



Toward a Split-Time Method for Estimation of Probability of Surf-Riding in Irregular Seas

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ABSTRACT

The paper reviews the status of a multi-year research effort for using the split-time method to calculate the probability of ship capsizing due to broaching-to in irregular waves. The present work focuses on extending the existing theory of surf-riding and broaching-to from regular waves to irregular waves and applying it to numerical simulation codes. Extending the theory for irregular waves leads to the formulation of a spatial-temporal framework for considering surf-riding where the celerity of irregular waves must be defined. An approximate metric for the likelihood of surf-riding in irregular waves has been proposed as the distance, in the phase plane, between the instantaneous position of a ship and the stable surf-riding equilibrium at that instant. Further work includes studying the properties of the surf-riding phase plane in irregular waves and statistics of surf-riding occurrences.

Keywords: *Probability, Surf-Riding, Celerity, Split-Time Method*

1. INTRODUCTION

Surf-riding and broaching-to are associated, complex phenomena linked through the generation of attraction towards a point of equilibrium located near a wave trough, appearing to be stable in surge but unstable in yaw. This yaw instability leads to a rapid, uncontrollable turn which can induce a large roll angle or capsize. The standing theory of surf-riding and broaching-to was proposed on the basis of the nonlinear dynamics theory (Spyrou 1996, 1997). It used an ordinary differential equations (ODE) model of ship motion in regular following/quartering waves. The minimal mathematical model for describing these phenomena includes surge-sway-yaw for modeling broaching-to plus a roll motion equation to model capsize. An auto-pilot equation also must be included for the directional control of the ship. The resulting system has a 10-dimensional phase space.

This already complex description becomes even more complex if one considers the fact that large-amplitude ship motions are described

by integro-differential equations, where wave excitation cannot be separated from stiffness and hydrodynamic memory effects, such as radiation and diffraction forces, are present. Furthermore, the irregularity of realistic ocean introduces new physical qualities to the phenomenon.

The challenge to include surf-riding and broaching into a probabilistic assessment of stability based on advanced hydrodynamic codes has been taken up by the US Office of Naval Research (ONR) project "A Probabilistic Procedure for Evaluating the Dynamic Stability and Capsizing of Naval Vessels". The project's main objective is to create a robust theory of probabilistic capsizing in irregular waves and a numerical procedure based on this theory.

The split-time method is the probabilistic framework of this project. The idea of the split-time method is to separate the complex problem of the probability of a rare, extreme event in two problems that may be less complex. The "non-rare" problem is associated with an intermediate random event, statistics of which can be obtained by running an advanced code for sufficient time. This "non-rare"



problem ensures a correct relation between time and probability in irregular waves. The “rare” problem is responsible for modeling the physics of the rare event. Its core is a metric of the danger of the severe event, for example capsizing or large roll angle caused by broaching, that is computed when the intermediate random event occurs. The procedure is repeated for each intermediate random event observed during the non-rare simulations. The sample of metric values is then statistically extrapolated to determine the likelihood of the rare event. Since the metric value is expected to include the physics of the extreme event, actual observations of the event are not required, so the method is expected to work for conditions (sea states, speed, loading) where extreme events are rare.

2. SURF-RIDING AND BROACHING-TO IN LAMP

2.1 Objective and Approach

As the current theory of surf-riding and broaching-to was developed using ODE, application to more general models raises certain questions. Can this theory be extended to the cases when ship motions are presented with integro-differential equations with 6 degrees of freedom and hydrodynamic memory effects? How well will the analysis tools of nonlinear dynamics work with the dynamical system represented by the advanced hydrodynamic code? The results of a study of these issues are presented in Spyrou *et al.* (2009) and Belenky *et al.* (2010).

While the Large-Amplitude Motion program (LAMP) was used as the main hydrodynamic code for this study, the results are meant to be code-independent and applicable to any code capable of reproducing surf-riding, broaching-to, and capsizing. LAMP is based on a boundary value problem for radiation and diffraction (potential flow), a body non-linear evaluation of the hydrostatic

and Froude-Krylov forces, and force models for vortex and viscous effects (Lin and Yue 1990). The use of LAMP for the simulation of ship maneuvering is described in Lin *et al.* (2006) and Yen *et al.* (2010).

The ship configuration used for demonstrating the surf-riding and broaching phenomena is the “tumblehome” form of the ONR Topsides Study (Bishop *et al.* 2005).

2.2 Time-Domain Simulations

The study first looked at the modeling of large heel angle (up to capsizing) caused by a sharp turn. Figure 1 shows predicted time histories of roll with two values of the transverse metacentric height (GM) after a 30° rudder application while sailing at 38 knots in calm water. At the higher GM value, the ship attains a large heel angle before recovering. At the lower GM value, the ship capsizes.

The study then looked at surf-riding in combinations of wave frequency and height and ship speed for which the theory predicts a co-existence of surging and surf-riding. Figure 2 shows different responses depending on the initial conditions. Systematic simulations were performed for a nominal Froude number (Fn) from 0.3 to 0.41 and commanded headings relative to the wave direction from 10 to 32 degrees. A summary of the results is presented in Figure 3.

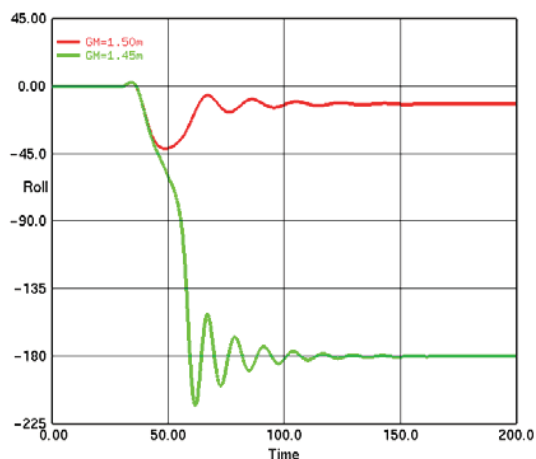


Figure 1: Large heel angle and capsizing due to sharp turn in calm water (Belenky *et al.* 2010)

