



# Prediction of Parametric Rolling of Ships in Single Frequency Regular and Group Waves

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## ABSTRACT

In this paper, a 3D nonlinear time domain numerical simulation method, which is based on the impulse response function concept, is applied to the investigation of parametric rolling of the ITTC-A1 ship. During the simulation, the hydrodynamic coefficients are determined by 3D panel code on the basis of linear potential theory, whereas several nonlinear terms are considered in the equations of motion, such as the excitation by large amplitude waves, the exact restoring forces and moments resulting from the actual wetting of the ship hull geometry and the semi-empirical nonlinear viscous damping. In addition, nonlinear inertia terms are retained when considering motions of large angles. The parametric rolling of the ship is predicted by simulating 6 degree of freedom (DoF) nonlinear motions in response to single frequency regular waves and triple frequency group waves. The obtained numerical results are compared with corresponding experimental measurements and good agreement has been observed.

**Keywords:** *numerical simulation, parametric resonance, roll motion, non-linear dynamics, ship safety*

## 1. INTRODUCTION

Parametric rolling is the induced roll motion of a ship due to the periodic change of the restoring characteristics as the ship advances in waves. This phenomenon is often observed on ships with excessive bow flares and very flat sterns, such as modern container ships, car carriers etc. Typically it takes place when the wave frequency of encounter is close to twice of the natural roll frequency of the ship and near the heave/pitch resonance frequency. Under such condition, the occurrence of the parametric roll phenomenon, which is a strongly nonlinear oscillatory motion phenomenon, is actually subject to the incident wave amplitude, the ship's loading condition, ship's speed and roll damping feature.

The prediction of parametric rolling has high practical value, as it can lead to not only the loss of cargo, but also to the loss of the ship, thus it is an important safety issue already considered in IMO regulations (Peters et al, 2011). It is, also, a popular research subject, because of the complexity of the associated nonlinear ship dynamics and hydrodynamic phenomena; thus the correct prediction of parametric rolling, in terms of likelihood of *occurrence and resulting roll amplitude*, in regular waves and irregular seas remains a challenge to state-of-the-art numerical simulation methods and software tools.

The investigation of parametric rolling by numerical and experimental methods has a long history, dating back to the 1930s (see Paulling, 2006 for historical review). The phenomenon attracted special interest only in the last few decades with the serious accidents on large



ships, for instance, the containership APL CHINA casualty in 1998 (France et al, 2003).

Several approaches are being employed to analyze and understand the parametric roll phenomenon, ranging from the uncoupled, one degree of freedom non-linear roll equation, adjusted with appropriate parameters, for instance, Paulling (1961), Francescutto (2002), Umeda et al. (2003), to models of multi degrees of freedom, where the roll motion and ship hydrodynamics are appropriately coupled with the other degrees of freedom, Ribeiro et al. (2005), Neves, (2005), Krueger (2006), Spanos and Papanikolaou (2006). Parametric rolling has been investigated for both regular and irregular seaways (Belenky, 2003), as well as for following and head seas conditions. A thorough review of the related literature has been carried out by ITTC (2005).

In this paper, a nonlinear time domain method based on the impulse response function concept (Liu et al., 2014), is applied to the simulation of parametric rolling. This method has been developed (independently of earlier work by Spanos and Papanikolaou, 2006) at the Ship Design Laboratory of National Technical University of Athens in the frame of NTUA-SDL's HYBRID software system, aiming to facilitate the analysis of the seakeeping performance and safety of ships in complex environmental and/or adverse sea conditions. In the framework of potential theory, the wave excitation is decomposed into Froude-Krylov, radiation and diffraction forces. Incident wave forces (both hydrodynamic and hydrostatic parts) are calculated through direct integration of the corresponding pressures over the instantaneous wetted surface, which is defined by the undisturbed incident wave and the instant position of the ship. The radiation forces are calculated using the added mass and damping coefficients calculated by a 3D frequency domain code NEWDRIFT (Papanikolaou et al. 1985, 1990) and transformed in the time domain by application of the impulse response function concept.

Diffraction forces are obtained in a similar manner, using corresponding results obtained by NEWDRIFT. Solving the six coupled nonlinear integro-differential equations of motion by a time integration method, the six DOF motions of the ship are obtained in the time domain.

