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The use of energy build up to identify the most critical heeling axis direction for stability calculations for floating offshore structures, review of various methods

Joost van Santen, *GustoMSC, the Netherlands*, joost.vansanten@gustomsc.com

ABSTRACT

For offshore structures like semi submersibles and jack-ups, hydrostatic stability is to be determined for what is called the weakest axis, which is not necessarily the same as the longitudinal axis of symmetry of the structure. When allowing trim to take place, the determination of the critical axis is complicated as free trimming leads to multiple solutions regarding the position for a given heel angle. It will be shown that for a freely floating structure, looking at the increase in potential energy can be used to identify those axis directions which are critical as well as realistic. The theoretical results will be illustrated with detailed data obtained for a two typical offshore structures using a standard stability program.

Keywords: *offshore, stability, energy, trim, twist, jack-up, semi submersible*

1. INTRODUCTION

Historically, determination of the stability offshore rigs is seen as extension of stability for ships. For ships the longitudinal axis is taken as the heeling axis, where trim is allowed to remove the trimming moment.

Early on, the offshore industry has recognized that the most critical axis is not necessarily the longitudinal axis. So, the wording critical axis was introduced, but as a kind of heritage one also had to consider free trim. This directly leads to a problem.

This problem is that a given position can be defined by infinite combinations of axis directions, heel angle and trim. Two angles suffice to uniquely define the position. Using three angles means that one is superfluous.

In the 80's several papers appeared introducing the use of pressure integration to replace the conventional way of slicing up the structure and calculation of the contribution of each slice, [1],[2]. In the publication of van Santen [2], the problems mentioned above were raised and a way to avoid them by using free twist was introduced. In this paper, also the concept of energy build up was looked at.

By Vassalos et all [3], free trim for a semi submersible was interpreted as selection of the heeling axis direction such that the trimming moment is nil. This is in contrast with normal practice for ships where the rotation around an axis perpendicular to the heeling axis is varied such that trimming moment is nil. In fact their free trim is equal to the free twist method.

More recently by Breuer and Sjölund [4], the problem was looked at again and using the build up of energy was proposed as a solution.

In the underlying publication, the increase in potential energy of a structure due to forced heel be looked at. Evaluation of stability according to free trim, free twist and steepest descent will be looked at. Examples for various structures will be shown.

This paper deals with freely floating structures. Effects due to mooring or dynamic positioning are specifically excluded.

2. EXAMPLE WITH A BARGE

For a rectangular barge (figure 1) with a raised forecastle, we can construct the righting arm curves for a range of axis directions (without trim) as shown in figure 2.

Applying free trim has dramatic consequences as is shown in figure 3. For some axis directions the righting arm curves stop prematurely. Why is this? In order to analyze this, the trimming moment as a function of trim angle is to be studied, see figure 4. The trim angle for which this moment is zero is the free trim angle belonging to a particular heeled situation.

For an axis direction of 80 deg, when heeled beyond 5.6 deg heel, the zero crossings disappear, meaning that there are no solutions with zero trim moment. When passing a certain position, continuation of the righting arm curve can only be achieved by reducing the heel whilst at the same time increasing the trim angle. This leads to the righting arm curve as shown in figure 5 and the trim angle as shown in figure 6.

For intact offshore rigs, some authorities require a range of positive stability up to the second intercept with the windarm of 30 deg. For a longitudinal axis direction this criterion would be met, but for an axis direction of 80 deg, the structure would fail. So, one could say that 80 deg axis direction is the most critical one and the rig would fail to comply with the range requirement.

Looking at the structure (figure 1) it is obvious that one should select an almost longitudinal axis as the most critical axis direction and not 80 deg. But for a more generic structure it is not that easy to identify the critical axis direction.

Why is it that from visual observations we reject 80 deg and accept 0 deg ? The key can be found in the rate at which the potential energy in the system increases with heel. For 0 deg axis direction this is far less than for 80 deg.

The potential energy is given by the negative of the vertical distance (VCB) between the centre of gravity CoG and the centre of buoyancy CoB.

E will be used to denote energy divided by displacement, so E = -VCB.

At the equilibrium position the energy is taken as the reference value, E_0. For a change in heel, the increase in energy is given as

$$\begin{aligned}\Delta E &= E(\varphi) - E_0 \\ &= VCB_0 - VCB(\varphi)\end{aligned} \tag{2}$$

The input of energy is the work done by the overturning heeling moment given by

$$\int_0^{\varphi} M(\varphi)d\varphi \tag{3}$$

Where M(φ) is the overturning moment divided by displacement. This results in:

$$\begin{aligned}-\left(VCB(\varphi) - VCB_0\right) &= \int_0^{\varphi} M(\varphi)d\varphi \\ &= \int_0^{\varphi} -(\text{righting arm}) \times d\varphi\end{aligned} \tag{4}$$

Note that for a free floating structure, a positive overturning moment results in a positive righting arm, but this means in fact that the counteracting restoring moment given by y_{cob} is negative.

The most critical axis can be viewed as the axis direction for which a given heel angle is reached with the least effort. In this way, the least energy is to be fed into the system in order to reach that particular heel angle.

Figure 8 shows the amount of energy fed into the barge depending on axis direction and heel angle. The trim is fixed to zero. From this figure it is seen that for an axis direction of about 0 deg the lowest amount of energy is needed to reach a given heel angle. This agrees with the subjective feeling that the most critical axis is almost longitudinal.

The problem now is how to determine the axis direction for which the energy increase is lowest. But before we do so, another example will be given, as found in the publication by Breuer and Sjöland [4].

3. ABS JACK-UP

Following discussions between MSC and ABS on the approval of a particular MSC jack-up, related to the Range of Stability (RoS) requirement [5], Breuer and Sjölund [4] published their paper in which they showed energy contour plots for a range of trim and heel angles.

For the damage as indicated in figure 9, the increase in energy depending on axis direction and heel angle for zero trim is given in figure 10. This shows that the lowest rate of increase is found for an axis direction of about 320 deg. For this direction, the righting arm is shown in figure 11 (free trim). Selecting another axis direction (280 deg) results in a vanishing righting arm curve, as shown in figure 12. Note the large trim angles related to the vanishing arm curve. Using an axis direction of 280 deg may lead to the false impression that ABS' RoS criterion is not. Actually it is met, but it is for an axis direction of 320 deg.

Figure 13 shows the righting arm curves obtained using free trim for a range of heeling axis directions. It is seen that the vanishing curve phenonenom is not exclusively for 280 degrees axis direction but occurs for other directions as well.

4. AXIS CONVENTION

When using heel, trim and twist, it is important to have a clear understanding of their meaning. For the remainder of this publication, the following convention is used when positioning a structure in a heeled condition (see figure 14):

- first the heeling axis direction is chosen,
- the structure is heeled around this axis
- the structure is trimmed around a transverse axis which was initially horizontal but changes direction due to heel.