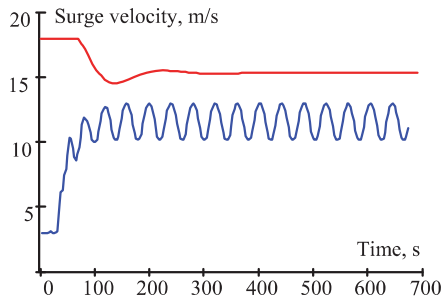


Figure 2: LAMP simulations showing coexistence of surging and surf-riding (Belenky *et al.* 2010)

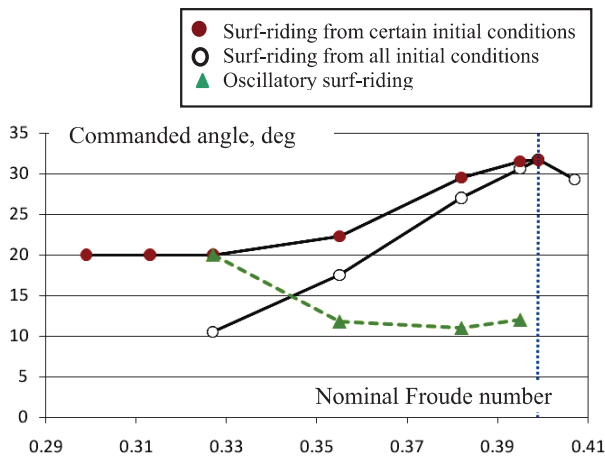


Figure 3: Boundaries of surf-riding in terms of commanded heading as a function of nominal Froude number (Spyrou *et al.* 2009)

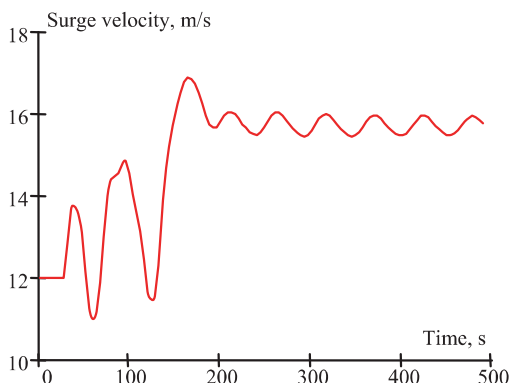


Figure 4: Capture into oscillatory surf-riding for commanded relative heading of 12 deg, $F_n=0.36$; notice the upward jump of mean speed (Spyrou *et al.* 2009)

A remarkable feature that appeared at the higher nominal speeds ($F_n > 0.33$) was a stable

oscillatory type of surf-riding, for which the time history of ship speed is shown in Figure 4. As the ship is carried along by a single wave, it is also oscillating up-and-down the wave face. This fascinating occurrence has been observed in the past and has been explained as being due to a Hopf bifurcation (Spyrou 1996).

2.3 The Continuation Method

The continuation method is a powerful approach for studying the behavior of nonlinear systems by mapping and characterizing equilibria. However, continuation techniques were developed for dynamical systems described by ODE, and the application of continuation with advanced hydrodynamic codes presents many challenges, mostly caused by hydrodynamic hysteresis (memory effect). The first use of the continuation method (DERPAR) with a potential flow code (LAMP) was described by Spyrou *et al.* (2009). To avoid excessive complexity associated with the memory effect, the diffraction and radiation forces have been approximated with constant added mass and damping coefficient.

A sample result from LAMP-based continuation analysis is given in Figure 5. The curve shows the positions of surf-riding equilibria in the coordinates as yaw vs. rudder angle, and follows the similar curve from Spyrou (1996). This curve contains both stable and unstable surf-riding equilibria. That is why the continuation method is necessary, as direct time-domain simulations are not capable of capturing unstable equilibria.

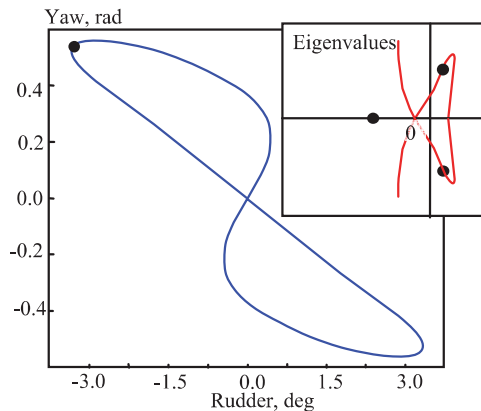


Figure 5: Curve of yaw vs. rudder angle for surf-riding equilibria calculation with DERP and LAMP (Belenky *et al.* 2010)

The stability of regions of the equilibrium curve was analyzed by examining the eigenvalues. The inset plot shows eigenvalues in the complex plane for a point on the curve at which the Hopf bifurcation (Figure 4) can be expected. Further development has led to the inclusion of hydrodynamic memory effects into the continuation analysis (Spyrou and Tigkas 2011).

2.4 The Theory and Code

The analysis carried out by Spyrou *et al.* (2009) demonstrated that the LAMP-simulated behavior is consistent with the current theory of broaching-to and surf-riding. It also confirmed the generic nature of the phenomena that had been identified independently and for a very different configuration in earlier research. In particular, the consideration of all six degrees does not change the qualitative picture of the phenomenon.

LAMP-based continuation analysis produced results that are consistent with direct simulation of the stable equilibria as well as the results of previous continuation research of a more theoretical nature.

These results lead to the conclusion that the current theory can be used to explain the outcomes of time-domain simulation with

advanced hydrodynamic codes and that advanced hydrodynamic codes can describe a nonlinear dynamical system.

3. SPATIAL-TEMPORAL FRAMEWORK FOR SURF-RIDING

In regular waves, a sliding coordinate system with its origin on the wave crest (or any other point fixed to the wave) allows a space-only consideration for surf-riding, as the next or previous wave in space and time is exactly the same. Figure 6 illustrates this approach; the upper part (Figure 6a) shows the balance of thrust and resistance plotted with the spatial representation of the surging wave force. The intersections of these two lines correspond to surf-riding equilibria, which define the topology of the phase plane shown in the lower part (Figure 6b). As the wave is regular, shifting coordinates in space corresponds to shifting coordinates in time, so only one coordinate – usually space – has to be considered.

