonlinear stochastic dynamical system concerning the roll motion:

$$\ddot{x}(t) + f\big(\dot{x}(t)\big) + g\big(x(t)\big) = u(t) \tag{1}$$

where $x(t)$ indicates a roll angle, the notation $(\cdot)$ and $(\cdot\cdot)$ indicate the 1st and the 2nd or-der deferential operator with time, $f(*)$ indi-cates the nonlinear mapping function concern-ing the damping force, $g(*)$ indicates the non-linear mapping function concerning the restor-ing force and $u(t)$ indicates an external distur-bance that is treated with the random variable, respectively. Note that $u(t)$ has the finite vari-ance, but is not white noise sequence. And Equation 1 can be written in the following vec-tor form:

$$\dot{\boldsymbol{x}}_t = \mathcal{F}\big(\boldsymbol{x}_t\big) + \boldsymbol{u}_t \tag{2}$$

where, as the notation (T) means the transpose,

$$\boldsymbol{x}_t = \big[\dot{x}(t), x(t)\big]^T,$$
$$\mathcal{F}\big(\boldsymbol{x}_t\big) = \Big(-f\big(\dot{x}(t)\big) - g\big(x(t)\big), \dot{x}(t)\Big)^T,$$
$$\boldsymbol{u}_t = \big[u(t), 0\big]^T.$$

According to the locally linearization method (Ozaki, 1986), Equation 2 can be discretized as follows:

$$\boldsymbol{x}_n = \mathrm{EXP}\big[\boldsymbol{K}_{n-1}\Delta t\big] \cdot \boldsymbol{x}_{n-1} + \boldsymbol{B}_{n-1}\boldsymbol{u}_n \tag{3}$$

where,

$$\boldsymbol{x}_n = \big[\dot{x}_n, x_n\big]^T,$$
$$\boldsymbol{K}_n = \frac{1}{\Delta t}\mathrm{LOG}\big(\boldsymbol{A}_n\big),$$
$$\boldsymbol{A}_n = \boldsymbol{I} + \boldsymbol{J}_n^{-1}\Big\{\mathrm{EXP}\big[\boldsymbol{J}_n\Delta t\big]\Big\}\boldsymbol{F}_n,$$
$$\mathrm{LOG}\big(\boldsymbol{A}_n\big) = \sum_{k=1}^{\infty}\frac{(-1)^k}{k}\big(\boldsymbol{A}_n - \boldsymbol{I}\big)^k,$$
$$\boldsymbol{J}_n = \frac{\partial\mathcal{F}\big(\boldsymbol{x}_n\big)}{\partial\boldsymbol{x}_n},$$
$$\boldsymbol{F}_n\boldsymbol{x}_n = \begin{pmatrix} -f(\dot{x}_n) & -g(x_n) \\ \dot{x}_n & 0 \end{pmatrix},$$

and, Δt indicates a discrete interval and $B_{n-1}u_n$ is a two-dimensional colored noise se-quence, which is obtained by the stochastic in-tegral.

In Equation 3, since the term of the noise is not the white noise sequence, it is necessary to transform the colored noise sequence into a white noise sequence in order to deal with the problem stochastically. To do the whitening, Yamanouchi (1956) showed how to use the discrete autoregressive process: in Equation 3, let

$$\boldsymbol{\varepsilon}_n \equiv \boldsymbol{B}_{n-1}\boldsymbol{u}_n. \tag{4}$$

Then this can be approximated by the following m–th order discrete autoregressive process.

$$\boldsymbol{\varepsilon}_n = \sum_{i=1}^{m} \boldsymbol{D}_i \boldsymbol{\varepsilon}_{n-i} + \boldsymbol{w}_n, \quad (\boldsymbol{\varepsilon}_n = \boldsymbol{w}_n \text{ for } i = 0), \tag{5}$$

where w_n is a 2×2 Gaussian white noise se-quence with $N(0, \mathrm{diag}(\sigma_1^2, \sigma_2^2))$ and D_n indi-cates a 2×2 autoregressive coefficient matrix. On the other hand, the following relation is ev-ident.