5. FREE TRIM VERSUS FREE TWIST

As is seen above, looking at energy increase during heel is a clear indicator for the determination of the critical axis direction. The next question is how to deal with trim. For ships, introducing free trim is clearly meant to obtain the lowest righting arm. This is easily seen when going from the free trim to the situation with nil trim. When going from the trimmed to the free trim situation, energy is dissipated, thus the righting arm is less than for the fixed axis direction and zero trim situation. The question is if this also works when varying the axis direction.

For this purpose consider a heeled position with both a zero trimming moment and a zero moment around the twist axis which is the initially vertical axis. When applying a small change in the trim angle (dθ) and in the axis direction (dψ) reactive moments will result.

For a constant displacement, using conventional hydrostatic considerations, these moments can be determined by looking at the horizontal and vertical components of the rotations. The horizontal component (α) causes a change in waterline shape and in CoB position. The combined effect translates into a reactive moment $-GM_L\,\alpha$.

The rotation (β) around the vertical axis causes a transverse movement of the CoB which causes a reactive moment $y_{cob}\,\beta$. Thus, for a change of trim, dθ, the externally imposed moment needed for this change in trim becomes:

$$My_t = \left(GM_L\cos\varphi - y_{cob}\sin\varphi\right)d\theta \qquad (5)$$

Note that the value of GM is the instantaneous metacentric height for the given heeled position.

Similarly, due to a change of axis direction dψ the externally imposed moment becomes:

$$My_a = \left(-GM_L\sin\varphi - y_{cob}\cos\varphi\right)d\psi \qquad (6)$$

These moments are around a horizontal axis. When considering energy input, the moment is to be taken along the axis around which the rotation takes place. For instance, for trim, the moment around the trim axis is $My_a\cos\varphi$. Thus, for trim the energy input starting from the free twist, zero trim, position is:

$$dE_t = \frac{1}{2}\cos\varphi\left(GM_L\cos\varphi - y_{cob}\sin\varphi\right)d\theta^2 \qquad (7)$$

Due to a change in axis direction the energy input starting from free twist is:

$$dE_a = \frac{1}{2}\sin\varphi\left(GM_L\sin\varphi + y_{cob}\cos\varphi\right)d\psi^2 \qquad (8)$$

The term between the brackets in equation 7 is in general positive. In equation 8, for small heel angles the term between the brackets can be approximated by $GM_L\varphi$- $GM_T\varphi\cos\varphi$. So, for small heel angles, it is in general small but also positive. For larger angles it is in general positive. So, in general, when starting from the free twist position with zero trim and applying either a small change in twist or in trim an increase in energy in the system is found. Thus, the free twist position is the position with the lowest potential energy for a given heel angle.

The question arises if there is a reduction in energy when going from a free twist position with zero trim to a new position (with a change in twist of dψ) and letting the rig free to trim with angle dθ. The energy input needed to twist is given by dE_a. By letting the rig free to trim, the energy recovered is – dE_t. So the total energy input is dE_a-dE_t. The twist angle is obtained by imposing a moment on the rig. This moment is nullified by letting it free to trim. So:

$$My_a = -My_t \qquad (9)$$

Using equations 5 and 6 gives the relation between dψ and dθ:

$$d\psi = \frac{GM_L\cos\varphi - y_{cob}\sin\varphi}{GM_L\sin\varphi + y_{cob}\cos\varphi}\cdot d\theta \qquad (10)$$

By substituting [10] in [8] the difference in energy input is found to be

$$\begin{aligned} dE &= dE_a - dE_t \\ &= -\frac{1}{2}d\theta^2 y_{cob}\frac{GM_L\cos\varphi - y_{cob}\sin\varphi}{GM_L\sin\varphi + y_{cob}\cos\varphi} \end{aligned} \qquad (11)$$

Generally, the following applies:

- GM_L is positive,
- y_{cob} is negative for a positive righting arm

Thus, for the weakest axis, the numerator ($GM_L\cos\varphi$-$y_{cob}\sin\varphi$) is in general positive. The denominator is also in general positive. So, when going from a free twist to a free trim situation the energy input is positive. This leads to the conclusion that the free twist situation contains less energy than fixing the axis direction combined with free trim. The free twist position equals the situation with a local minimum energy as both moments My_t and My_a are zero.

For the jack-up show in figure 9, the numerical values for small variations in trim and twist around the free twist point are shown in figure 15. These are based on the calculations with a stability program as well as on the approximations given above.

Also, the calculated relation according to equation [10] between axis and trim for which energy E_a is exchanged with E_t is shown ("lowest envelope"). This figure shows the validity of the theoretical formulations.

It is important to be aware that in the free twist method, trim is always zero. Hence, the heel angle is always equal to the steepest slope of an initial horizontal plane (like a deck).

6. CHANGE IN AXIS DIRECTION DUE TO A CHANGE IN HEEL ANGLE

In section 5 it is shown that the free twist method results in the lowest amount of potential energy build up for a given heel angle. For a free twist situation, the moment around the initial vertical axis is zero. When making a small step in the heel angle, the change in axis direction can be estimated using this condition. For a constant displacement a heel increase $d\varphi$ results in moment around the twist axis of $-C_{xy} \cdot d\varphi$, where C_{xy} = cross moment of inertia of the waterline for a rotation around its centre of floatation. Making this equal but opposite to the moment due to a small change in twist (My_a, equation [6]) the change in axis direction results in:

$$d\psi = \frac{-C_{xy} / \text{volume}}{GM_L \sin\varphi + y_{cob}\cos\varphi} d\varphi \qquad (12)$$

It is seen that the axis direction is influenced by both the metacentric height (GM_L) and the righting arm (which equals $-y_{cob}$).

7. STABILITY OF THE SOLUTION

The free twist position is characterized by the twist moment My_a being zero. It is not necessarily a position with the lowest energy, but it can also be a position with the highest energy. For a given heel angle, there are several axis directions for which the energy is either minimum or maximum. The maximum is by definition an unstable position, the minimum is a stable position. Stability is indicated by the energy input equations 8 (for twist) and 7 (for trim). If the term $GM_L\sin\varphi+y_{cob}\cos\varphi$ is negative, the position is unstable in twist. For an intact structure at a small heel angle, the following approximations apply:

$y_{cob} = -GM_t\ \varphi$
$\sin(\varphi) = \varphi$
$\cos(\varphi) = 1.0$

$$\begin{aligned}&\text{stability term:}\\ &\left(GM_L - GM_t\right)\varphi\end{aligned} \qquad (13)$$

Clearly, for small angles the position is stable when heeling is around the axis with the smallest GM value. For larger angles, the actual values of GM_L and y_{cob} are to be considered.

When the structure is in free to trim, also the stability for trim should be looked at. For trim to be unstable, the term $GM_L\cos\varphi-y_{cob}\sin\varphi$ is to be negative. Keeping in mind that $-y_{cob}$ is the righting arm, it is seen that for a positive righting arm this is the case as long as GM_L, being the metacentric height for the given heeled position, is positive.

