A Time-Efficient Approach for Nonlinear Hydrostatic and Froude-Krylov Forces for Parametric Roll Assessment in Irregular Seas

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ABSTRACT

In the context of second generation stability criteria assessments, the present paper introduces a fast time domain algorithm for parametric roll assessment in irregular seas. Two features can be distinguished in the present proposal: a) it involves the essential heave-roll-pitch nonlinear coupling and b) it is a pre-calculated *derivative* model, convenient for the required systematic Monte Carlo simulations.

The main features of the model are described. The proposed methodology is based on a 3D panel method in which hydrostatic and wave-induced forces are computed on the actual body surface considering a set of systematic pre-defined hull positions. This set of data is preprocessed through polynomial fitting and the coefficients of the derivative model, corresponding to a Taylor series expansion defined up to the third order, are obtained. The methodology is applied to a container ship in head seas. The model is capable of reflecting the non-ergodicity of the head seas parametric rolling. At the same time, the heave and pitch motions display "weakly ergodic" responses.

KEYWORDS: *Head seas; Derivative model; Second generation stability criteria; Parametric rolling; Irregular seas.*

1. INTRODUCTION

Nowadays, several numerical methodologies exist for predicting the nonlinear behavior of parametric rolling of ships in waves in time domain. These procedures are usually categorized into fully nonlinear and weakly nonlinear codes. The first type considers nonlinearities in all the involved forces, whereas the second one introduces nonlinearities basically in the restoring and incident wave forces in a time domain integration scheme. Both approaches demand high computational effort and, as a consequence, appear prohibitive in terms of simulation time when assessing parametric roll motions in stochastic seas.

The first two authors have proposed an analytical derivative model which was validated in regular waves, Neves and Rodríguez (2006) . In order to enhance the capabilities of the *derivative* model, i.e., by considering more general body geometries, the present paper uses an improved approach for the computation of the nonlinear restoring and Froude-Krylov coefficients. The proposed methodology is based on a 3D panel method in which hydrostatic and wave-induced forces are computed on the actual body surface considering a set of systematic pre-defined hull positions. This set of data is preprocessed through polynomial fitting and the coefficients of the derivative model, corresponding to a Taylor series expansion defined up to the third order are obtained. Such strategy does lead to an adequate hydrodynamic modeling capable of taking into consideration the essential nonlinear coupling effects between heave, roll and pitch, see Rodríguez et al. (2007).

In recent years, other authors have proposed mathematically similar approaches to achieve fast time domain algorithm in the context of second generation stability criteria assessments. Song and Kim (2011) used Fourier series decomposition applied to a one-degree-of-freedom roll model. Alternatively, Weems and Wundrow (2013) used a volume-based heave-roll-pitch hybrid model. Finally, Somayajula and Falzarano (2014) used a Volterra series model, again applied in a one degree of freedom model.

The present derivative model, derived in the context of Taylor series expansion is a very fast and reliable time domain algorithm which may be useful in the context of second generation stability criteria assessment. In the present paper the polynomial approach is described and applied in the context of irregular seas. The potentialities of the model are highlighted and some numerical results are presented. It is anticipated that one of the main advantages of the proposed methodology is that after all the derivatives are pre-computed, the equations of motion may be integrated in a very effective and fast way.

2. MATHEMATICAL MODEL

In order to simulate parametric rolling in irregular waves it is necessary to express forces and moments acting on the ship as time series. As these forces and moments depend on the submerged geometry which is governed by a random wave profile and irregular motions of the ship, the problem becomes very complex and costly in computer time. The “exact” approach of the problem implies in not only solving the ship hydrodynamic problem for the random submerged geometry but also the nonlinear hydrostatics in time domain, and additionally, to solve implicitly and iteratively the equations of motion.