$$\begin{aligned}
\boldsymbol{\varepsilon}_n &= \boldsymbol{x}_n - \boldsymbol{A}_{n-1}\boldsymbol{x}_{n-1} \\
\boldsymbol{\varepsilon}_{n-1} &= \boldsymbol{x}_{n-1} - \boldsymbol{A}_{n-2}\boldsymbol{x}_{n-2} \\
&\vdots \\
\boldsymbol{\varepsilon}_{n-m} &= \boldsymbol{x}_{n-m} - \boldsymbol{A}_{n-m-1}\boldsymbol{x}_{n-m-1}.
\end{aligned} \tag{6}$$

Therefore, by substituting Equations 6 into Equation 5, we can obtain the following two dimensional $(m + 1)$–th order time-varying autoregressive model.

$$\boldsymbol{x}_n = \sum_{i=1}^{m+1} \boldsymbol{C}_i \boldsymbol{x}_{n-i} + \boldsymbol{w}_n. \tag{7}$$

Here C_i $(i = 1, \cdots, m + 1)$ is the time-varying autoregressive coefficient matrix, which is expressed as follows:

$$\begin{aligned}
\boldsymbol{C}_1 &= \boldsymbol{D}_1 + \boldsymbol{A}_{n-1}, \\
\boldsymbol{C}_2 &= \boldsymbol{D}_2 - \boldsymbol{D}_1\boldsymbol{A}_{n-2}, \\
&\vdots \\
\boldsymbol{C}_m &= \boldsymbol{D}_m - \boldsymbol{D}_{m-1}\boldsymbol{A}_{n-m}, \\
\boldsymbol{C}_{m+1} &= -\boldsymbol{D}_m\boldsymbol{A}_{n-m-1}.
\end{aligned}$$

Moreover, by using the following relation

$$\dot{x}_n \cong \frac{1}{\Delta t}\big(x_n - x_{n-1}\big), \tag{8}$$

Equation 7 can be approximated by the following the $M(= m + 2)$–th order scalar time-varying autoregressive model

$$x_n = \sum_{i=1}^{M} a_{n,i} x_{n-i} + w_n. \tag{9}$$

where $a_{n,i}$ indicates time-varying autoregres-sive coefficients, w_n is the Gaussian white noise sequence with $N(0, \sigma_2^2)$. Now, since $a_{n,i}$ is time-varying autoregressive coefficients, sup-pose that the following relation

$$\sum_{i=1}^{M} a_{n,i} \cong \sum_{i=1}^{M} \big\{\phi_i + \pi_i \exp[-\gamma x_{n-1}^2]\big\} \tag{10}$$

where φ_i is a linear term of autoregressive coefficients, π_i is a time-varying term of autoregres-sive coefficients and γ is a scaling parameter. Thus, Equation 9 can be written as follows:

$$x_n = \sum_{i=1}^{M} \big\{\phi_i + \pi_i \exp[-\gamma x_{n-1}^2]\big\} x_{n-i} + w_n \tag{11}$$

This time series model, which is called an exponential autoregressive(ExpAR) model, was first introduced by Ozaki & Oda (1978). And then characteristics are investigated by Hag-gan & Ozaki (1981). According to Haggan & Ozaki (1981), consider the following character-istic equations of Equation 11:

$$\lambda^M - \phi_1\lambda^{M-1} - \cdots - \phi_{M-1}\lambda - \phi_M = 0 \tag{12}$$

$$\lambda^M - \big(\phi_1 + \pi_1\big)\lambda^{M-1} - \cdots - \big(\phi_{M-1} + \pi_{M-1}\big)\lambda - \big(\phi_M + \pi_M\big) = 0 \tag{13}$$

If all roots of these equations lie inside of the unit circle, then the nonlinear stochastic dynamical system is stationary and stable. Moreover, when the real part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluated as nonlinear for the damping force. And also, when the imaginary part of the characteristic root changes from positive/negative to negative/positive, the dynam-ical system for the roll motion can be evaluated as nonlinear for the restoring force.