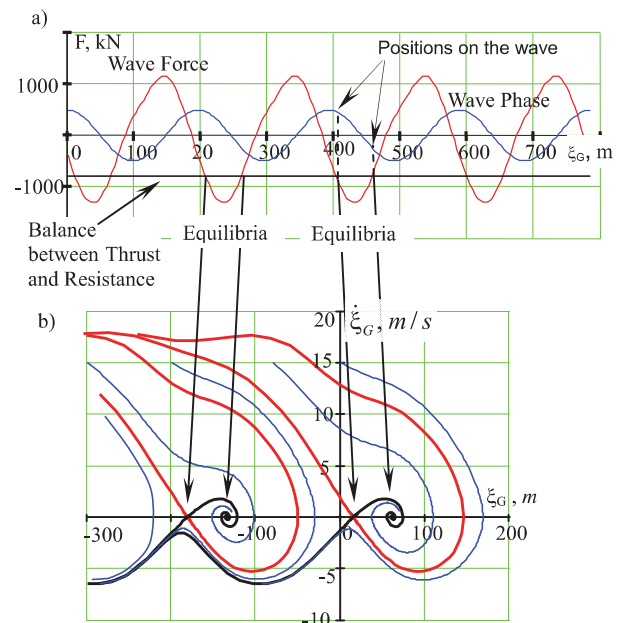


Figure 6: Surf-riding in regular waves – space consideration only: (a) balance of forces and (b) phase plane

In irregular seas, the waves are changing in both space and time. Consider an Earth-fixed coordinate system and imagine Figure 6 slowly changing in space for a fixed instant of time. The picture then changes slightly for the next time instant. As the waves change, surf-riding equilibria (actually quasi-equilibria) appear and disappear at different instances of time and points in space. This qualitative description is illustrated in Figure 7.

This rather qualitative consideration produces two main outcomes:

- Celerity of irregular waves must be defined in order to find the equilibria
- All elements of the mathematical model of surf-riding in irregular waves must be functions of position (x-coordinate) and time.

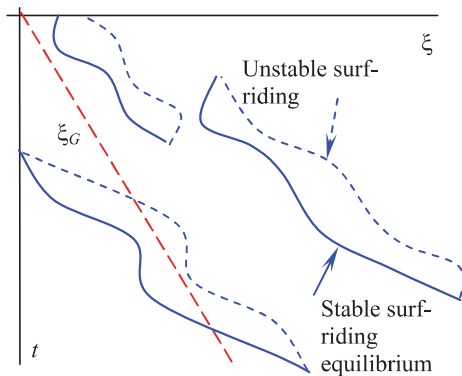


Figure 7: Spatial-temporary framework for surf-riding in irregular seas (Belenky *et al.* 2011)

4. CELERITY OF IRREGULAR WAVES

4.1 Velocity of a Profile

The most evident idea for irregular wave celerity was to find characteristic points in the profile, track them in time, and take their velocity as an approximation of wave celerity at these points. Belenky *et al.* (2012) proposed the tracking of zero-crossing points as illustrated in Figure 8. Each curve in Figure 9 is a spatial wave profile plotted at a time instant. Three zero-crossing points, marked by circles,

are tracked. The celerity calculated by this method is generally reasonable, but at times can become negative or jump to a very large value.

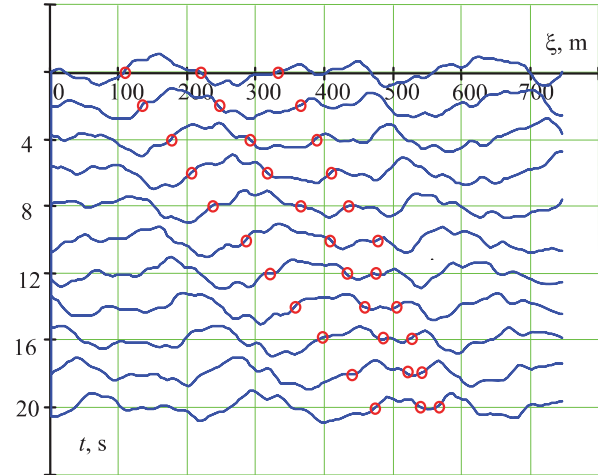


Figure 8: Celerity based on zero-crossing points (Belenky *et al.* 2012)

To gain a better understanding of the nature of this behavior, simplified cases were created with only two and three components (bi- and tri-chromatic waves), as shown in Figures 9 and 10. The tendency for large peaks can be seen even for two frequencies, and is even more dramatic for three frequencies.

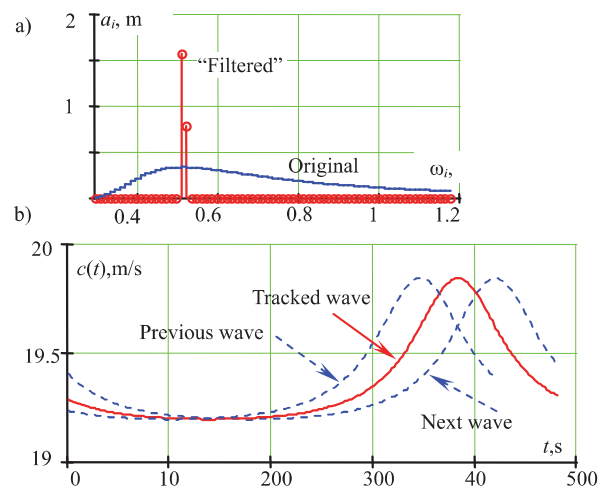


Figure 9: Bi-chromatic waves: (a) spectrum and (b) time history of celerity (Belenky *et al.* 2012)

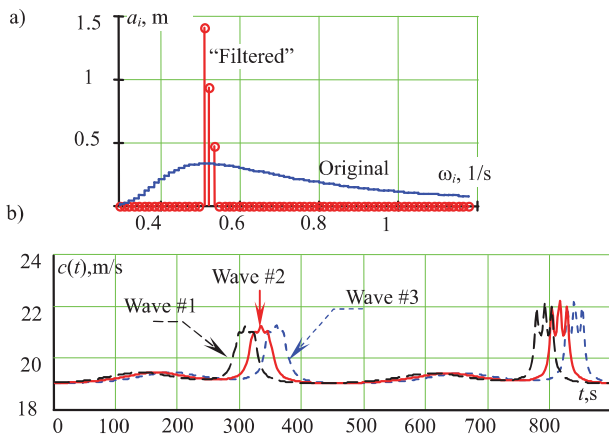


Figure 10: Tri-chromatic waves: (a) spectrum and (b) time history of celerity (Belenky *et al.* 2012)

Spyrou *et al.* (2012) had preferred to track the points of constant wave slope, proposing the concept of instantaneous celerity which can be calculated at the exact ship position at each time step. Figure 11 shows the celerity curves calculated for seaway derived from a JONSWAP spectrum. In a further development, they also proposed calculating a characteristic local celerity value corresponding to the point of maximum wave slope that is found in the vicinity of the ship. All these methods converged to the finding that the peaks in the celerity curve are intrinsic to the problem.