## 2. MATHEMATICAL MODEL

In order to study the nonlinear ship motion problem, three coordinate systems are defined: the earth-fixed  $OXYZ$  system, a system  $O'X'Y'Z'$  travelling with the mean ship speed, always parallel to  $OXYZ$  and a body-fixed  $Gxyz$  system, with its origin  $G$  at the center of gravity. It is assumed that at  $t=0$  both  $O$  and  $O'$  coincide with  $G$ . The two coordinate systems,  $O'X'Y'Z'$  and  $Gxyz$  are connected by the three Euler angles:  $\theta$  (roll),  $\psi$  (pitch), and  $\phi$  (yaw). If  $O'X'Y'Z'$  is rotated by the three Euler angles, it becomes parallel with  $Gxyz$ . The order of rotation is  $\theta$ ,  $\psi$ , and  $\phi$ . A vector  $\bar{x}$  in the  $Gxyz$  system may be expressed as  $\bar{x}'$  in  $O'X'Y'Z'$  system as follows:

$$\bar{x}' = \mathbf{T}\bar{x} \quad (1)$$

where  $\mathbf{T}$  is the transformation matrix:

$$\mathbf{T} = \begin{bmatrix} \cos \psi \cos \phi & \sin \theta \sin \psi \cos \phi - \cos \theta \sin \psi \cos \phi + & -\cos \theta \sin \psi \cos \phi + \\ & -\cos \theta \sin \phi & +\sin \theta \sin \phi \\ \cos \psi \sin \phi & \sin \theta \sin \psi \sin \phi + \cos \theta \cos \phi & \cos \theta \sin \psi \sin \phi - \\ & +\cos \theta \cos \phi & -\sin \theta \sin \phi \\ -\sin \psi & \sin \theta \cos \psi & \cos \theta \cos \psi \end{bmatrix} \quad (2)$$

The ship is assumed travelling on the free-surface with a mean speed  $\vec{v}_{O'} = [U, 0, 0]^T$  parallel to the  $OX$  axis, subject to incident regular waves. The location of the ship in the  $OXYZ$  system is expressed by the location of the center of gravity ( $G$ ) and the three Euler angles. The location of the center of gravity is defined by  $\bar{x}_G(t) = [X_G(t), Y_G(t), Z_G(t)]^T$  and it's velocity  $\vec{v}_G(t)$  by the time derivative of  $\bar{x}_G(t)$ . The relationship between the absolute velocity of the ship and the relative velocity (both



expressed in the earth-fixed coordinate system) is:

$$\vec{V}_G^R = \vec{V}_G - \vec{V}_{O'} = \vec{V}_G - [U, 0, 0]^T \quad (3)$$

The angular velocities about the ship-fixed coordinate axes given by  $\vec{\omega}$  are related to the time derivatives of the Euler angles as follows:

$$\vec{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\psi \\ 0 & \cos\theta & \sin\theta\cos\psi \\ 0 & -\sin\theta & \cos\theta\cos\psi \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{\phi} \end{bmatrix} = \mathbf{B} \begin{bmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{\phi} \end{bmatrix} \quad (4)$$

where:

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & -\sin\psi \\ 0 & \cos\theta & \sin\theta\cos\psi \\ 0 & -\sin\theta & \cos\theta\cos\psi \end{bmatrix} \quad (5)$$

Let  $\vec{a}_G$  be the (total) acceleration vector of the center of gravity  $G$ , expressed in the body-fixed system;  $\vec{a}_G$  may be expressed as follows:

$$\vec{a}_G = \dot{\vec{v}}_G + \vec{\omega} \times \vec{v}_G \quad (6)$$

The first term in the above equation corresponds to the rate of change of the translational velocity of the ship, while the second one takes into account the effect of rotation of the body-fixed coordinate system.

The equations of motion are given by application of Newton's second law:

$$m(\dot{\vec{v}}_G + \vec{\omega} \times \vec{v}_G) = \vec{F} \quad (7)$$

$$\mathbf{I}\dot{\vec{\omega}} + \vec{\omega} \times \mathbf{I}\vec{\omega} = \vec{M} \quad (8)$$

In the above equations, the external forces and moments are expressed in the body-fixed system of coordinates and they consist of the gravitational, radiation, diffraction, incident wave force, restoring forces and possible viscous terms, while  $\mathbf{I}$  is the moment of inertial matrix of the ship.