8. STEEPEST DESCENT METHOD

As shown above, the free twist method easily identifies those combinations of axis direction and heel angle, which follow the minimum energy path in the energy plot. The paths thus identified are limited in number.

By Breuer and Sjöland [4] a variation of the free twist method is proposed by considering a range of paths covering the energy plot. For each point on the path, the *next* position is found as follows:

- For a given position determine the direction of the restoring moment vector
- set the overturning moment vector opposite to the restoring moment vector
- set the rig to a *new* position by applying a small rotation around the restoring moment vector
- repeat the aforementioned steps till either the maximum or minimum energy level is reached

As the rotation axis and direction of the moment vector are in line, the energy input is maximized. This is equivalent to finding the position with the *largest* increase in energy for a given small rotation $\overline{R}$ around a horizontal axis with a varying direction.

For a given overturning moment vector $\overline{M}$ and a small rotation $\overline{R}$ the energy input is given by multiplying the overturning moment with the rotation:

$$\begin{aligned} dE &= \overline{M} \times \overline{R} \\ &= M_x R_x + M_y R_y \end{aligned} \qquad (14)$$

Where M_x, M_y, R_x and R_y are the components along the horizontal x and y axis.

For a given rotation step R in direction μ, the rotation vector becomes
$R_x = R \cos \mu$
$R_y = R \sin \mu$

The components of the overturning moment divided by displacement along the x and y axis can be approximated by

$$M_x = -y_{cob} + GM_t\,R\cos\mu$$
$$M_y = x_{cob} + GM_\ell\,R\sin\mu \qquad (15)$$

An approximation of the energy input when applying a small rotation R_s is given by:

$$dE = \int_0^{Rs} M_x R\cos\mu\,dR + \int_0^{Rs} M_y R\sin\mu.dR$$

$$dE = 0.5\left(-2\,y_{cob}\,R_S\cos(\mu) + R_S^2\,GM_t\cos^2(\mu)\right) + 0.5\left(2\,x_{cob}\,R_S\sin(\mu) + R_S^2\,GM_l\sin^2(\mu)\right) \qquad (16)$$

Where μ is the direction of the rotation axis. The direction μ is the solution of equation 15 where the gain in energy for a given step R_s is maximized. This direction can be found by solving:

$$\frac{dE}{d\mu} = 0 \qquad (17)$$

For small angles, this is equivalent to selecting the rotation axis in line with the moment vector given by:

$M_x = -y_{cob}$
$M_y = x_{cob}$

In the Steepest Descent Method for a given increase in rotation, dR, the increase in potential energy is maximized.

In paper of Breuer and Sjöland, the starting point is selected at the level where the energy is at its maximum, the "watershed contour" and the path is downhill to the equilibrium position, thus the name steepest "descent".

Streamlines.

In the steepest descent method, the direction of the instantaneous rotation vector is parallel to the instanteous moment vector as given above. Thus the rotation vector is given by:

$R_x = -y_{cob}$
$R_y = x_{cob}$

So, the rotation path, expressed in combinations of heel and trim, can also be constructed by calculating the streamlines for the energy plot. Commercial programs like MATLAB can do this easily.

Observations

From the above, the following observations are made regarding SDM:

In contrast with either free trim or free twist, SDM cannot calculate a single position. Each position depends on the previous position. As such it is not deterministic.

Due to the fact that each position depends on the previous position, any errors like numerical truncations etc will cause the path to deviate from the exact path. This makes comparing the results obtained from independent calculations difficult to compare.

For a given increase in rotation, the energy increase is maximized which is in contrats with the free twist method where for a given *increase* in heel the energy increase is minimized.

The energy plot has negative and positive extremes. The negative extremes correpond with the equilibrium condition; the positive ones correspond with the point where the rig will capsize when pushed further . At these extremes the direction for which the energy increase is at its maximum will be ill conditioned as the arms (x_{cob} and y_{cob}) are close to zero. This is also seen in equation (14) where the second order terms become important for small moments.

The method finds those paths in the directions of the largest gradient. Thus, the majority of the path will go to the maxima in the energy plot. Only a very small number (equal to the number of saddle points in the energy plot) will go to the minimum energy points.

Due to the nature of the method, going uphill will be difficult for cases close to the free twist path. Any numerical error will cause the path to quickly deviate from the ideal path through the valley. It is easier to go downhill, starting at the maxima (watershed contour in [4]).

For each path found using SDM (of which in fact an infinite number are present) the criteria are to be evaluated. However, this means that also unstable positions like a transverse axis direction for the barge example are evaluated. This is in fact an undesirable situation as a rig in such a position will move from such an unstable position to a stable one.

In the SDM method, the notation of heel versus righting arm loses its meaning as it is replaced by rotation (around the instantaneous righting moment vector) and righting moment itself. This makes it difficult to plot a righting arm versus heel angle as the starting heel angle (first upcrossing with arm=0) is ill defined. A way around might be to take the inclination of an initially vertical axis as the starting rotation. But one should be aware that for some paths for an increasing rotation, the inclination of such a vertical axis first decreases after which it increase again.

Because the meaning of heel is lost, evaluation of some criteria like ABS' RoS or 30 degree range to the second intercept with the wind overturning arm is difficult. An alternative would be to define heel as the steepest inclination of an initially horizontal surface and calculate the righting arm based on the increase in energy between successive heel angles. This would also solve the problem mentioned in the previous paragraph.

Some paths in SDM are opposite to the expected path. By that, it is meant that for some paths, the damaged rig heels such that the damaged area gets *out* of the water. In a BMT study, sponsored by the European Community [6], it was observed that capsize for a jack-up was always with the damaged compartment moving down, irrespective of the direction of wind and waves.

9. CASE STUDIES

For two structures, analyses have been done into the increase in potential energy during progressive heeling. These structures are:

- ABS jack-up,
- a simplified semi submersible

9.1 ABS jack-up

Intact. This structure, see fig 9, has been analyzed for both the intact and damaged condition. For the intact condition, figure 16 shows the energy surface for a range of heel angles (0-24 deg) and a range of axis directions (0-360 deg).

It is seen that for larger heel angles 3 axis directions are present following the local minimum path. These are 90^0, 210^0 and 330^0. The 210 and 330 deg directions are in fact identical due to the symmetry of the structure.

From the graph, it is also seen that the 330 (or 210) degree yields the lowest energy increase. For angles exceeding about 6 degrees heel there are 3 maxima and 3 minima in the plot. What is not clear is that for smaller heel angles there are only 2 maxima and 2 minima. The extremes at around 35 deg and 145 deg disappear for small heel angles. This is shown in figure 17 which shows the axis directions which have a zero twist moment depending on the heel angle.

Damaged. The damage case has a damaged compartment as indicated in figure 9. For a VCG of 23.45 m, the range of stability criterion of ABS is just met. This range should be 7 deg + 1.5 x steady heel (2.59 deg) = 10.89 deg.

The plot of the energy (figure 10) shows a minimum value for an axis direction of around 320 deg. Figure 18 shows the axis direction for which free twist is satisfied, i.e. where the moment around the y axis is indeed zero.