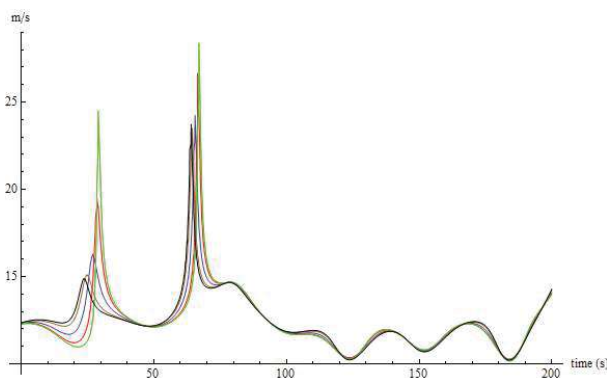


Figure 11: Celerity curves corresponding to wave slopes 1/75, 1/100, 0, -1/100, -1/75 (Spyrou *et al.* 2012)

The peaks appear to be related to changes in the local wave features, as when one wave overtakes another. As this occurs, a local wave

feature may cease to exist and the tracked points disappear and appear in another place, leading to a “spike” in the velocity of those points. Several candidate wave features were considered as points to track, and the smoothest behavior was found for the point of the maximum wave slope on the forward face. As this point can also be associated with the maximum surging force on a ship, it has a physical meaning with regard to surf-riding.

The calculation of wave celerity by tracking the points of maximum wave slope has been implemented in LAMP. In order to consider oblique and short-crested (multi-directional) seas, the algorithm searches for zeros of the derivative of the wave slope in the ship’s direction of travel,

$$\frac{\partial^3 \zeta_W(\xi_{\alpha \max}, t)}{\partial \xi^3} = 0 \quad (1)$$

with the condition that they are on the down-slope:

$$\frac{\partial^2 \zeta(\xi_{\alpha \max}, t)}{\partial \xi^2} < 0 \quad (2)$$

ζ_W is wave elevation, ξ is a coordinate in the mean direction of travel, and $\xi_{\alpha \max}$ is a position of maximum wave slope angle. ξ is related to this Earth-fixed coordinate system as:

$$\begin{aligned} x &= x_G(t) + \xi \cos \psi \\ y &= y_G(t) + \xi \sin \psi \end{aligned} \quad (3)$$

(x_G, y_G) is the current position of the center of gravity of the ship and ψ is the mean heading angle with respect to the global frame.

For efficiency, the algorithm computes the elevation and its derivatives on a line of points within a characteristic wave length of the ship, then iterates within intervals containing a maximum slope point. The local maximum slope points are tracked in time and the propagation speed of the point closest to the ship provides a practical celerity.

Figure 12 shows a snapshot of a LAMP simulation for the ONR tumblehome hull form running in long-crested irregular waves. The plot shows the wave profile at that time instance along the ship's travel direction with marks for the points of maximum down slope and elevation (crest). The wave in this case is derived from a Bretschneider spectrum with $H_s=7\text{m}$ and $T_m=12.0\text{s}$. A portion of the time history of the ship speed and the local wave celerity is shown in Figure 13.

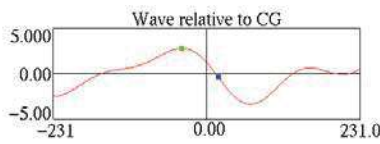


Figure 12: LAMP simulation of ONR Topsides tumblehome hull in irregular following seas (Spyrou *et al.* 2012)

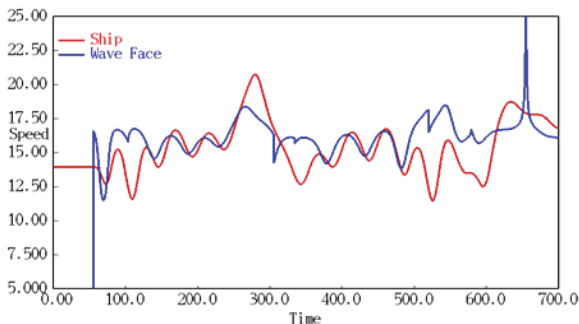


Figure 13: Ship speed and wave celerity for case with significant surf-riding in irregular waves (Spyrou *et al.* 2012)

The comparison of ship speed and local wave celerity clearly shows periods of surf-riding (e.g. $t=380-480$) and periods where the wave is overtaking the ship (e.g. $t=480-600$). While the initial wave celerity jump at 60 seconds is an artifact of ramping-up the wave, the “spike” in wave speed at $t=675$ is a case where the phasing of the wave component produces a very rapid local translation of the point of maximum slope and is related to a “merging” of wave faces. However, other discontinuities in the celerity, such as the one at $t=530$, are simply cases where the closest

point of maximum slope switches from one face to another as the waves overtake the ship.

In general, the problem of celerity in irregular waves and how it can affect ship motion is a very deep problem. Spyrou *et al.* (2014) provided a comprehensive theoretical analysis with an emphasis on the detection of surf-riding.

4.2 Celerity Based on Instantaneous Frequency

Looking for a method capable of producing a smoother celerity curve, Spyrou *et al.* (2014a) proposed an alternate scheme based on the instantaneous frequency derived from an envelope presentation of the wave elevation:

$$\zeta(t, \xi) = A(t, \xi) \cos(\vartheta(t, \xi)) \quad (4)$$

For a case where the wave elevation ζ_W is presented as typical cosine series, the amplitude $A(t, \xi)$ and phase $\vartheta(t, \xi)$ functions are computed from the wave elevations ζ_W and the Hilbert transform of the wave elevations $H(\zeta_W)$:

$$A(t, \xi) = \sqrt{\zeta_W^2 + (H(\zeta_W))^2} \quad (5)$$

$$\vartheta(t, \xi) = \arctan \frac{\zeta_W}{H(\zeta_W)}$$

$$\zeta_W(t, \xi) = \sum_{i=1}^{N_\omega} a_i \cos(k_i \xi - \omega_i t + \varphi_i) \quad (6)$$

$$H(\zeta_W(t, \xi)) = \sum_{i=1}^{N_\omega} a_i \sin(k_i \xi - \omega_i t + \varphi_i)$$

a_i , ω_i , and k_i are the amplitude, frequency, and wave number of the i^{th} component, while φ_i is a random phase shift.