## 2.1 Diffraction forces

For weakly nonlinear motions, assuming the ship in the upright/mean position when calculating the diffraction forces due to the incoming waves is a reasonable approach. However, as the motions increase and particularly when the dimensions of the ship are small compared to the wave length (e.g. the case of a small boat in large waves), the effect of the oscillatory ship motions on the diffraction forces increases, and their calculation assuming the ship at its instantaneous position may be considered. On the other hand, in this latter case, diffraction effects will tend to zero, due to the small disturbance to the incoming waves, caused by the presence of the ship.

In the current study, diffraction forces and moments are calculated assuming the ship in the upright/mean position using the velocity potential results obtained by NEWDRIFT. This code is based on a 3D panel method for the evaluation of the responses (ship motions, structural loads and drift forces) of arbitrarily shaped marine structures and shiplike bodies subject to the excitation of incident regular waves; it has been widely benchmarked over the last 35 years and applied to a variety of problems and ship types by marine industry professionals and university researchers. NEWDRIFT was initially developed for the zero speed case (Papanikolaou, 1985), based on the distribution of zero-speed pulsating Green sources over the mean wetted body surface to express the radiation and diffraction potentials and was subsequently extended to the case of shiplike bodies advancing with forward speed in waves (Papanikolaou et al. 1990).

## 2.2 Incident wave forces

The Froude-Krylov and restoring forces/moments are calculated by integrating the pressure over the *instantaneous* wetted surface of the ship. The incident wave pressure



is herein defined by the sum of linear dynamic pressure and hydrostatic pressure:

$$p = -\rho \frac{\partial \Phi_I}{\partial t} - \rho g Z \quad (9)$$

According to the linear wave theory, the dynamic pressure is assumed constant from the mean free surface to the actual free surface, whereas the hydrostatic component increases linearly from zero at the actual water surface and is being added to the dynamic component.

### 2.3 Radiation forces

Following Cummins (1962), the radiation forces and moments in the body-fixed coordinate system are evaluated by:

$$F_i(t) = -\sum_{j=1}^6 \left[ A_{ij} \dot{v}_j + \int_0^t L_{ij}(t-\tau) v_j(\tau) d\tau \right] \quad i = 1 \sim 6 \quad (10)$$

where  $A_{ij}$  are the added mass coefficients,  $v_j$  stand for the velocities of the ship in 6DOF, while the kernel functions  $L_{ij}$  may be calculated from the damping coefficients  $B_{ij}$ :

$$L_{ij}(\tau) = \frac{2}{\pi} \int_0^{\infty} B_{ij}(\omega) \cos \omega \tau d\omega \quad (11)$$

The added mass and damping coefficients appearing in (10) and (11) are calculated by NEWDRIFT.

### 2.4 Time domain integration

In order to smoothly introduce the incident wave disturbance into the numerical scheme and to mitigate the effect of initial transients on the steady response to an incident regular wave, the velocity potential of the incident wave is defined as following:

$$\Phi_0 = \frac{g \zeta_a}{\omega} e^{kz} \sin[k(x \cos \beta + y \sin \beta) - \omega t] \quad (12)$$

$$\zeta = \begin{cases} \zeta_a \left[ \frac{1}{2} \left( 1 - \cos \frac{t\pi}{nT} \right) \right]^2 & t < nT \\ \zeta_a & t \geq nT \end{cases} \quad (13)$$

where  $\omega$  is the wave frequency,  $\zeta_a$  the wave amplitude,  $k$  the wave number,  $\beta$  the angle of incidence (with 180deg corresponding to head waves) and  $n$  is a pre-defined integer parameter.

During the numerical simulation of the motion of the ship, the determination of the instantaneous wetted surface, considering the ship motions as well as the actual wave elevation is required. This is done according to the following procedure:

1. Prepare a panelization for the ship, including the part of the outer shell above the waterline, up to and including the deck;
2. At each time instant, check the position of each panel in relation to the instantaneous wave surface, mark the panels that are fully wetted, partially wetted, or non-wetted;
3. The non-wetted panels are skipped. Regarding the partially wetted panels, if only one node is immersed, a new triangular panel will be formed; if 2 nodes are immersed, a new rectangular panel will be formed; if 3 nodes are immersed, the wetted area will be split into 2 panels.