To overcome these difficulties and make the solution of the parametric roll problem more practical, the paper proposes a hybrid method which combines some hypothesis from classical seakeeping with the solution of the nonlinear equations in time domain. The equations to be integrated are (Neves and Rodríguez, 2006):

$$\begin{aligned}&(m+Z_{\ddot{z}})\ddot{z}+Z_{\ddot{\theta}}\ddot{\theta}+Z_{\dot{z}}\dot{z}+Z_{\dot{\theta}}\dot{\theta}+\\&Z_z z+Z_\theta\theta+\frac{1}{2}Z_{zz}z^2+\frac{1}{2}Z_{\phi\phi}\phi^2+\frac{1}{2}Z_{\theta\theta}\theta^2+\\&Z_{z\theta}z\theta+\frac{1}{6}Z_{zzz}z^3+\frac{1}{2}Z_{zz\theta}z^2\theta+\frac{1}{2}Z_{\phi\phi z}\phi^2 z+\\&\frac{1}{2}Z_{\phi\phi\theta}\phi^2\theta+\frac{1}{2}Z_{\theta\theta z}\theta^2 z+\frac{1}{6}Z_{\theta\theta\theta}\theta^3+Z_{\zeta z}(t)z+\\&Z_{\zeta\theta}(t)\theta+Z_{\zeta\zeta z}(t)z+Z_{\zeta zz}(t)z^2+Z_{\zeta z\theta}(t)z\theta+\\&Z_{\zeta\varphi\varphi}(t)\varphi^2+Z_{\zeta\zeta\theta}(t)\theta+Z_{\zeta\theta\theta}(t)\theta^2\\&\qquad\qquad=Z_w(t)\end{aligned}\tag{1}$$

$$\begin{aligned}&(J_{xx}+K_{\ddot{\varphi}})\ddot{\varphi}+K_{\dot{\varphi}}\dot{\varphi}+K_{\dot{\varphi}|\dot{\varphi}|}\dot{\varphi}\left|\dot{\varphi}\right|+K_\varphi\varphi+K_{z\varphi}z\varphi+\\&K_{\varphi\theta}\varphi\theta+\frac{1}{2}K_{zz\varphi}z^2\varphi+\frac{1}{6}K_{\varphi\varphi\varphi}\varphi^3+\frac{1}{2}K_{\theta\theta\varphi}\theta^2\varphi+\\&K_{z\varphi\theta}z\varphi\theta+K_{\zeta\varphi}(t)\varphi+K_{\zeta\zeta\varphi}(t)\varphi+K_{\zeta z\varphi}(t)z\varphi+\\&\qquad K_{\zeta\varphi\theta}(t)\varphi\theta=K_w(t)\end{aligned}\tag{2}$$

$$\begin{aligned}
&(J_{yy}+M_{\ddot{\theta}})\ddot{\theta}+M_{\ddot{z}}\ddot{z}+M_{\dot{z}}\dot{z}+\\
&M_{\dot{\theta}}\dot{\theta}+M_{z}z+M_{\theta}\theta+\frac{1}{2}M_{zz}z^{2}+\\
&\frac{1}{2}M_{\phi\phi}\phi^{2}+\frac{1}{2}M_{\theta\theta}\theta^{2}+M_{z\theta}z\theta+\frac{1}{6}M_{zzz}z^{3}+\\
&\frac{1}{2}M_{zz\theta}z^{2}\theta+\frac{1}{2}M_{\phi\phi z}\phi^{2}z+\frac{1}{2}M_{\phi\phi\theta}\phi^{2}\theta+\\
&\frac{1}{2}M_{\theta\theta z}\theta^{2}z+\frac{1}{6}M_{\theta\theta\theta}\theta^{3}+M_{\zeta z}(t)z+M_{\zeta\theta}(t)\theta+\\
&M_{\zeta\zeta z}(t)z+M_{\zeta zz}(t)z^{2}+M_{\zeta z\theta}(t)z\theta+M_{\zeta\phi\phi}(t)\phi^{2}+\\
&M_{\zeta\zeta\theta}(t)\theta+M_{\zeta\theta\theta}(t)\theta^{2}=M_{w}(t) \qquad (3)
\end{aligned}$$

Definition of coefficients appearing in the set of equations (1-3) and how to obtain them have been described in Neves and Rodríguez (2007). The same paper demonstrates the good accuracy of the model in regular waves when compared to experimental results for a modern container ship.