The figure also shows the estimated axis direction based on equation [12]. In this estimate, the start value at zero heel is taken from the calculations with the stability program. The other values are based on the summed increments. A very good fit is seen between the estimated and the actual axis directions.

For the damaged case, at larger heel angles, there are two paths following a local extreme in the energy surface. For small heel angles there is only a single path.

Figure 19 shows the results using the Steepest Descent method. Instead of a single path following the lowest gradient (like the free twist), a (infinite) number of possible paths are present. Most of them will go to the 3 maxima and only 3 will go to the minimum energy values. As is seen, the calculated paths (the red lines) follow the streamlines closely. Looking into more detail, it was found that the free twist path does not correspond exactly with a SDM path. The reason for this needs further analysis.

Also for SDM, unstable regions are present where the path follows a local maximum (ridge) instead of a local minimum (valley). This can be seen in figure 20 which is like figure 19 but with iso-energy contours superimposed. The curvature of the iso-energy contour indicates stability of the path . The paths going through the valleys are stable throughout. Those going to the peaks become unstable at a certain point.

Figure 21 shows the righting arms curves for the SDM and free twist method. When constructing these curves, a particular problem related to the SD method emerges which is that at the equilibrium (starting) point the rotation is not yet defined since the rotation is the sum of the rotation steps taken. This makes the starting point ill defined in terms of heel angle. To overcome this, a pragmatic approach is taken by setting the starting rotation angle equal to the maximum inclination of the initially vertical axis. But this defies logic the more so as for most of the cases, the inclination of an initially vertical axis does not increase steadily.

For some cases, the inclination axis first decreases after which it decreases but in the opposite direction. This is shown in figure 22 where the righting arms are plotted as a function of the inclination. Note that the righting arms are still those of figure 21 which is not correct as we now have only transformed the x-axis data without transforming the arm data. Performing a transformation where both rotation angle and SDM righting arm are converted to inclination and righting arm results in figure 23. The righting arms in this figure are based on the gain in energy for each inclination step. The peaks in the figure are due to a (SDM) rotation which results in a change in the *direction* of an initially vertical axis, whilst inclination itself is not or hardly changed.

9.2 Semi-submersible

Figure 24 shows a semi-submersible which at the given displacement has an intact draft of 9.106 m. Both the intact and damage condition pose a challenge when calculating the stability.

Intact. Figure 25 shows the energy surface plot for the intact condition. Note that because of symmetry, the data for an axis direction of α is also found for axis direction α + 180. It is seen that for moderate heel angles, the minimum energy is found for an axis direction of 180 deg (and 0 deg). This is even better seen in the righting arm plot, figure 26.

When for 180 deg axis direction, heel is increased beyond 25.5 deg, the path follows a local unstable maximum instead of a local stable minimum. Instead, for larger heel angles, the minimum energy is found for axis directions of about 135 and 225 deg. A more detailed analysis can be done by looking at the twist moment depending on heel and axis direction. Figure 27 shows this as a surface plot of x_{cob} versus heel and axis direction. The locations where $x_{cob}=0$ are those satisfying free twist. The combination of heel and axis directions for which the twist moment (or x_{cob}) is zero are identified by the change in color from green to blue in figure 27. The stable and unstable positions can be identified by looking at the slope $dx_{cob}/ d\psi$ for a given heel angle. The stable and unstable paths for which there is an extreme in the energy build up are as given in figure 28 for axis directions between 120 and 240 degrees, see also figure 29 where these paths are plotted in the energy plot.

Starting with zero heel, a gradual increase in the overturning moment will result in the rig to follow a seemingly erratic path:

heel	axis direction
0 – 1.6	180 deg
1.6 – 4.6	151-154 deg or 209 – 206 deg
4.7 – 25.4	180 deg
25.5 –50	133-138 deg or 227 – 222 deg

At hindsight, the unstable area for very small heel angles is also seen in figure 26 as a barely noticeable hill for axis 180 and heel 4 deg.

Damaged. When damaging the rig by removing the pontoon corner part as indicated in figure 24, the energy surface looks simpler than for the intact rig. The axis direction for minimum energy increase is around 140 deg, see figure 30. Still, also in this case, the axis direction changes considerably for increasing heel as shown in figures 31 and 32

When actually increasing the heel angle in a stepwise manner, the rig will suddenly change position when passing the 12 and 23 degrees heel, see figures 31. At 12 degrees heel, it will change from axis direction about 140 deg to about 166 degree. At 23 degree heel it will change back to about 130 deg axis direction. Further study showed that this behavior hardly depends on the VCG.

10. USE OF MINIMUM ENERGY PATH IN EVALUATING STABILITY CRITERIA

When evaluating stability criteria, a distinction can be made between those without and with external influence. An example of the first group is ABS's range of stability criterion for jack-ups. An example of the second group is the well known requirement on ratio between the area under the restoring moment and the wind overturning moment curves. For the first group, the minimum energy path can be followed as being the most critical, i.e. it requires the least effort to reach a particular heel angle.

For the second group, both the magnitude of the restoring moment and of the overturning moment is to be considered. This highly complicates the task of selecting the most critical heeling axis direction. When following the minimum energy path, it is relatively easy to adapt the wind overturning moment to the instantaneous axis direction. But, this still assumes that the direction of the wind overturning moment follows that of the restoring moment. In general, it is well possible that this is not the case *and* that the wind overturning moment has a trimming component as well. But, this raises the question if one should absorb this by letting the unit trim or if the axis direction should be modified. It is also possible that the wind overturning moment for other directions is higher and thus more governing.

Traditionally, only the overturning effect of the wind is considered. Any trimming or twisting moment is ignored. This means that the positions for zero trim or zero twist moment remain the same. Also the wind direction is usually taken in line with the heeling direction; the wind heeling axis being the same as the critical heeling axis.

A possible way to include the wind in the determination of the critical axis is to subtract the energy input due to wind for a fixed axis direction from the plot where energy is given depending on axis direction and heel angle (like fig 10). This would result in a deformed energy plot. The critical (varying) axis direction is found by following the lowest valley, similar to the case without wind.

When including possible downflooding, the complexity of the calculation increases further.

Apart from the calculation difficulties, the major issue is the nature of the calculation. When evaluating criteria, the effect of wind for an otherwise calm sea is looked at. The effect of waves and resulting motions can be important, especially for a damaged structure [6]. Also, the calculation of the wind overturning effect is usually simplified in that the reactive force is assumed to work at the lateral center of resistance and that the forces due wind and the reactive forces do not introduce a yawing moment.

Including the effect of mooring or DP makes the analysis even more complex. In view of this, there seems to be no justification to focus on one detail whilst ignoring other effects which may be much more important.

In the end, we should not forget that the criteria are quite abstract. Being abstract, they lend themselves to consistency and reproducibility. The end results are not so much influence by the details of the method used. As such, they fulfill the requirement of a criterion of which the evaluation is clearly defined and can be reproduced independently in a quick manner.