## 3. THE EXPERIMENT

In order to investigate the occurrence of roll resonance and its dependence on the basic parameters of the problem, tank tests have been conducted within SAFEDOR project for the validation of numerical simulation methods in an international benchmark study, coordinated by NTUA-SDL (Spanos and Papanikolaou, 2009).

This benchmark study was based on parametric roll investigations of the



standardized containership ITTC-A1. The principal particulars and body plans of these ships are shown in Table 1 and Figure 1. The two ship loading conditions investigated are shown in Table 2, while in Table 3 the matrix of simulated tests which will be described and discussed in the following section is presented.

The ship model has been investigated with a constant forward speed in head waves. The model was free to move in 3 DoF, namely, in heave, roll and pitch. Tests were independent of any course keeping mechanism. The model was tested both in regular and irregular waves. Free decay tests were also performed in order to evaluate the damping properties of the ship model. The motion of the model, the restraining forces and moments and the wave elevation have been measured. This type of semi-captive tests can be considered “ideal” for the benchmarking of numerical simulation methods (even deviating from the actual, free to move ship performance), as the uncertainties related to the speed and course keeping are suppressed. The experimental results as well as the numerical results of the benchmark study have been extensively reported by Spanos D. (2009) within SAFEDOR framework.

Table 1 Main Particulars of ITTC-A1 ship

<i>Items</i>	<i>Ship</i>
length : $L_{pp}$	150.0 m
breadth : $B$	27.2 m
depth : $D$	13.5 m
draught at FP : $T_f$	8.5 m
mean draught : $T$	8.5 m
draught at AP : $T_a$	8.5 m
block coefficient : $C_b$	0.667
prismatic coefficient : $C_p$	0.678
water plane coefficient : $C_w$	0.787
wetted surface area : $S$	5065 m <sup>2</sup>

Table 2. Loading conditions tested in benchmark study

Tests 01 ~ 11	GM = 1.38 m
	$i_{xx}$ = 10.33 m
	$i_{yy}$ = 37.5 m
Tests 12 ~ 22	GM = 1.00 m
	$i_{xx}$ = 10.33 m
	$i_{yy}$ = 38.2 m

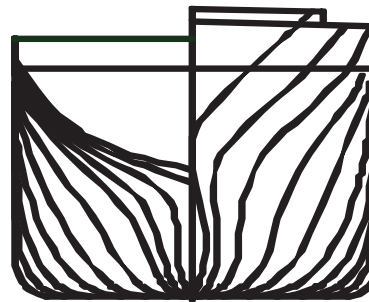


Figure 1. Body plan of ITTC-A1 Ship

Table 3. Matrix of simulated tests from SAFEDOR benchmark study

TEST	GM (m)	Heading (deg)	Fn	H (m)	T (sec)	Comment
T01	1.38	180	0.00	-	-	Roll decay
T02	»	»	0.08	3.6	10.63	Regular
T03	»	»	»	5.7	»	»
T04	»	»	0.12	3.6	»	»
T05	»	»	»	5.7	»	»
T06	»	»	»	2.4 2.4 2.4	10.63 9.66 11.55	Group
T07	»	»	»	4.0 1.0 1.0	10.63 9.66 11.55	»
T09	»	160	»	3.6	»	Regular
T10	»	»	»	5.7	»	»
T11	»	»	»	4.0 1.0 1.0	10.63 9.66 11.55	Group
T20	1.00	180	0.08	5.0	12.12	Regular
T21	»	»	0.12	5.0	»	»
T22	»	»	0.08	4.0 1.0 1.0	12.12 10.77 13.47	Group



#### 4. NUMERICAL RESULTS AND DISCUSSION

In this section we apply the 6 DoF model described, outlined in section 2, to the simulation of parametric rolling phenomenon.

Before proceeding to the parametric rolling simulation, it is necessary to calibrate the roll damping and gyration properties of the model according to the decay test results. Figure 2 shows the tuned decay simulation for Test 01. Using this result, the actual roll viscous damping may be adjusted, as well as the radius of gyration.

For simulating the parametric roll phenomenon in a longitudinal, symmetric wave load condition, we need to introduce a small roll disturbance (by a small initial roll displacement of 1-2 degrees), upon the excitation of which the ship will start rolling with a continuously decreasing roll amplitude, unless the condition for the occurrence of a parametric roll instability is met. This initial disturbance corresponds in practice to the excitation by a sudden side wind or other acting transverse force, while the ship is sailing in exactly head or following sea condition.