3. METHODOLOGY FOR IRREGULAR SEAS

In irregular seas, functions that govern wave excitations lose their harmonic character, becoming random functions in time domain. One way of expressing these direct excitation forces and moments, as well as wave restoring actions associated to a given sea spectrum is through the transfer functions amplitude of excitations (or restoring) in regular waves in frequency domain. Therefore, for example, for the heave excitation force we have:

$$S_{Z_w}(\omega_e)=\left[RAO_{Z_w}(\omega_e)\right]^2 S_{\zeta}(\omega_e) \qquad (4)$$

and for the roll wave restoring coefficient $K_{\zeta\phi}(t)$:

$$S_{K_{\zeta\phi}}(\omega_e)=\left[RAO_{K_{\zeta\phi}}(\omega_e)\right]^2 S_{\zeta}(\omega_e) \qquad (5)$$

For the exciting forces and moments in the other degrees of freedom and for the other wave restoring coefficients the same logic as given in equations (4) and (5) follows. As observed in the structure of eqs. (1-3) the hydrodynamic coupling between the three modes is described by eight time-dependent contributions in the heave equation ($Z_{\zeta z}(t)$, $Z_{\zeta\theta}(t)$, $Z_{\zeta\zeta z}(t)$, $Z_{\zeta zz}(t)$, $Z_{\zeta z\theta}(t)$, $Z_{\zeta\phi\phi}(t)$, $Z_{\zeta\zeta\theta}(t)$, $Z_{\zeta\theta\theta}(t)$), four contributions in the roll mode ($K_{\zeta\phi}(t)$, $K_{\zeta\zeta\phi}(t)$, $K_{\zeta z\phi}(t)$, $K_{\zeta\phi\theta}(t)$) and eight contributions in the pitch mode ($M_{\zeta z}(t)$, $M_{\zeta\theta}(t)$, $M_{\zeta\zeta z}(t)$, $M_{\zeta zz}(t)$, $M_{\zeta z\theta}(t)$, $M_{\zeta\phi\phi}(t)$, $M_{\zeta\zeta\theta}(t)$, $M_{\zeta\theta\theta}(t)$). So, in total there are twenty functions to be pre-computed.

Added mass and damping in the directly excited modes (heave and pitch) may be computed using convolution (Cummins, 1962). However, based on Celis (2008) – which reports small influence of memory effects on the development of parametric rolling in regular waves – and given the aim of simplifying the proposed methodology, added mass and damping are computed at the frequency value corresponding to the peak value, as done by many authors. For roll mode these hydrodynamic coefficients are computed at the roll natural frequency, introducing also nonlinearities in roll damping (adopting the same approach proposed by the Authors in previous articles (Neves and Rodríguez, 2007) for parametric roll in regular waves. Hydrostatic restoring is introduced directly into the equations, as the respective coefficients are independent of time. These are pre-calculated for different hull positions defined around the hull mean position.

In summary, the proposed procedure for parametric roll simulation in irregular waves consists in:

a) Define a sea state, i.e. specify a sea spectrum $S_{\zeta}(\omega)$ for given basic parameters like significant wave height (H_s) and peak period (T_p).

b) Transform sea spectrum defined for wave frequency ω and heading χ into encounter frequency domain ω_e (see Bhattacharyya, 1978):

$$S_{\zeta}(\omega_e) = \frac{S_{\zeta}(\omega)}{1 - \frac{2\omega U}{g}\cos\chi} \qquad (6)$$

c) Compute amplitude operators of wave exciting forces and all wave restoring coefficients in frequency domain. The amplitude operators of wave exciting forces may be computed using well known softwares like WAMIT® or HANSEL. In the present paper, all twenty wave restoring coefficients are computed using DSSTAB (based on numeric calculation of numerical restoring coefficients based on polynomial fitting, as described in Rodríguez et al., 2007). These operators are computed for unit wave amplitude, analogous to ship RAOs.