Today's calculation power allows for refinements which were not possible when the criteria were developed. Refinements in the calculation (just because we can) without rethinking the criteria themselves do not make much sense.

By Couser [7] the difficulty in programming the calculation of the AVCG curves as limited by the various requirements was mentioned. The examples shown here indicate that apart from the vagueness in the criteria, the structure itself can cause problems. For the semi-submersible case at transit draft it is almost impossible to do a proper AVCG calculation. A possible way out is to use a fixed axis direction with nil trim. For a righting arm curve constructed in this way to be acceptable, it should show only modest trimming moments as this indicates that the energy depletion by letting it free to twist or trim is small.

11. CONCLUSIONS

1. Extending the free trim approach as used for ships to offshore units, in combination with a varying heeling axis direction, may lead to severe interpretation problems.

2. A free trim or free twist approach is sensible as this generally leads to the slowest build up of potential energy for increasing heel angles. Thus the lowest righting arm curve is achieved.

3. For the evaluation of stability criteria, the rig should be stable in trim or twist up to the maximum heel angle needed for the evaluation of a particular criterion. Preferably the rig should be stable up to the capsize point.

4. The free twist approach with zero trim leads to the lowest gain in potential energy during heeling. This is based on both theory and data obtained from direct calculations. Thus when left free to trim and free to twist, the result is that the twist angle varies and that the trim angle remains zero.

5. Application of the lowest gain in energy approach is an unambiguous way to define the most critical or weakest axis.

6. For the determination of the critical axis direction, other effects are to be considered as well. When wind overturning is in the criterion, axes other than the weakest based on energy, may have to be considered. Also when looking at openings, other axis directions have to be looked at. For these complex cases, it is strongly suggested to perform the calculations for *any* axis direction and with the trim fixed at zero.

7. The Steepest descent method has a number of disadvantages which makes it less suitable for evaluation of stability criteria. Among these, being non deterministic, aiming at the maximizing the energy increase and exploring unrealistic inclination paths are the most important.

8. Class regulations should consider energy build up as important tool in identifying critical axis directions and they should give considerations to stability in trim or twist during the heeling process.

12. REFERENCES

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13. NOMENCLATURE

Δ	=	displacement (t)
μ	=	direction of a small rotation
φ	=	heel angle
ψ	=	axis direction (twist angle)
θ	=	trim angle
C_{xy}	=	cross moment of the waterline plane for rotation around its centre $\approx\int x.y.dA$
CoB	=	Centre of Buoyancy
CoG	=	Centre of gravity
E	=	potential energy divided by displacement
GM_L	=	metacentric height for trim at a particular heel angle
GM_T	=	metacentric height for heel at a particular heel angle
M_x	=	moment around the longitudinal horizontal axis divided by displacement
M_y	=	moment around the transverse horizontal y axis divided by displacement
R	=	small rotation around the restoring moment vector
VCB	=	vertical centre of buoyancy above the centre of gravity
x	=	horizontal axis along the initial heeling axis, forward positive
x_{cob}, y_{cob}	=	location centre of buoyancy relative to CoG
y	=	horizontal axis perpendicular to the x axis (right handed system)