3 FITTING OF THE ExpAR MODEL AND PARAMETER ESTIMATION

As to the estimation of the model order M and the coefficients γ, (φ_i, π_i; i = 1, · · · , M) in the ExpAR model, for simplicity, by fixing the parameter γ at one of a grid of values, we estimated the model order M and the corresponding φ_i, π_i parameters as well as Haggan & Ozaki (1981). As N is the total number of observations, after fixing $\gamma = \gamma_0$, the ExpAR model for $n = M + 1, \cdots, N$; $i = 1, \cdots, M$ can be written as follows:

$$x_n = \sum_{i=1}^{M} \{\phi_i + \pi_i \exp[-\gamma_0 x_{n-1}^2]\} x_{n-i} + w_n. \quad (14)$$

So the matrix form of Equation 14 can be written as

$$\boldsymbol{X}^{(n)} = \boldsymbol{H}\boldsymbol{\beta} + \boldsymbol{w}. \quad (15)$$

where, $n = N - M, \cdots, N$ and

$$\boldsymbol{X}^{(n)} = \big(x_n, x_{n-1}, \cdots, x_{n-(N-M-1)}\big)^T,$$
$$\boldsymbol{Y}^{(n)} = \big(\exp[-\gamma_0 x_n^2] x_n, \exp[-\gamma_0 x_n^2] x_{n-1}, \cdots, \exp[-\gamma_0 x_n^2] x_{n-(N-M-1)}\big)^T,$$
$$\boldsymbol{H} = \big(\boldsymbol{X}^{(n-1)}, \boldsymbol{Y}^{(n-1)}, \boldsymbol{X}^{(n-2)}, \boldsymbol{Y}^{(n-2)}, \cdots, \boldsymbol{X}^{(n-i)}, \boldsymbol{Y}^{(n-i)}\big),$$
$$\boldsymbol{\beta} = \big(\phi_1, \pi_1, \phi_2, \pi_2, \cdots, \phi_i, \pi_i\big)^T,$$
$$\boldsymbol{w} = \big(w_n, w_{n-1}, \cdots, w_{M+1}\big)^T,$$

so that the normal equations for β become $X^{(n)} = H\beta$, hence β can be found from

$$\hat{\boldsymbol{\beta}} = \big(\boldsymbol{H}^T\boldsymbol{H}\big)^{-1}\boldsymbol{H}\boldsymbol{X}^{(n)}. \quad (16)$$

The model order M of the fitted model is se-lected by using the Akaike Information Cri-terion(AIC) for nonlinear time series(Ozaki & Oda, 1978)

$$\mathrm{AIC}(M) = (N - M)\log\hat{\sigma}_{2,M}^2 + 2(2M + 1), \quad (17)$$

where,

$$\hat{\sigma}_{2,M}^2 = \frac{\left(\hat{w}_N^2 + \hat{w}_N^2 + \cdots + \hat{w}_M^2\right)}{N - M} \quad (18)$$

is the least squares estimate of the residual variance of the model.

4 STATISTICAL PREDICTION OF ROLL WITH LARGE AMPLITUDE

As mentioned before, we can evaluate the stability of the dynamical system by using the characteristic roots calculated from Equation 12 and 13. However, it is impossible to un-derstand the absolute amount of roll ampli-tude based on this method, and we cannot give the accurate information concerning the future roll motion to officers. Therefore, we propose a novel procedure to solve this prob-lem. In this procedure, We firstly consider a predictive probability distribution calculated from a stochastic simulation based on the op-timum ExpAR model determined by Equation 17. Then, as the results we can estimate the upper and lower endpoint of the probability distribution, and can use them as evaluation index concerning the absolute amount of roll amplitude.

The concrete procedure is as follows:

Step 1:
Fit the ExpAR model to the data of N samples, and the model order, and estimate parameters such as the model order, the scaling factor and so on based on Equation 16 to 18.

Step 2:
Reduce the model order M from the data of N samples.

Step 3:
Prepare the $N - M$ sequence, and give the data of $N - M$ samples as the ini-tial values of their sequence. After that, perform the stochastic simulation of roll motion based on the ExpAR model deter-mined by the AIC.

Step 4:
Calculate the histogram by using the obtained realizations at [Step 1]. Estimate the predictive probability distribution of roll motion by normalization of the his-togram. In this case, the the upper and lower endpoint of the probability distri-bution are simultaneously obtained.

Step 5:
Evaluate the safety level of the roll amplitude based on the upper and lower end-point of the probability distribution. If values exist within the safety level of the roll amplitude then return to [Step 1] to consider next data set. Otherwise, in-form officers the information in which the present state is danger.

Step 6:
Return to [Step 1] to consider next data set.