d) Transform domains of amplitude operators defined in step (c) from wave frequency to encounter frequency:

$$\omega_e = \omega - \frac{\omega^2 U}{g}\cos\chi$$

e) Calculate spectra of exciting forces and all twenty restoring coefficients, as indicated in equations (4) and (5).

f) Use Fourier analysis to generate time series of all spectra defined in step (e). In the Fourier analysis, a general time series $\gamma(t)$ may be obtained from a given spectrum $S_{\gamma}(\omega)$ using the following well known expressions:

$$\gamma(t) = \sum_{n=1}^{N} \gamma_{a_n} \cos(k_n x - \omega_n t + \varepsilon_n) \qquad (7)$$

where: $\gamma_{a_n} = \sqrt{2 S_{\gamma}(\omega_n).\delta\omega}$

g) Solve nonlinear equations of motion in time domain using, for example, the classical 4th order Runge-Kutta routine.

4. NUMERICAL SIMULATIONS

Now the above presented methodology is applied. A container ship is considered, here denominated SAFEDOR (ITTC A1), described in Spanos and Papanikolaou (2009). 3D lines of ship hull are shown in Figure 1.

Figure 1 SAFEDOR ship hull.

SAFEDOR hull was tested for a JONSWAP spectrum with significant height H_s = 5.00 m, peak period T_p = 10.63 s, γ = 3.3 , wave incidence is 180° (head seas). Ship speed corresponds to Fn = 0.12. Sea spectrum and transformed spectrum to Fn = 0.12 are shown in Figure 2.

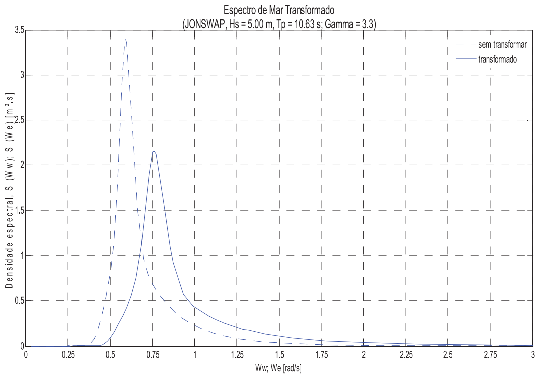

Figure 2 Sea spectrum, wave and encounter frequency domains

Transfer functions of external wave exciting force in heave and moment in pitch are given in Figures 3 e 4, respectively.

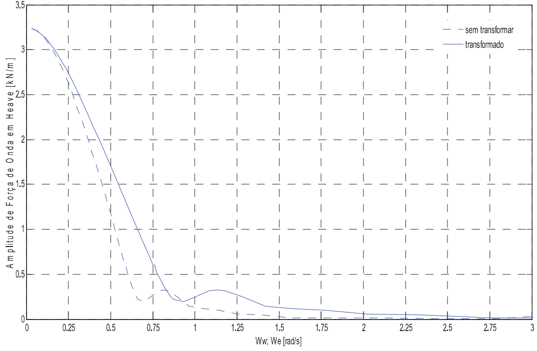

Figure 3 Transfer function, heave wave exciting force, Fn = 0.12

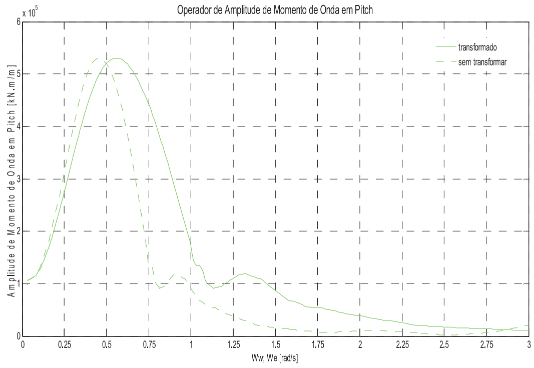

Figure 4 Transfer function, pitch wave exciting moment, Fn = 0.12

Transfer functions corresponding to roll wave restoring coefficients $K_{\zeta\varphi}$, $K_{\zeta z\varphi}$ and $K_{\zeta\varphi\theta}$ are shown in Figures 5 to 7.