The instantaneous frequency ω_W and the instantaneous wave number k_W are then computed as:

$$\omega_w(t, \xi) = -\frac{\partial \vartheta(t, \xi)}{\partial t} \quad (7)$$

$$k_w(t, \xi) = \frac{\partial \vartheta(t, \xi)}{\partial \xi} \quad (8)$$

This leads to the following definition of the wave celerity:

$$c(t, \xi) = \frac{\omega_w(t, \xi)}{k_w(t, \xi)} \quad (9)$$

For numerical calculations, it is convenient to substitute equations (5, 7, 8) into equation (9) and perform differentiation:

$$c(t, \xi) = -\frac{\frac{\partial H(\zeta_w)}{\partial t} \zeta_w - \frac{\partial \zeta_w}{\partial t} H(\zeta_w)}{\frac{\partial H(\zeta_w)}{\partial \xi} \zeta_w - \frac{\partial \zeta_w}{\partial \xi} H(\zeta_w)} \quad (10)$$

Equation (10) presents the wave celerity as a function of time and space that can be computed everywhere. The result can be compared against the instantaneous celerity calculated through the tracking method. Spyrou *et al.* (2014a) shows a series of comparisons between the “wave profile” and “instantaneous frequency” calculation schemes, an example of which is reproduced in Figure 14. The two calculations are generally very similar and both show “spikes” in celerity, although there is significant difference at the second spike at 440 s. What causes the “spikes” in formula (10)?

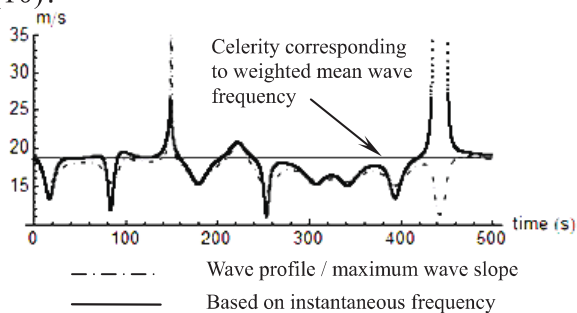


Figure 14: Instantaneous celerity for the bandwidth limited to 20% of the peak period (Spyrou *et al.* 2014a)

Figure 15 shows the spatial profile of the wave celerity calculated for Sea State 7 (Significant wave height 7.5 m, modal period 15 s) using a full-bandwidth Bretschneider spectrum and random phases. At this instant in time, there are four spikes at around 1500 m.

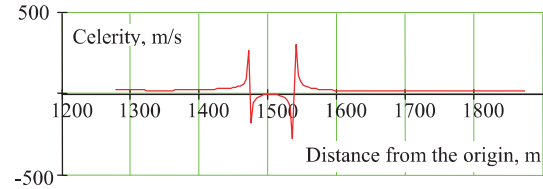


Figure 15: Spatial profile of wave celerity

One of the advantages of the present scheme of celerity calculation is that the origin of a spike can be analyzed. Consider the instantaneous wave number, which is the denominator in formula (9):

$$k_w(t, \xi) = \frac{\partial \vartheta(t, \xi)}{\partial \xi} = \frac{\frac{\partial H(\zeta_w)}{\partial \xi} \zeta_w - \frac{\partial \zeta_w}{\partial \xi} H(\zeta_w)}{\zeta_w^2 + (H(\zeta_w))^2} \quad (11)$$

The spatial profile of the instantaneous wave number is shown in Figure 16. It has a minimum around 1500 m and this minimum is negative. The curve of the instantaneous wave number crosses zero twice, which results in spikes in the value of instantaneous wave celerity. Figure 17 plots the spatial profile of wave elevations and shows that the minimum of the wave number and spikes of the celerity occur near the secondary maximum, *i.e.* the local maximum without crossing the line of calm water.

The secondary maxima and minima are related to the origination of new waves. If the local maximum at 1500 m raises and crosses the calm water line, the new wave will appear. Appearance of the new wave will lead to appearance of a new point of maximum of the wave slope and new zero-crossing. At the instance the new wave appears, the tracked points will make a finite “jump” in the infinitely small period of time causing a

theoretically infinite “spike” in celerity. Limiting the smallest value of the instantaneous wave number will limit the value “spike” and will make the celerity equation (10) into a practical formula for calculation.

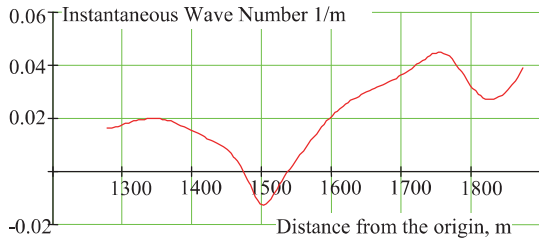


Figure 16: Spatial profile of the instantaneous wave number

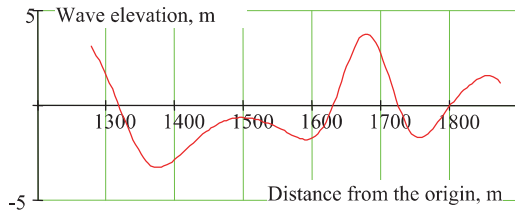


Figure 17: Spatial profile of the wave elevation

5. SURF-RIDING IN IRREGULAR WAVES

5.1 Simple Mathematical Model of Surging and Surf-Riding in Irregular Waves

A simple model for one-degree-of-freedom nonlinear surging was proposed in Belenky *et al.* (2011):

$$(M + A_{11})\ddot{\xi}_G + R(\dot{\xi}_G) - T(\dot{\xi}_G, n) + F_X(t, \xi_G) = 0 \quad (12)$$