5 VERIFICATION

To verify the proposed procedure, we analyzed the two kinds of data of the paramet-ric roll resonance concerning a container model ship obtained by Hashimoto *et. al.* (2005). First one is regular waves, and other one is ir-regular waves. They were measured at sam-pling interval 0.1[sec] when the ship was run-ning in head seas.

Figure 1 shows a time series of roll motion in regular waves. We analyzed 1,200 samples every 300 samples, namely every 30[sec], con-cerning this data. In this figure, we defined them as form "Data set 1" to "Data set 4". As shown this figure, the amplitude of roll motion is small in "Data set 1", and becomes gradually large after "Data set 1".

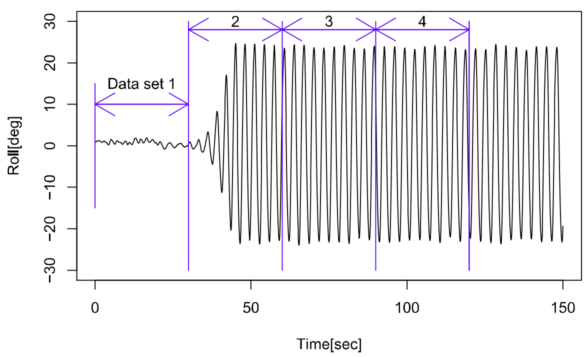

Figure 1 Time series in the case of regular waves

Results of the ExpAR modeling are summa-rized in Table 1. The probability distribution are calculated based on values shown in this table.

As to the regular waves shown in Figure 1, the predictive probability distribution of next data set calculated from optimum ExpAR model determined by AIC concerning present data set is shown in from Figure 2 to Figure 5.

Table 1 Results of the ExpAR modeling concerning the data of Fig.1

Data	1	2	3	4
Order	3	6	2	2
AIC	-4462.8	-3309.4	-2555.1	-2507.9
γ	8321.8	49.6	51.1	51.7
$\hat{\sigma}^2$	1.04×10^{-7}	4.32×10^{-6}	6.68×10^{-5}	7.82×10^{-5}
ϕ_1	2.79	2.31	1.88	1.86
π_1	0.0419	-0.0545	-0.0344	-0.0358
ϕ_2	-2.70	-1.15	-0.935	-0.933
π_2	-0.0827	0.177	0.0322	0.0332
ϕ_3	0.910	-0.809	–	–
π_3	0.0428	-0.164	–	–
ϕ_4	–	0.373	–	–
π_4	–	0.00175	–	–
ϕ_5	–	0.653	–	–
π_5	–	0.0519	–	–
ϕ_6	–	-0.395	–	–
π_6	–	-0.0110	–	–

In these figures, the horizontal axis indicates the roll angle and the vertical axis indicates the density of the probability, respectively. As you can see from Figure 2, the predictive probabil-ity distribution of ”Data set 2” calculated from optimum ExpAR model of ”Data set 1” shows that the amplitude of roll motion has danger of growing in the future, since the density of the probability exists in the range of from -30[deg] to 30[deg]. In actual, the amplitude of roll motion is growing in ”Data set 2”. There-fore, it is considered that it is possible to predict the absolute amount of roll amplitude based on this method. As to the results of from Figure 3 to Figure 5, the predictive probability distribu-tion is nearly normal distribution, since time series is stationary, although their amplitude are large.

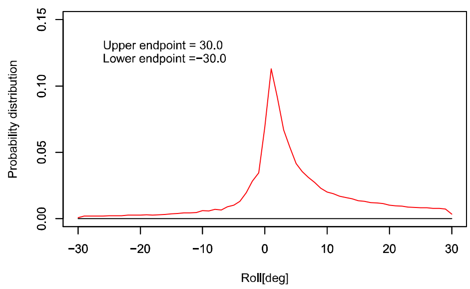

Figure 2 Probability distribution in Data 2 predicted by using ”Data set 1” of Fig.1

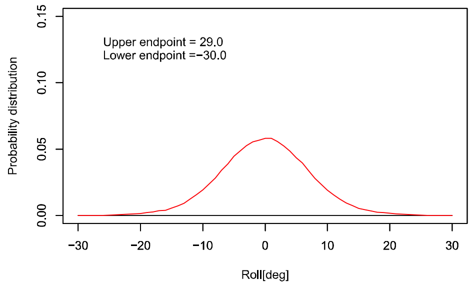

Figure 3 Probability distribution in Data 3 predicted by using ”Data set 2” of Fig.1