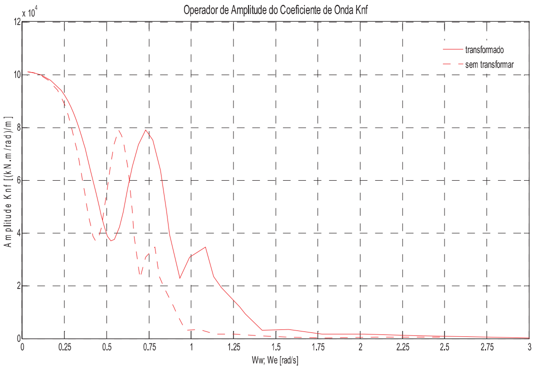

Figure 5 Transfer function, coefficient $K_{\zeta\varphi}$

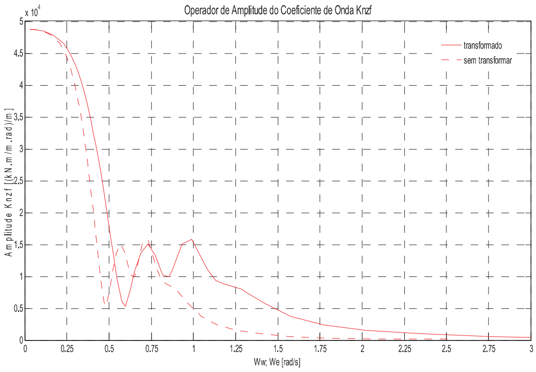

Figure 6 Transfer function, coefficient $K_{\zeta z\phi}$

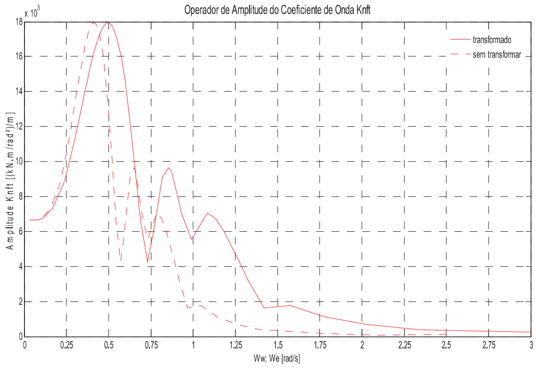

Figure 7 Transfer function, coefficient $K_{\zeta\phi\theta}$

With sea spectrum given in Figure 2 and the transfer functions the corresponding spectra for the specified sea condition are computed, as exemplified in equations (4) and (5). Some of these spectra are shown in Figures 8 to 10. Fourier analysis applied to the various spectra, corresponding time series are generated. Samples are given in Figures 11 to 13.