When analysing the roll amplitude time records, the mean roll amplitude of the stationary response is used for the quantification of the performance of the numerical method in terms of the predictability of the roll motion magnitude, which is the mean value of successive amplitudes.

Figure 3 shows the overall results for the mean roll amplitude from the present method against the tank tests. In general, numerical predictions of the roll amplitude are consistently higher than experimentally measured amplitudes. Examining the testing conditions, it is noted that going from Test 02 to Test 03, or from Test 04 to Test 05, or from Test 09 to Test 10, the only difference has been the *increase of wave amplitude*, while the wave frequency remained the same. The results for

such an *increase of incident wave amplitude* were the *decrease of resonance amplitude*, both in the experimental and numerical simulation. *This is a clear indication of a rare nonlinear motion phenomenon, for which an increase of the excitation amplitude leads to a decrease of the response amplitude* (or even to zero response, see Spanos D. and Papanikolaou A. 2009b). Commenting on conducted Test 20 and Test 21, the only difference in testing conditions is the increase of speed from  $F_n=0.08$  to  $F_n=0.12$ ; this change has induced the disappearance of the roll resonance, as could be also numerically predicted.

Figure 4-11 show the time histories and tank results of several test cases. It is observed that for regular waves, after the initial transient time (depending on the numerical set-up), the ship response is fully developed and a stationary rolling behaviour (constant amplitude) is achieved. Another observation is that the period of roll resonance is twice that of the encounter wave period, which shows the essentially different mechanism of the parametric rolling from a normal (prime resonance) rolling motion excited by incident waves.

At last in Figure 12 the mean roll amplitude results based on the present method are plotted against the simulations of the four best performing methods from the SAFEDOR benchmark study together with the experimental data (full diamond symbols). Interestingly the present method correctly simulated the outcome of Test 21, where no parametric rolling has been observed in tank test, but even the four best performers of SAFEDOR benchmark participants failed to properly simulate it. Comparing the conditions for Test 20 and Test 21, which only differ slightly with respect to ship's speed of advance, but are associated with the initiation of parametric rolling or not, it proves that the sensitivity of prediction methods with respect to the ensuing parameters is very crucial for the reliable prediction if the parametric roll phenomenon.

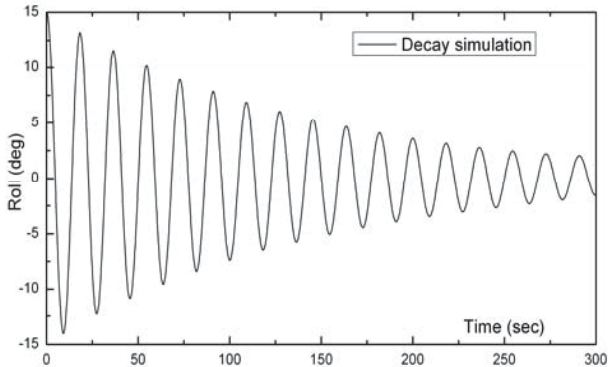


Figure 2. Roll decay simulation GM=1.38m

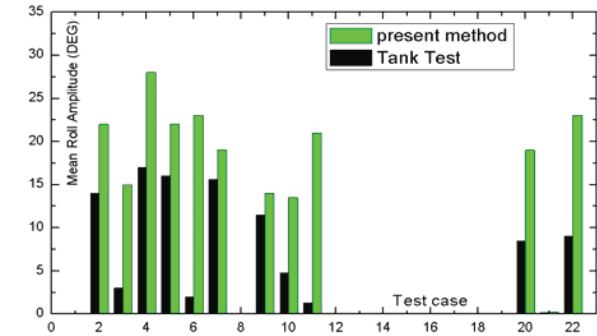


Figure 3. Predicted mean roll amplitude by present method against the experimental results