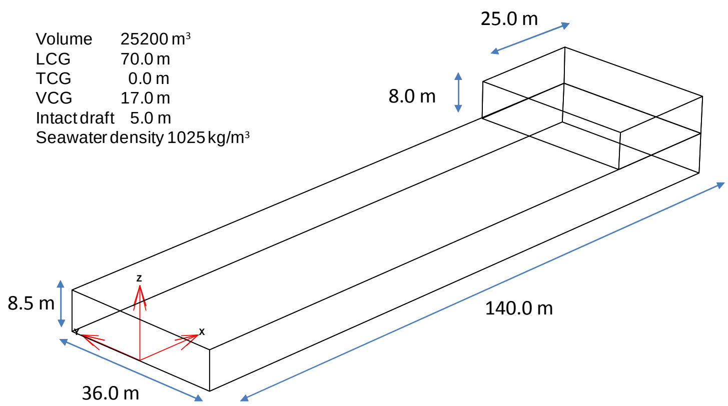


Figure 1 Barge dimensions

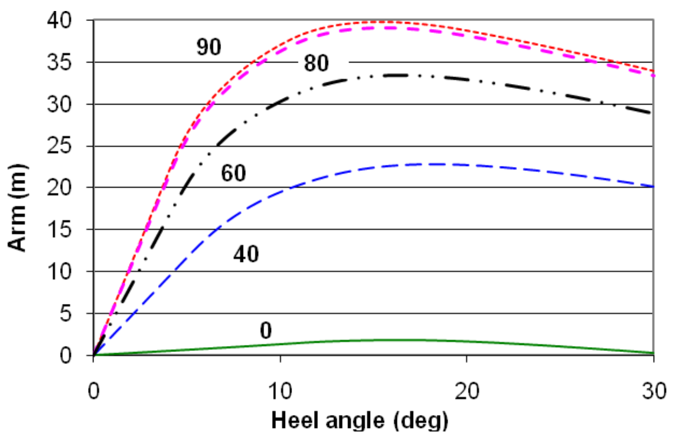


Figure 2 Righting arm curves, no trim

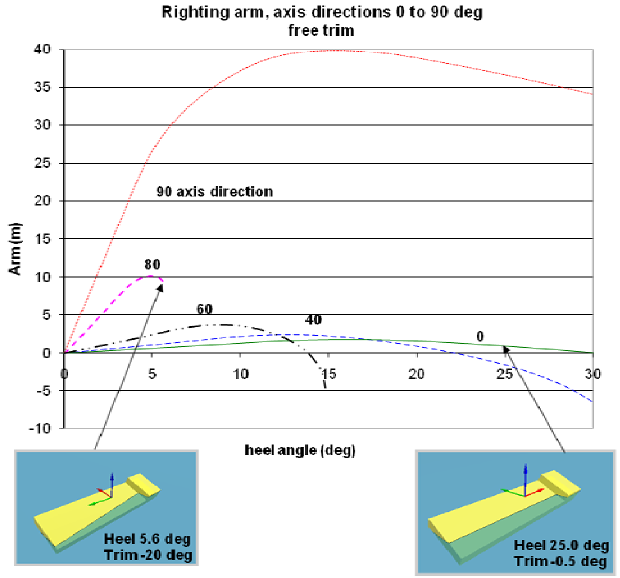


Figure 3 Righting arm curves, free trim

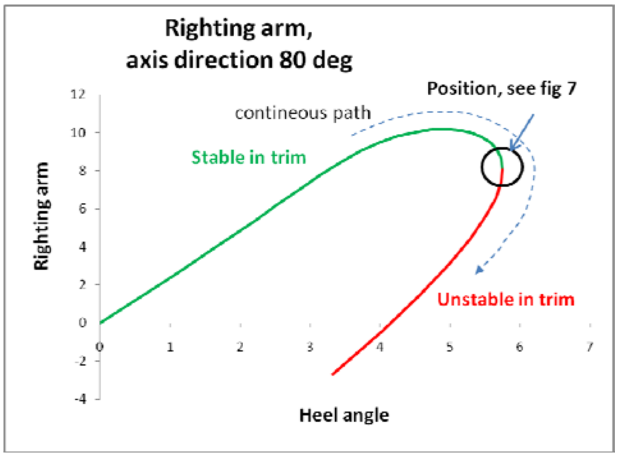


Figure 5 Continuous righting arm curve, free trim

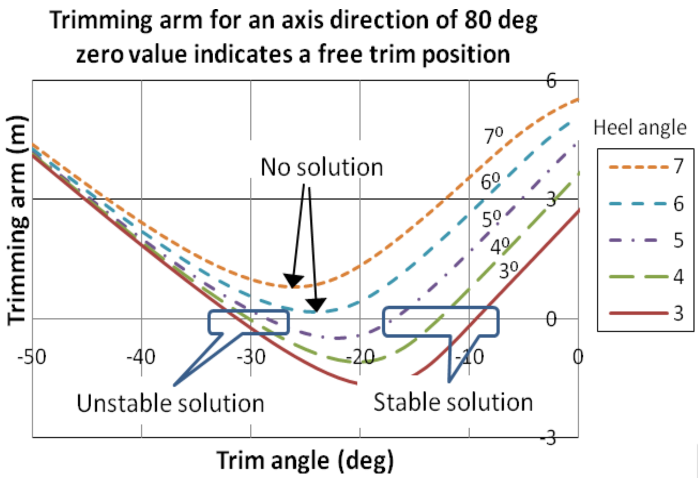


Figure 4 Trimming arm curves

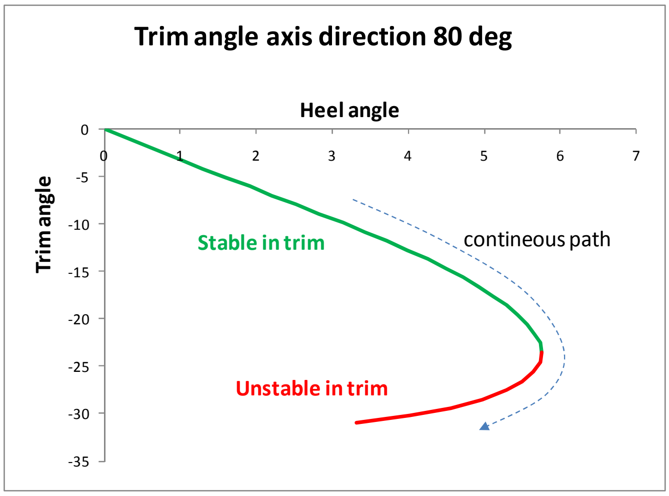


Figure 6 Trim angle for continuous heel

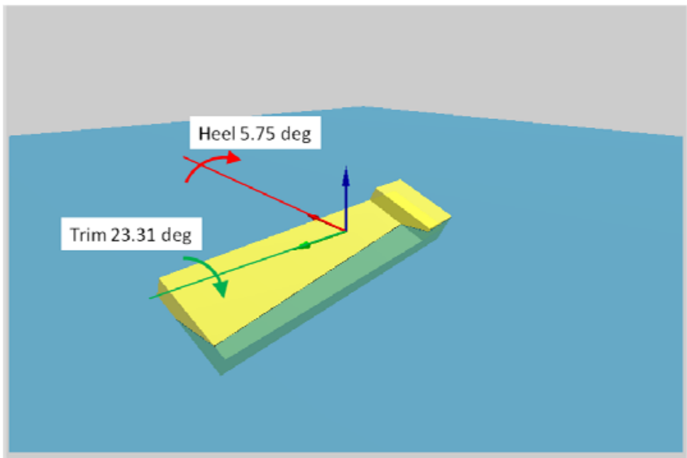


Figure 7 position at maximum heel angle, axis direction 80 degrees