M is mass of the ship, A_{11} is the longitudinal added mass, R is resistance in calm water, T is the thrust in calm water, n is the propeller rotation rate, F_X is the surging component of the Froude-Krylov wave force, and ξ_G is longitudinal position of the center of gravity in the Earth-fixed coordinate system. The dot above the symbol indicates temporal

derivative. Following Spyrou (2006), a polynomial approximation for thrust and resistance are used:

$$\begin{aligned} R(U) &= r_1 U + r_2 U^2 + r_3 U^3 \\ T(U, n) &= \tau_1 n^2 + \tau_2 n U + \tau_3 U^2 \end{aligned} \quad (13)$$

As the model is meant at this stage to be qualitative, a linear wave-body formulation seems to be appropriate for the case. Therefore:

$$F_X(t, \xi_G) = \sum_{i=1}^N A_{Xi} \cos(k_i \xi - \omega_i t + \varphi_i + \gamma_i) \quad (14)$$

As a body-linear formulation is adopted, the amplitude A_{Xi} and phase shift γ_i are available via response amplitude and phase operators:

$$A_{Xi} = a_i RAO(k_i) \quad (15)$$

$$\begin{aligned} RAO(k_i) &= \rho g k_i \left(\left(\int_{-0.5L}^{0.5L} C(x, k_i) \cos(k_i x) dx \right)^2 \right. \\ &\quad \left. + \left(\int_{-0.5L}^{0.5L} C(x, k_i) \sin(k_i x) dx \right)^2 \right)^{1/2} \end{aligned} \quad (16)$$

$$C(x, k_i) = 2 \int_{-d}^0 \exp(k_i z) b(x, z + d) dz \quad (17)$$

Here x and z are measured in the ship-fixed coordinate system (positive forwards of amidships and upward from the baseline), $b(x, z)$ is the molded local half-breadth, and d is the draft amidships. The surging phase shift is expressed as:

$$\gamma_i = \arctan \left(\frac{\int_{-0.5L}^{0.5L} C(x, k_i) \sin(k_i x) dx}{\int_{-0.5L}^{0.5L} C(x, k_i) \cos(k_i x) dx} \right) \quad (18)$$

The mathematical model described by equations (12 through 18) is essentially an

extended time domain version of the model presented in Spyrou (2006). It is expected to reproduce nonlinear surging and surf-riding, which is illustrated in Figure 18.

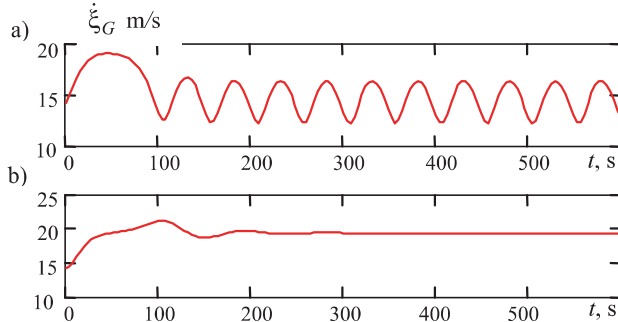


Figure 18: Solution in regular waves: co-existence of (a) surging and (b) surf-riding (Belenky *et al.* 2011)

5.2 Identification of Surf-Riding Instances in Irregular Waves

There are two aspects to the problem of identifying instances of surf-riding. The first is that surf-riding in irregular seas is not visually evident from the ship speed, as was illustrated in Figure 13. Without the wave celerity plotted alongside, an observer sees just a periodic stochastic process. The second aspect is to see how well definition of the wave celerity in irregular seas can explain the observed behavior. In a sense, the identification of surf-riding can be used as qualitative validation of the celerity calculation scheme.

A very basic example of such identification is described by Belenky *et al.* (2012), using tri-chromatic waves and zero-crossing wave celerity (see Figure 10). The time histories of surging speed and celerity are shown in Figure 19. The evolution of the surging speed for the first hundred seconds suggests attraction to a surf-riding equilibrium. But is this really the case?

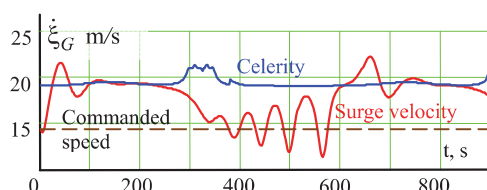


Figure 19: Time histories of surging velocity and celerity for tri-chromatic waves (Belenky *et al.* 2012)

Figure 20 shows a “spatial snapshot” of the forces (surging force vs. balance of thrust and resistance) superimposed with a spatial profile of the wave and its zero-crossing points. The time instant is $t=150$ s. The instantaneous position of the ship is indicated via the x-coordinate of the diamond, while the balance between thrust and resistance is indicated by its y-coordinate. The diamond is located at the intersection of the surging force and the balance of the thrust and resistance at the wave celerity. This means that the ship is in equilibrium and surf-riding is observed.

This simple example shows how wave celerity allows an interpretation of the observed motion and establishes the fact of surf-riding. The example also verifies the physical relevance of the calculated wave celerity. The method of calculating celerity based on maximum wave slope was successfully tested for identification of surf-riding by Spyrou *et al.* (2012, 2014). Spyrou, *et al.* (2014a) used both methods to calculate the celerity; see Figure 21.

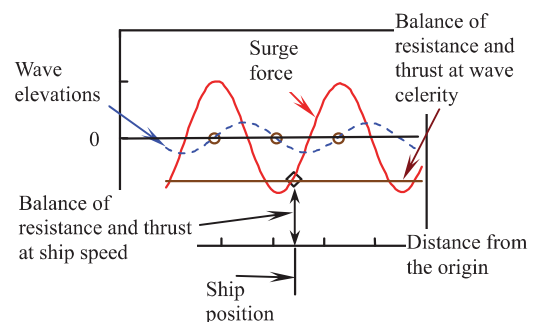


Figure 20: “Spatial snapshot” for $t=150$ s (Belenky *et al.* 2012)

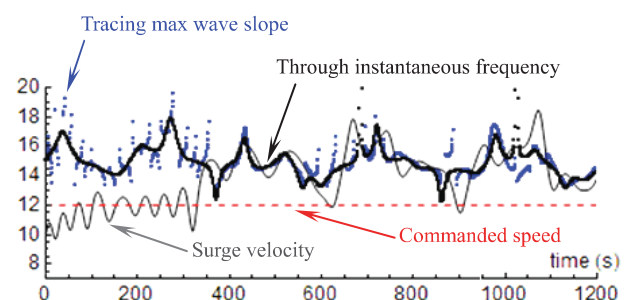


Figure 21: Time histories of surge velocity and celerity calculated with max wave slope and instantaneous frequency (Spyrou *et al.* 2014a)

Both methods of the wave celerity calculation seem to identify surf-riding instance in the same way, which is not surprising considering the similarity in the results of the two methods (Figure 14).