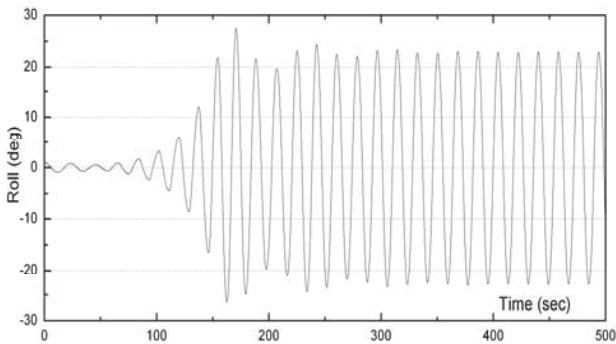


Figure 4. Simulated roll angle history, Test 02

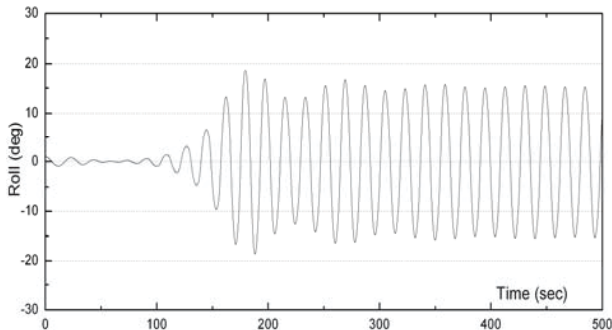


Figure 5. Simulated roll angle history, Test 03

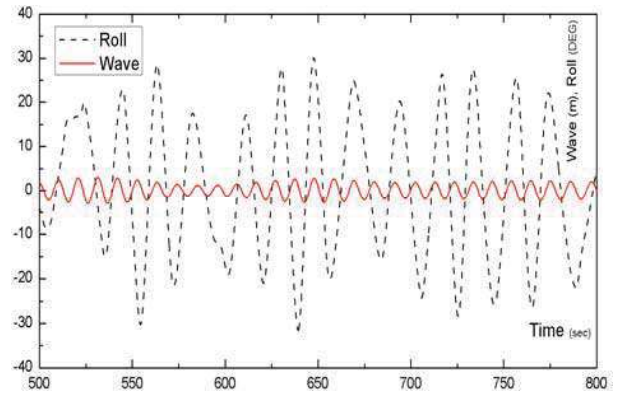


Figure 6. Time history of roll angle simulation in test case T07, head sea, group wave

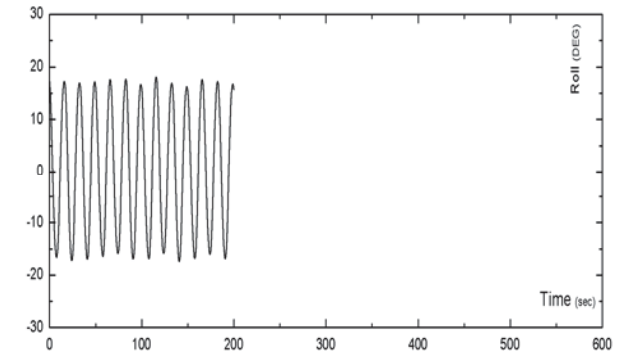
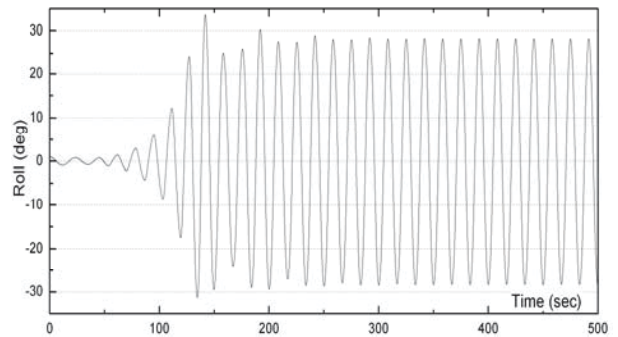
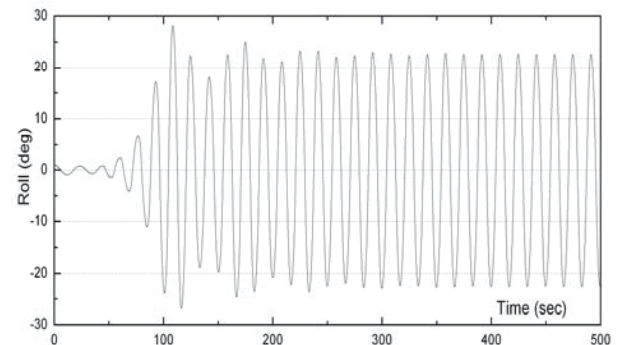


Figure 7. Time history of roll angle of Test 04, simulation (above) vs experiment (below)