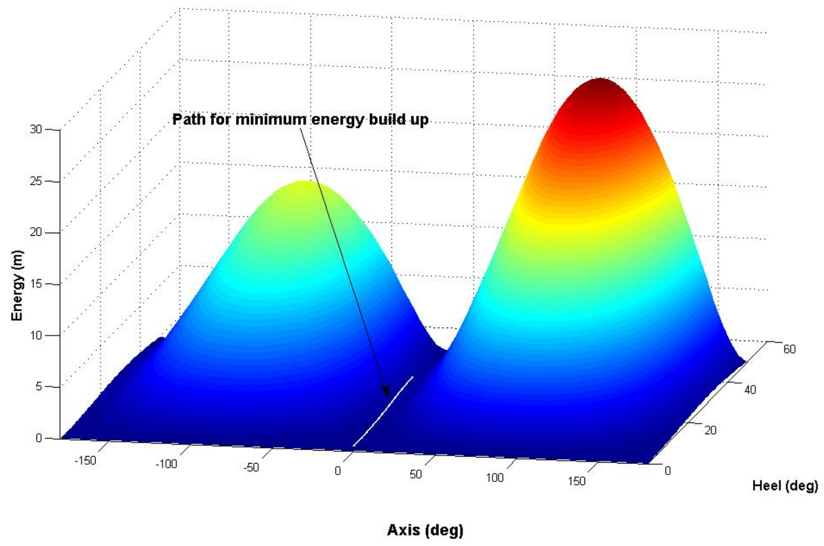


Figure 8 Energy surface for the barge

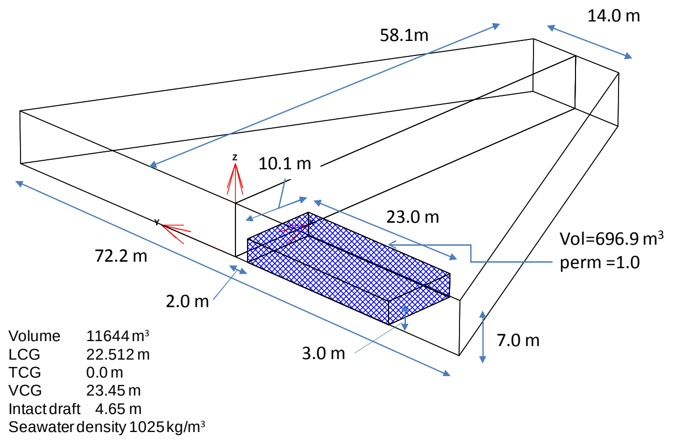


Figure 9 Dimensions of the studied jack-up

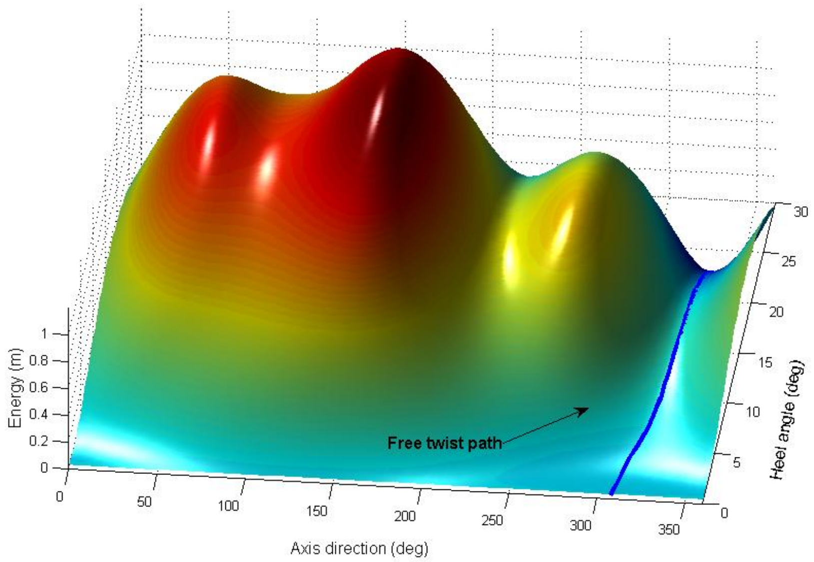


Figure 10 Energy surface for the jack-up, damaged condition

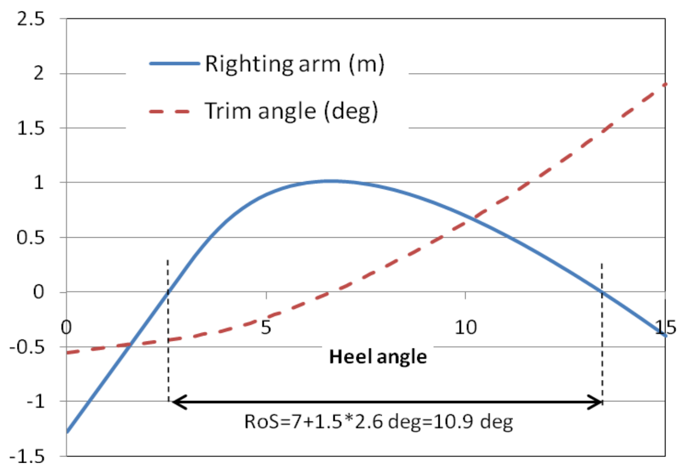


Figure 11 Righting arm curve jack-up damaged, axis direction 320 deg

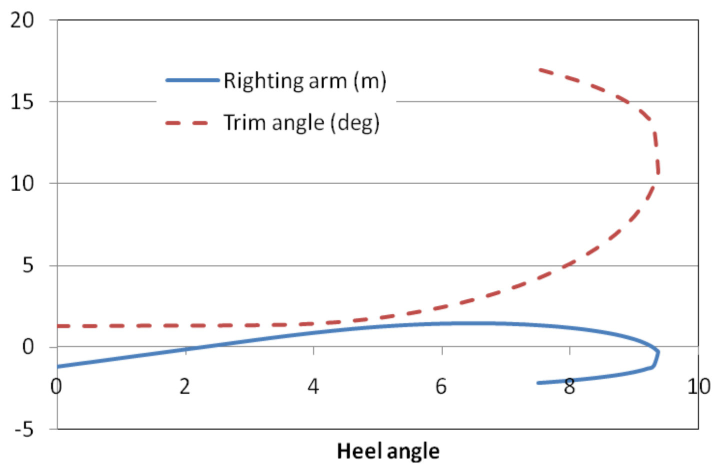


Figure 12 Righting arm curve jack-up damaged, axis direction 280 deg

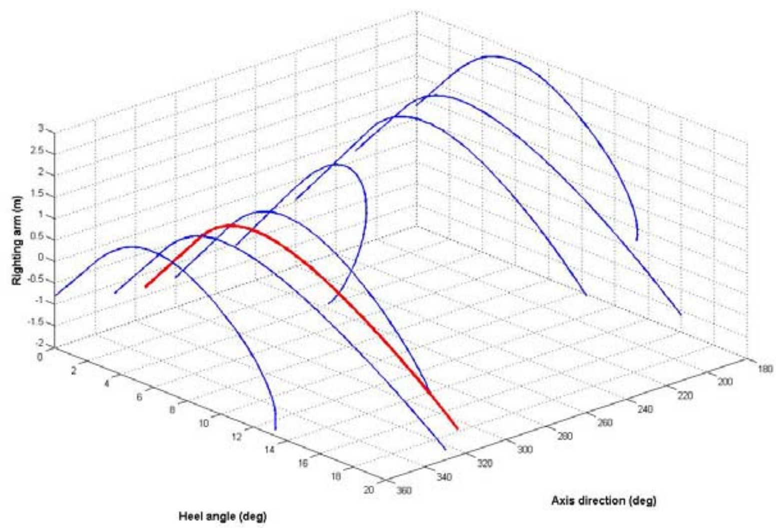