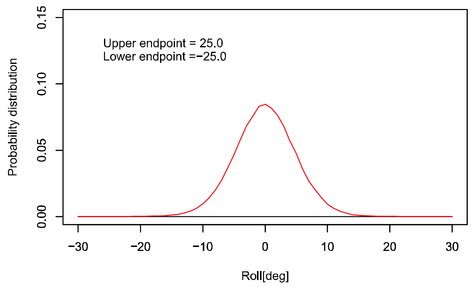

Figure 4 Probability distribution in Data 4 predicted by using ”Data set 3” of Fig.1

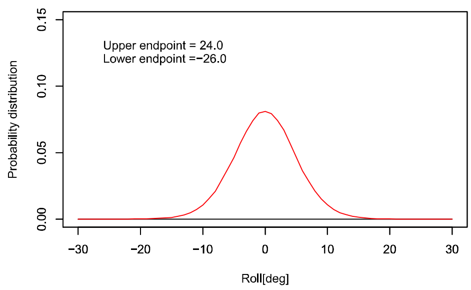

Figure 5 Probability distribution in after Data 5 predicted by using ”Data set 4” of Fig.1

Figure 6 shows a time series of roll motion in irregular waves. We analyzed 1,500 samples every 300 samples, namely every 30[sec], con-cerning this data. In this figure, we defined them as form ”Data set 1” to ”Data set 5”. As shown this figure, the amplitude of roll motion becomes large in after ”Data set 5”.

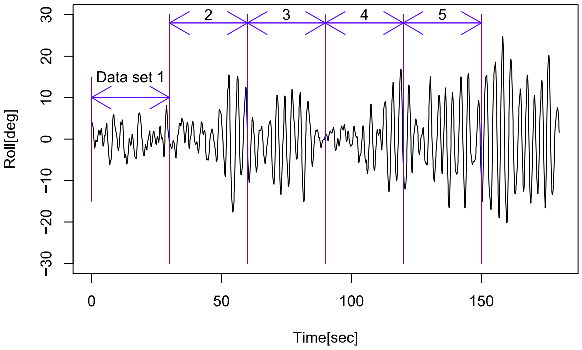

Figure 6 Time series in the case of irregular waves

Results of the ExpAR modeling are summa-rized in Table 2. The probability distribution are calculated based on values shown in this table.

Table 2 Results of the ExpAR modeling con-cerning the data of Fig.6

Data	1	2	3	4	5
Order	4	4	6	4	5
AIC	-3306.1	-3389.7	-3335.4	-3255.9	-3172.385
γ	458.7	97.7	132.1	106.4	109.5
$\hat{\sigma}^2$	4.85×10^{-6}	3.66×10^{-6}	3.96×10^{-6}	5.75×10^{-6}	7.25×10^{-6}
ϕ_1	3.02	3.21	2.97	3.06	2.69
π_1	0.0162	-0.0361	-0.00426	0.0373	-0.0273
ϕ_2	-3.61	-4.09	-3.14	-3.66	-2.16
π_2	-0.00978	0.126	-0.0263	-0.0749	0.125
ϕ_3	2.06	2.48	0.824	2.05	-0.391
π_3	-0.0184	-0.150	0.128	0.0258	-0.219
ϕ_4	-0.487	-0.617	1.02	-0.468	1.41
π_4	0.0125	0.0607	-0.181	0.0148	0.178
ϕ_5	–	–	-0.918	–	-0.574
π_5	–	–	0.105	–	-0.0567
ϕ_6	–	–	0.222	–	–
π_6	–	–	-0.0207	–	–

As to the irregular waves shown in Fig-ure 6, the predictive probability distribution of next data set calculated from optimum ExpAR model determined by AIC concerning present data set is shown in from Figure 7 to Figure 11. In these figures, as well as the case of regular waves, the horizontal axis indicates the roll an-gle and the vertical axis indicates the density of the probability, respectively. As you can see from Figure 11, the predictive probability dis-tribution in after "Data set 5" calculated from optimum ExpAR model of "Data set 5" shows that the amplitude of roll motion has danger of growing in the future, since the density of the probability exists in the range of from -30[deg] to 30[deg]. It means that it is possible to predict the absolute amount of roll amplitude based on this method even the case of irregular waves. As to the results of from Figure 7 to Figure 10, the upper and lower endpoint of the probabil-ity distribution is from about -20[deg] to about 20[deg]. Thus, we can judge that this experi-mental condition is overall dangerous. For of-ficers, this information is very important from the view point to remain safe navigation.