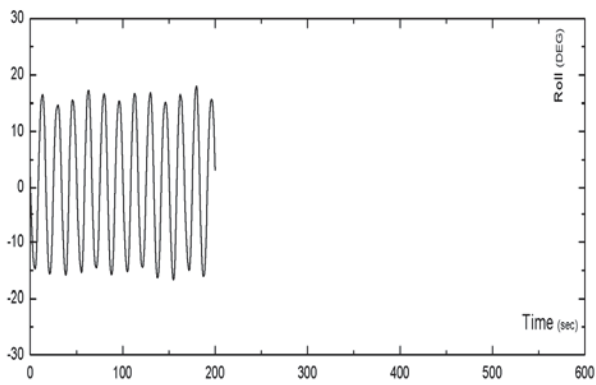


Figure 8. Time history of roll angle of Test 05, simulation (above) vs experiment (below)

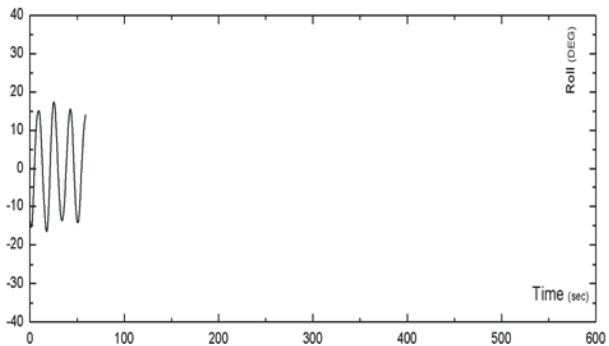
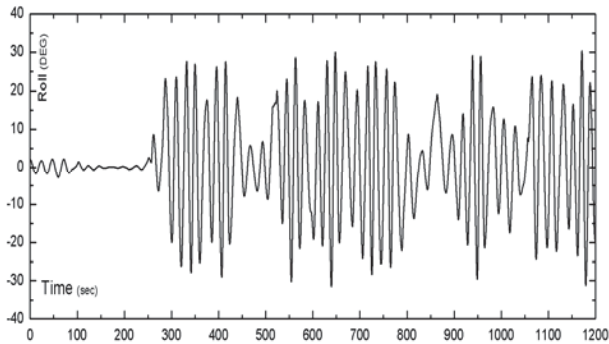


Figure 9. Time history of roll angle of Test 07, simulation (above) vs experiment (below)

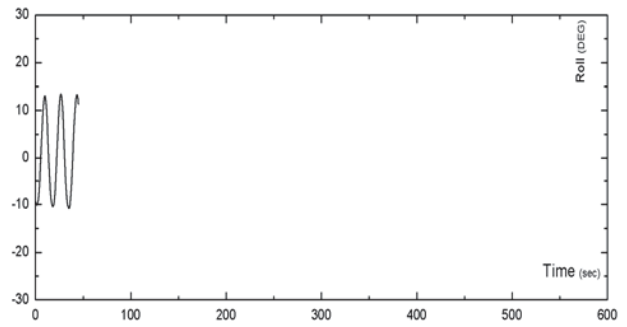
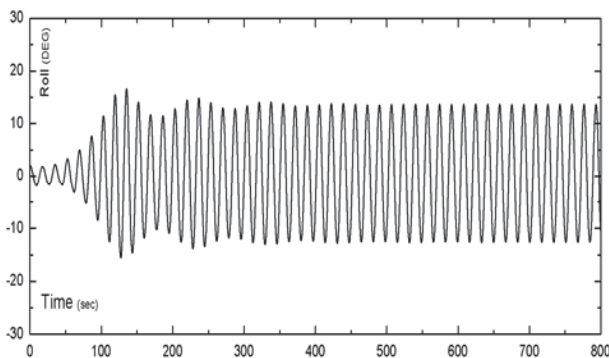


Figure 10. Time history of roll angle of Test 09, simulation (above) vs experiment (below)