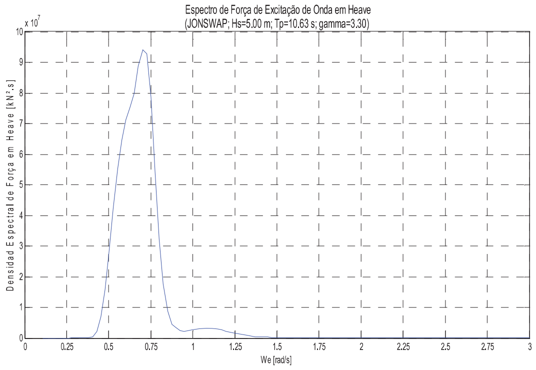

Figure 8 Heave exciting force spectrum

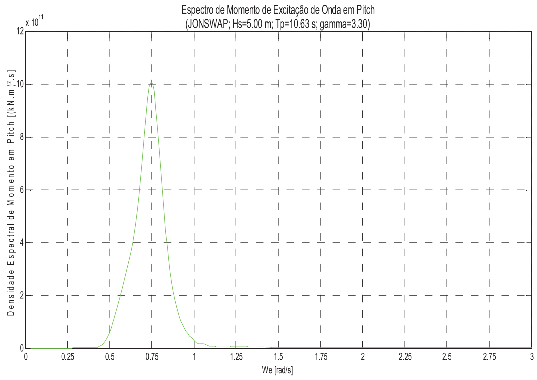

Figure 9 Pitch exciting moment spectrum

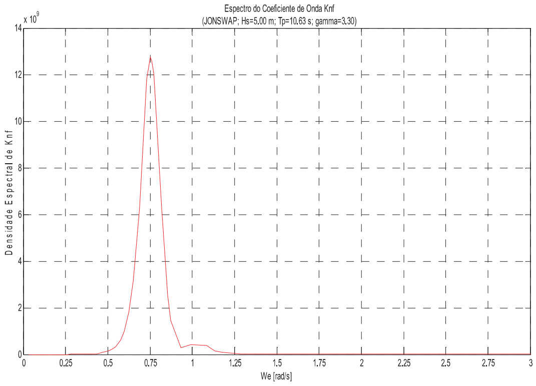

Figure 10 Roll restoring spectrum of $K_{\zeta\phi}$

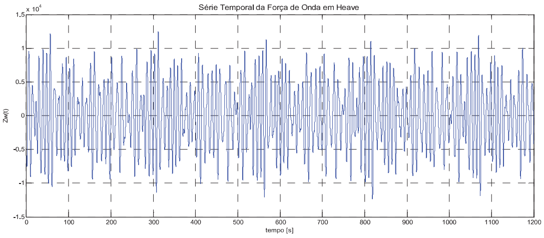

Figure 11 Heave force time series

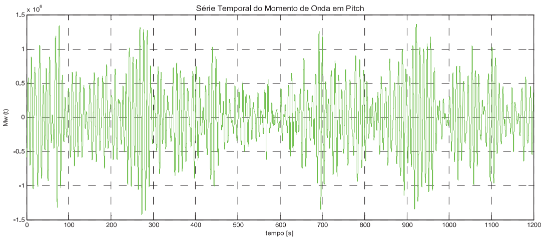

Figure 12 Pitch moment time series

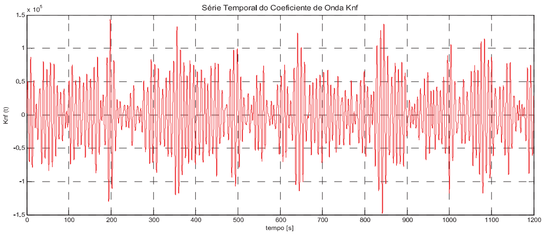

Figure 13 Wave coefficient $K_{\zeta\phi}$ time series

Time series obtained for the excitation forces and moments and all wave restoring coefficients are incorporated into the non linear equations of motion, eqs. (1-3), from which the time series of the heave-roll-pitch ship responses to the specified sea conditions result after integration.

To assess the ergodicity of responses in heave-roll-pitch modes, three realizations have been simulated (obtained from three different time series of wave excitations) for the specified sea conditions. These are shown in Figures 14 to 16.

Wave elevation is typically an ergodic process; additionally, transfer functions are linear. Therefore, even considering different time series (realizations) sufficiently long, wave excitation is statistically equivalent for the different realizations. In all cases the analyzed time series had a duration of 20', which in practice may be considered representative enough to describe the effects resulting from a given stochastic sea state.