5.3 Phase Space in Irregular Waves

The position and type of the equilibria defines the topology of the phase plane, so the evolution of surf-riding equilibria in space and time is a logical starting point for analyzing surf-riding in irregular waves. Figure 22 shows the calculated loci of equilibria for the trichromatic case described above. The result is remarkably consistent with the notional topology presented in Figure 7.

The accuracy of the evaluation of the equilibrium stability was not always sufficient near the points where the equilibria appeared or disappeared, so the stability status of those point was corrected based on geometric considerations. Correct points are shown as two-color symbols. The inset in Figure 22 shows a close-up where the geometrical correction was applied.

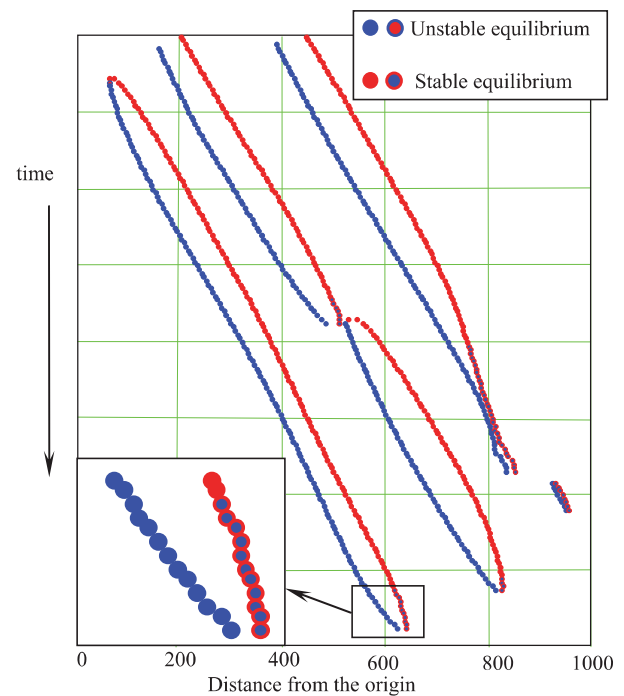


Figure 22: Calculated traces of equilibria

The unsteady motion of the equilibria qualitatively changes the phenomenon. A frame of reference moving with the equilibrium is no longer inertial. Also, the dynamical system cannot stay at the equilibrium position, even if the initial conditions correspond exactly to the equilibrium, because the equilibrium will move away. As an analogy, imagine a small heavy ball in a wine glass that is being moved in a circular motion. The ball will continuously “chase” a quasi-equilibrium point that moves around the inside of the glass.

The phase plane is also changing with time; strictly speaking, the phase plane by itself does not make sense beyond the “spatial snapshot”. The phase trajectory becomes a 3D line in hybrid phase-time coordinates (Kontolefas and Spyrou 2015). Projecting to a plane, the set of trajectories behaves as a non-stationary fluid flow. Spyrou *et al.* (2014a) describe an application of the concept of a Feature Flow Field (Theisel and Seidel 2003), which addresses the problem of feature tracking in non-stationary flow fields. The Feature Flow Field (FFF) method has been proposed for the tracking of a variety of different local features, including critical points of vector fields (such

as, in our case, the surf-riding equilibria). Figure 23 shows an example plot for surf-riding, where “saddle and focus-like” structures are present.

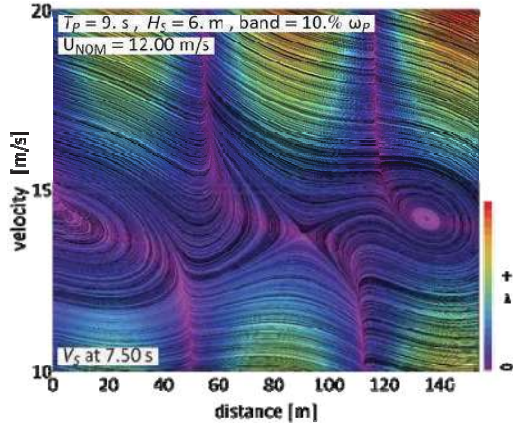


Figure 23: Streamlines and magnitude of forward velocity; JONSWAP Spectrum (Spyrou *et al.* 2014a)

Further developments towards the understanding of this time-dependent phase plane are described in Kontolefas and Spyrou (2015). Here is discussed a combined consideration of the feature flow field concept for tracking surf-riding states with the concept of hyperbolic Lagrangian Coherent Structures which can be seen as the finite-time generalization of the manifolds.

5.4 Statistics of High-Runs

The study of surf-riding in irregular waves requires the capability to characterize observations from simulations or model tests. However, the identification of surf-riding events in irregular waves is not trivial, particularly if the wave celerity cannot be calculated, which will generally be the case in a model test. It therefore makes sense to also look at the statistics of significant exceedances of the nominal speed by the surge velocity, also known as “high-runs”. Themelis *et al.* (2015) studied the statistics of high-runs depending on spectrum and sea state.

The idea of high-runs is not new. As pointed out in that work, Grim (1963) had

looked into the probabilistic quantification of the occurrence and duration of high-runs (“long run”) in a following irregular sea, taking into account the strongly nonlinear character of surge motion when the phenomenon occurs. Themelis *et al.* (2015) also examined the relation between the high-run occurrences and the instantaneous wave celerity (Figure 24).

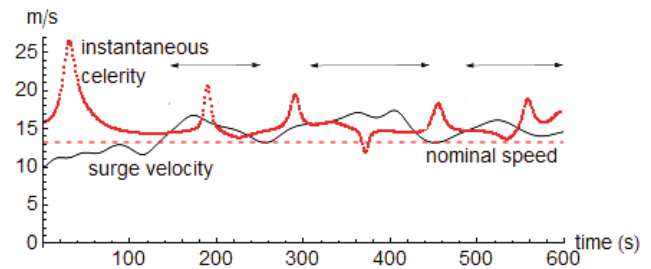


Figure 24: Schematic definition of high-run with the superimposed instantaneous wave celerity (Themelis *et al.* 2015)

The conclusion was that the velocity of the high-run shows good correlation with the mean instantaneous celerity when an error metric combining errors of amplitude and phase is applied.