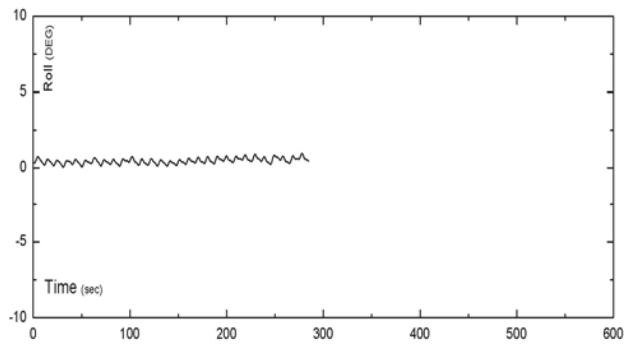
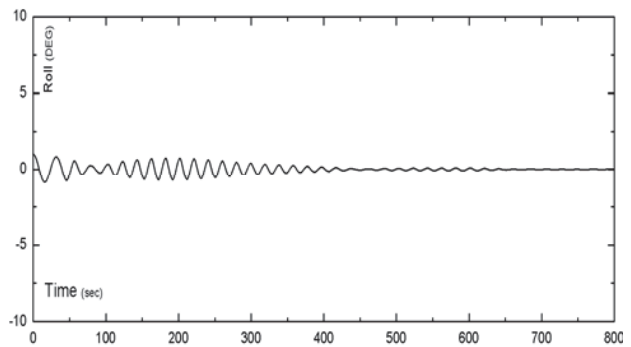


Figure 11. Time history of roll angle of Test 21, simulation (above) vs experiment (below)

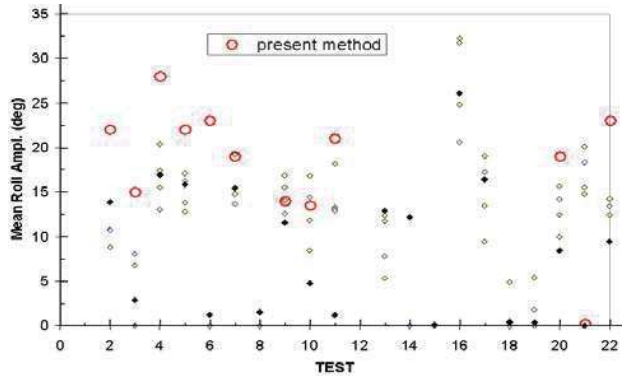


Figure 12 Mean roll amplitude as estimated from current method and the best performing





(four) methods of SAFEDOR benchmark, as well as experimental results

## 5. CONCLUSIONS

The prediction of parametric roll resonance is inherently a difficult subject, as revealed by the large scattering of numerical results presented in the SAFEDOR benchmark study. Hence it is a good test case for any under development numerical method and software tool to test its performance in such conditions. The quality of the numerical results should be judged by two criteria, as suggested by the SAFEDOR benchmark study:

1. Is the occurrence of resonance correctly captured?
2. Is the amplitude correctly predicted?

Based on these two criteria, we may draw a preliminary conclusion regarding the performance of the herein presented method.

1. The current method has been successfully applied to the prediction of parametric rolling phenomenon of ships in regular waves and triple-frequency group waves. For the 8 tested regular wave cases and 4 tested triple-frequency group wave cases, parametric rolling phenomenon has been correctly predicted in all tested cases (100%).
2. The importance of proper prediction in triple frequency wave group excitation is highlighted, noting that the predictability of parametric roll in natural irregular seas is inherently related to the occurrence of dominating wave groups within the multi-frequency wave ensemble.
3. The amplitude of roll resonance has been in general over-predicted. This is due to the employed linear viscous roll damping model. Despite this, the fact is,

as shown in Test Cases 2 3, Test Case 4 and 5, that the gradient of predicted amplitude change against the incident wave amplitude, has been correctly captured.

3. The correction of the potential theory linear damping by an equivalent viscous damping that may be deduced from roll decay tests, appears to have little effect on the correct prediction of the parametric rolling amplitude. Hence, it is advisable to test higher order roll damping models and to consider in addition the enhanced damping in other modes of motion (heave and pitch), before concluding.

## 6. ACKNOWLEDGMENTS

The work presented in this paper is supported by the Collaborative Project (Grant Agreement number 605221) SHOPERA (Energy Efficient Safe **SH**ip **OP**ERation) co-funded by the Research DG of the European Commission within the RTD activities of the FP7 Thematic Priority Transport / FP7-SST-2013-RTD-1/ Activity 7.2.4 Improving Safety and Security / SST.2013.4-1: Ships in operation. The European Community and the authors shall not in any way be liable or responsible for the use of any knowledge, information or data of the present paper, or of the consequences thereof. The views expressed in this paper are those of the authors and do not necessary reflect the views and policies of the European Community.

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