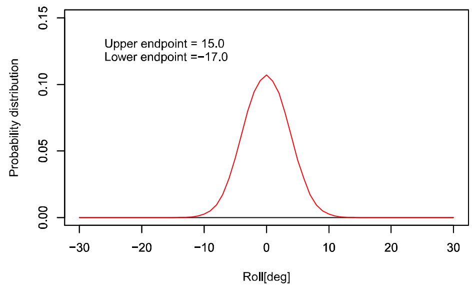

Figure 7 Probability distribution in Data 2 pre-dicted by using "Data set 1" of Fig.6

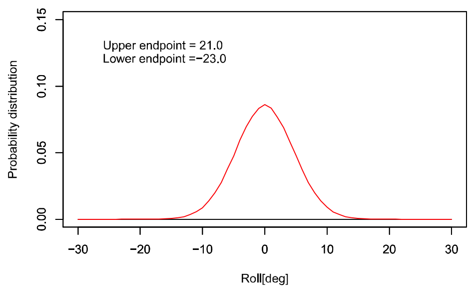

Figure 8 Probability distribution in Data 3 pre-dicted by using "Data set 2" of Fig.6

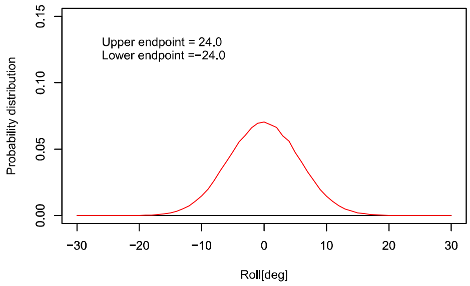

Figure 9 Probability distribution in Data 4 predicted by using "Data set 3" of Fig.6

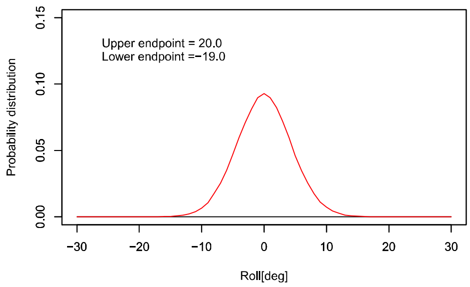

Figure 10 Probability distribution in Data 5 predicted by using "Data set 4" of Fig.6

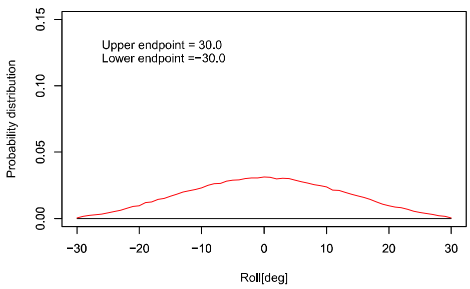

Figure 11 Probability distribution in after Data 5 predicted by using "Data set 5" of Fig.6

6 CONCLUSIONS

In this study, we attempt to establish a novel statistical prediction method, which uses an upper and lower endpoint of a predic-tive probability distribution calculated from a stochastic simulation based on the exponential autoregressive (ExpAR) model, in order to give the significant information concerning the roll motion to officers. To confirm the effectiveness of the proposed method, we analyzed the data of the parametric roll resonance. Main conclu-sions are summarized as follows:

1. As to the regular waves, it is roughly possible to predict the absolute amount of roll amplitude based on this method.

2. It is roughly possible to predict the absolute amount of roll amplitude based on this method even the case of irregular waves.

As mentioned before, it is very important for officer to obtain the predictive information concerning the parametric roll resonance from the view point to remain safe navigation, and we confirmed that the proposed procedure is possible to realize it. Therefore, we consider that the proposed procedure is practical use-fulness, and can be used as a powerful tool to remain safe navigation.

Note that we need to verify the proposed procedure more concerning many kind of ships as future task.

7 ACKNOWLEDMENTS

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