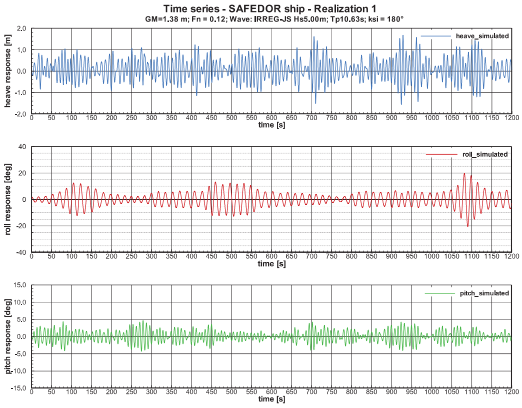

Figure 14 Heave, roll and pitch time series, realization #1

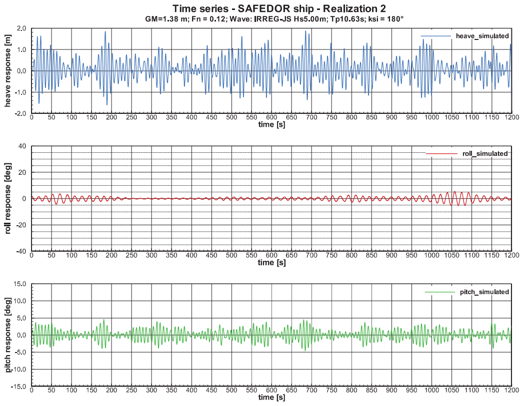

Figure 15 Heave, roll and pitch time series, realization #2

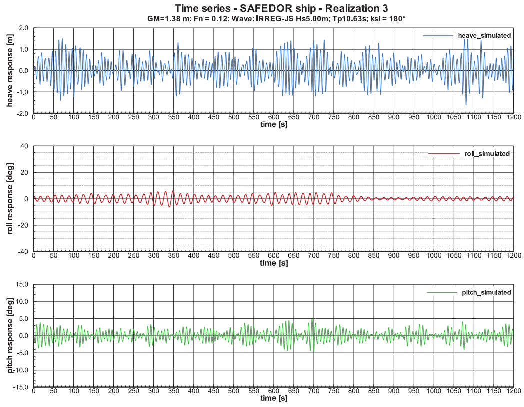

Figure 16 Heave, roll and pitch time series, realization #3

5. DISCUSSION OF NUMERICAL RESULTS

Despite the large frequency band with wave energy content, mainly the range of encounter frequency from 0.50 rad/s to 1.25 rad/s, wave excitation and restoring coefficients spectra are concentrated in a smaller band (0.50 to 0.85 rad/s) around the peak frequency of sea spectrum (0.76 rad/s) and outside the region of natural periods in heave (ω_{n3} = 0.82 rad/s), pitch (ω_{n5} = 0.85 rad/s) and roll (ω_{n4} = 0.33 rad/s).

Roll time series are distinct for the same tested condition. The first realization has moderate development of roll amplification, whereas the second and third realizations present quite small roll amplifications.

Roll responses have a larger period, corresponding typically to the characteristic 2:1 tuning, whereas both heave and pitch respond near encounter period.

Heave, roll and pitch spectra for the three realizations considered (see Figures 17 to 19) show the prevailing frequencies in each degree of freedom: heave and pitch present responses in the region of frequencies between 0.50 and 0.85 rad/s, which coincides with the main band of sea spectrum considered. On the other hand, roll responses are concentrated in a quite narrow band around the roll natural frequency, thus evidencing the occurrence of parametric rolling.

Roll spectra confirm the distinct character between the three realizations (see Figure 18). So, it is concluded that corresponding to the tested condition there two types of dynamic responses involved in roll: occurrence of parametric roll (realization #1) and non-occurrence of parametric roll (realizations #2 and #3). Thus, evidencing the influence of nonlinearities and the non-ergodicity in the responses.

Nonlinearities discussed in the above paragraph are also visible, but less intense in the heave and pitch spectra. It may be observed that spectral densities are smaller when there is parametric rolling (realization #1), whereas spectral densities are slightly larger in cases where parametric roll is very small (realizations #2 and #3). These results evidence the "weakly ergodic" character of heave and pitch motions, as has been pointed out by distinct authors (Belenky et al., 2003, Ogawa, 2007, Bulian et al., 2008).