5.5 Metric for Likelihood of Surf-Riding

As described at the beginning of this paper, a central element in the implementation of the split-time method is developing a metric for the likelihood of a rare event which can be evaluated at the occurrence of an intermediate event of some sort. An initial proposal for a metric of the likelihood of surf-riding can be found in Spyrou *et al.* (2014a) and is illustrated in Figure 25. The idea is to compute a “critical distance” in the phase plane between the ship’s state (position and velocity) at up-crossing and a critical state (point in the phase plane) from which the ship would be captured into surf-riding.

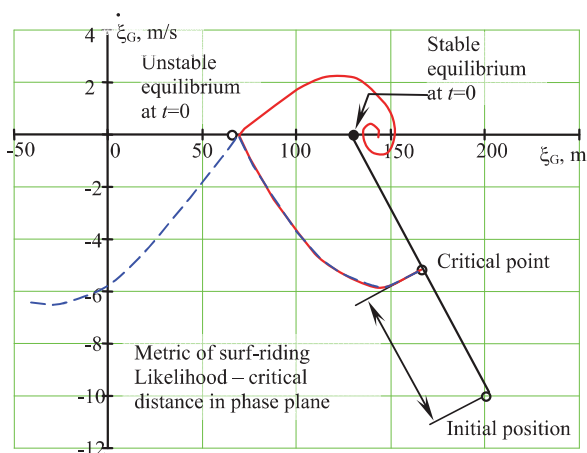


Figure 25: Initial idea for the metric of the likelihood of surf-riding

The critical point would lie on the line between the initial position of a ship (state at upcrossing) and the position of the stable quasi-equilibrium at the initial instant. A series of “rare” simulations would be performed for initial conditions corresponding to points along this line, and these simulations would determine the conditions that led to surf-riding.

This metric was initially tested with a slowly changing regular wave for which it is possible to explicitly consider the motion of the equilibria. The critical distance could be calculated, but the point to which the system was attracted and, once the critical point was reached, captured into surf-riding was different from the stable equilibrium at the initial moment.

A method of calculating wave celerity based on instantaneous frequency allowed the metric to be tried in irregular waves. The observed picture was more complex. The acceleration of the equilibria is not small and, as a result, the actual attraction does not occur. Instead, the dynamical system moves around the equilibrium path, similar to the picture on Figures 13 and 24. Figure 26 shows the phase plane computed for a moving frame of reference. The velocity of this frame of reference is constant and corresponds to a

velocity of the stable equilibrium at the initial moment.

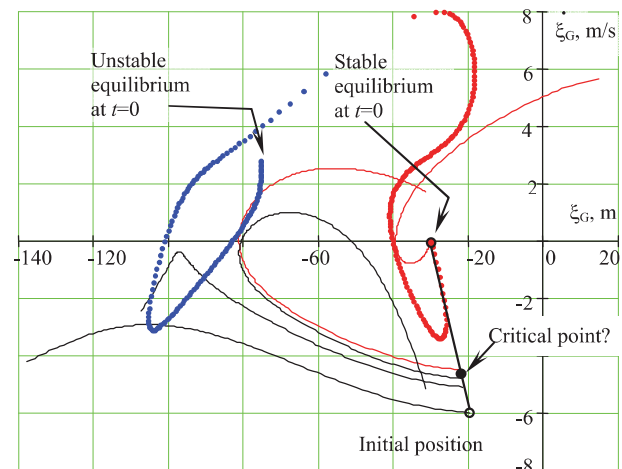


Figure 26: Phase plane trajectories for the critical point determination in calculating a metric for the likelihood of surf-riding in irregular waves

The main difference is that the dynamical system does not follow the stable equilibrium, even if it was placed exactly in the equilibrium position at the initial moment. Furthermore, the equilibria can move toward each other and disappear, releasing the ship from the surf-riding. This means that the criterion for attraction to equilibrium is not as evident as in the case shown in Figure 25. While it is possible to find the critical point visually, the development of a formal criterion of attraction remains for the future work.

6. CONCLUSIONS AND FUTURE WORK

This paper has reviewed work related to surf-riding and broaching-to that has been and continues to be performed under the ONR project entitled “A Probabilistic Procedure for Evaluating the Dynamic Stability and Capsizing of Naval Vessels”. The present work focusses on how irregular waves influence the phenomenon. The review addressed three major areas:



- Simulation of surf-riding and broaching-to by advanced hydrodynamic codes and the applicability of nonlinear dynamics tools
- Celerity of irregular waves
- Properties of time-dependent phase space of surging and surf-riding in irregular seas and their characterization.

The results of the studies may be summarized in the following conclusions:

- Surf-riding and broaching-to responses predicted via hybrid hydrodynamic codes are consistent with theoretical results, and the concepts and tools developed from ODE-based analysis can be applied to simulation tools as well
- The introduction of irregular waves leads to qualitative changes in surf-riding and broaching-to, including:
 - Problem must be considered in space and time
 - Surf-riding equilibria moves in an unsteady (accelerating) manner, and can appear and disappear in time
 - Surf-riding modes exist for a limited period of time
 - The system attracted to an equilibrium cannot stay with the equilibrium, but instead moves around it.

Problems to be addressed in the next stage of the research project include:

- Formulation of convergence criteria for the metric of likelihood of surf-riding
- Inclusion of sway and yaw into the simplest mathematical model, and study of the uncontrolled turn in irregular waves
- Formulation of the metric of likelihood of broaching-to and capsizing caused by broaching-to.

Solution of these problems is aimed at the developing the procedure for a physics-based statistical extrapolation using a limited data set from nonlinear time-domain numerical simulation. It is envisioned that the procedure will consist of the following steps:

- Prepare an extrapolation data set of simulation data
- Set an intermediate threshold providing a reasonable number (thousands) of upcrossings to be observed
- For each upcrossing, compute a metric of the likelihood of surf-riding, broaching-to, or capsizing due to broaching
- Fit a Generalized Pareto Distribution (GPD) to the metric data and evaluate the estimate of the capsizing rate and its confidence interval from the extrapolation of the metric to the level at which capsizing is inevitable.

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