It is important to notice here the great advantage in computing time when the derivative model is employed: running 50 time simulations of 10800 seconds with the corresponding 3100 frequency components (in order to avoid the *Self-Repeating Effect*) would take about 4 hours, whereas using full state-of-the art advanced hydrodynamic code DSSTAB in the same conditions would require about 3 days just to obtain a single realization of the random process on a normal desktop computer. On the other hand, running 20 simulations of 30 minutes with 220 frequency components, after pre-processing the derivative model will take nothing more than 12 seconds.

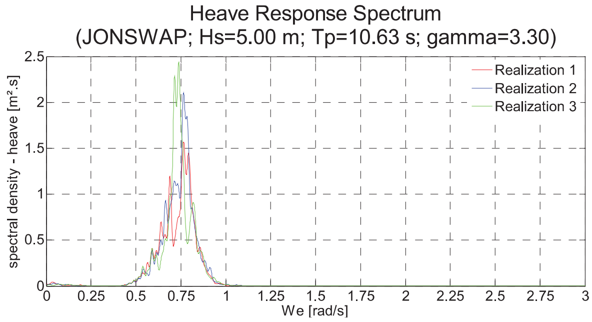

Figure 17 Heave motion spectra

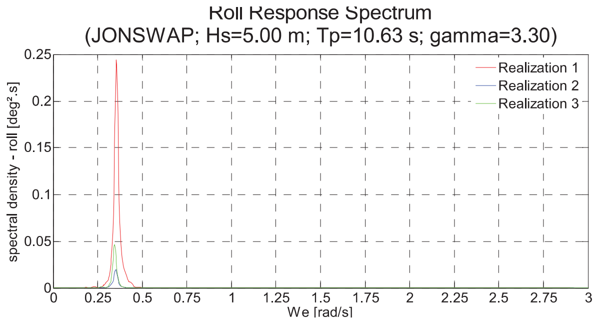

Figure 18 Roll motion spectra.

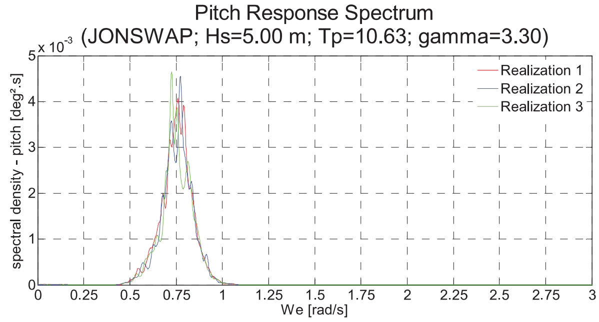

Figure 19 Pitch motion spectra

6. CONCLUSIONS

As previously mentioned, large series of time-domain simulations are required for realistic assessment of coupled heave, roll and pitch motions leading to extreme roll motion in irregular seas.

Some conclusions may be extracted based on the numerical results for the tested condition of SAFEDOR hull in head seas irregular seas.

It is shown that parametric roll affects the irregular responses in heave and pitch – as evidenced by the respective response spectra. In roll, the peak of responses is in the natural frequency, whereas in heave and pitch three peaks (or contributions) are observed, the larger at the encounter frequency and other two (much smaller) at the wave frequency and at the roll natural frequency.

Distinct realizations corresponding to a given test condition of SAFEDOR have produced qualitatively different responses with regard to occurrence of parametric rolling. This aspect confirms the capability of the proposed methodology to deal with strong nonlinearities, thus evidencing the non-ergodic character of roll responses when parametric rolling takes place.

For the heave and pitch modes the responses are "weakly ergodic".

The derivative model requires much less simulation time than full state-of-the art advanced hydrodynamic codes such as the DSSTAB code. This fact allows systematic Monte Carlo simulations in order to perform a realistic probabilistic assessment of parametric rolling of vessels.

7. ACKNOWLEDGMENTS

The present investigation has been supported by CNPq, CAPES and ANP. Authors greatly acknowledge this crucial support.

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