Figure 13 Righting arm curve damaged jack-up, axis direction 280 deg, free trim

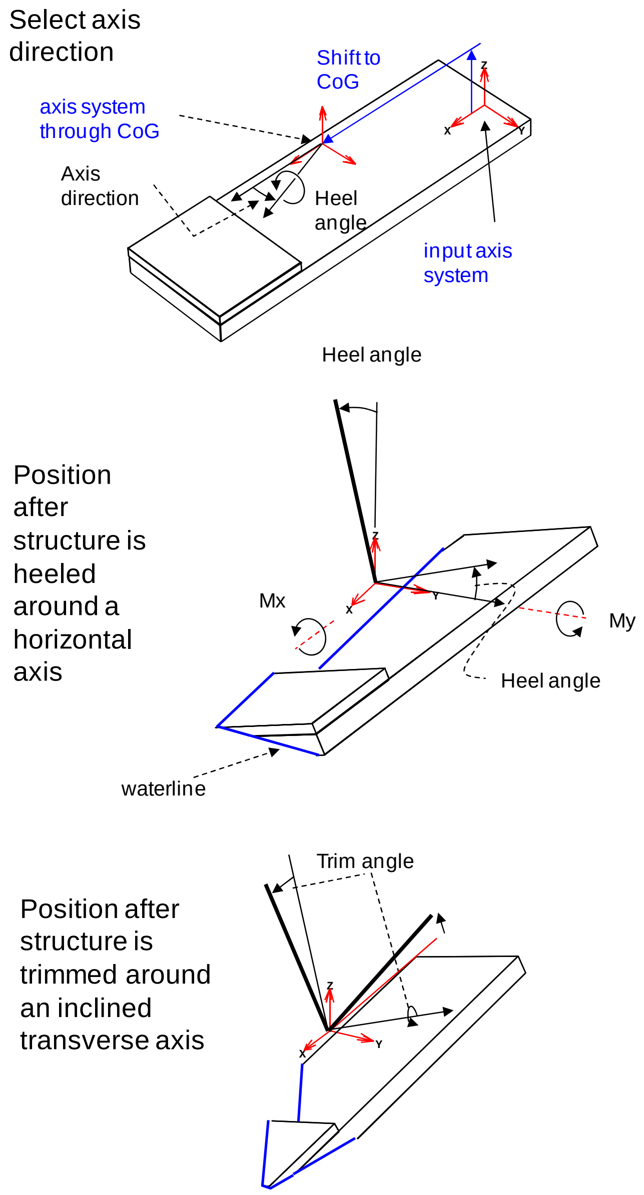


Figure 14 Application of twist, heel and trim

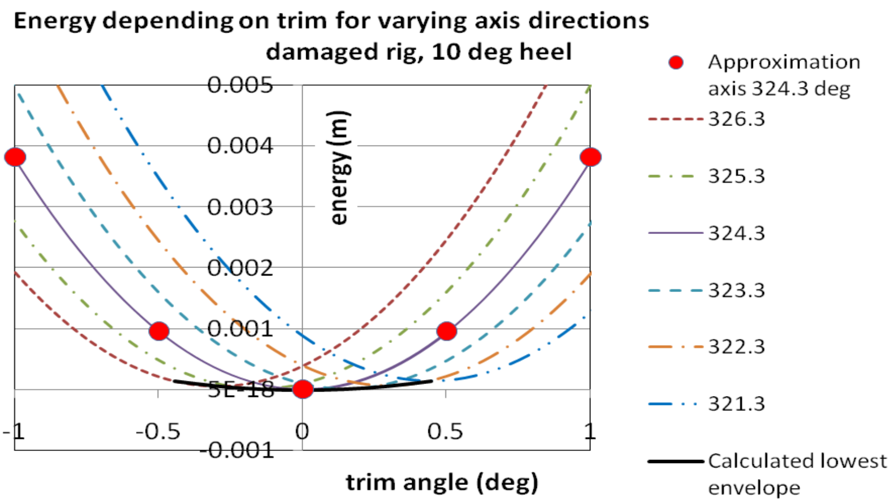


Figure 15 Energy depending on trim

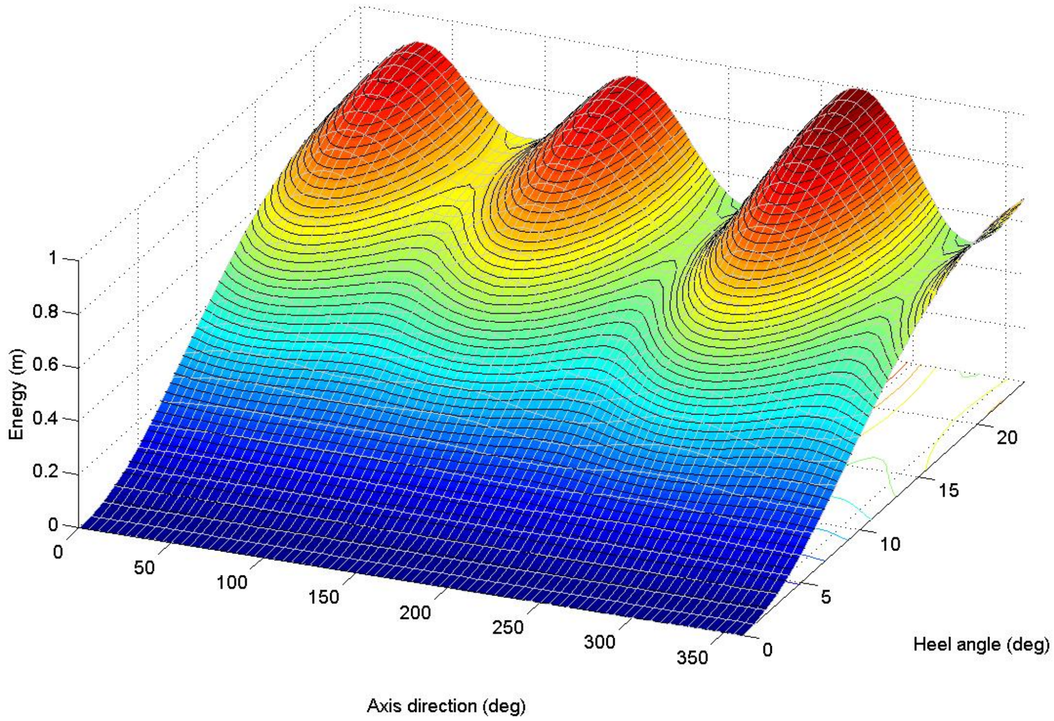


Figure 16 Energy surface jack up intact

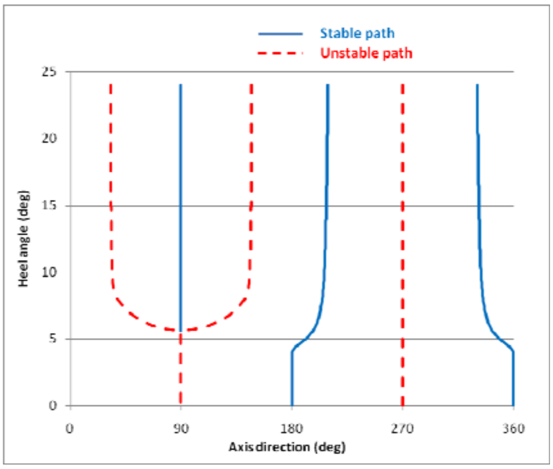


Figure 17 Axis directions for minimum energy build up jack-up intact

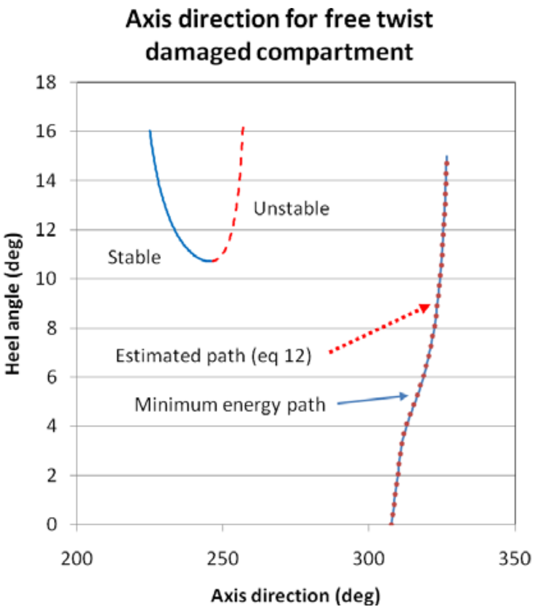
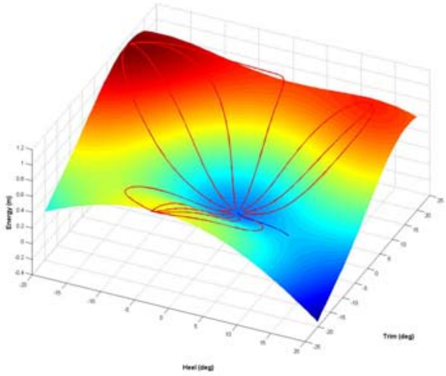


Figure 18 Axis directions for minimum energy build up, jack-up damaged



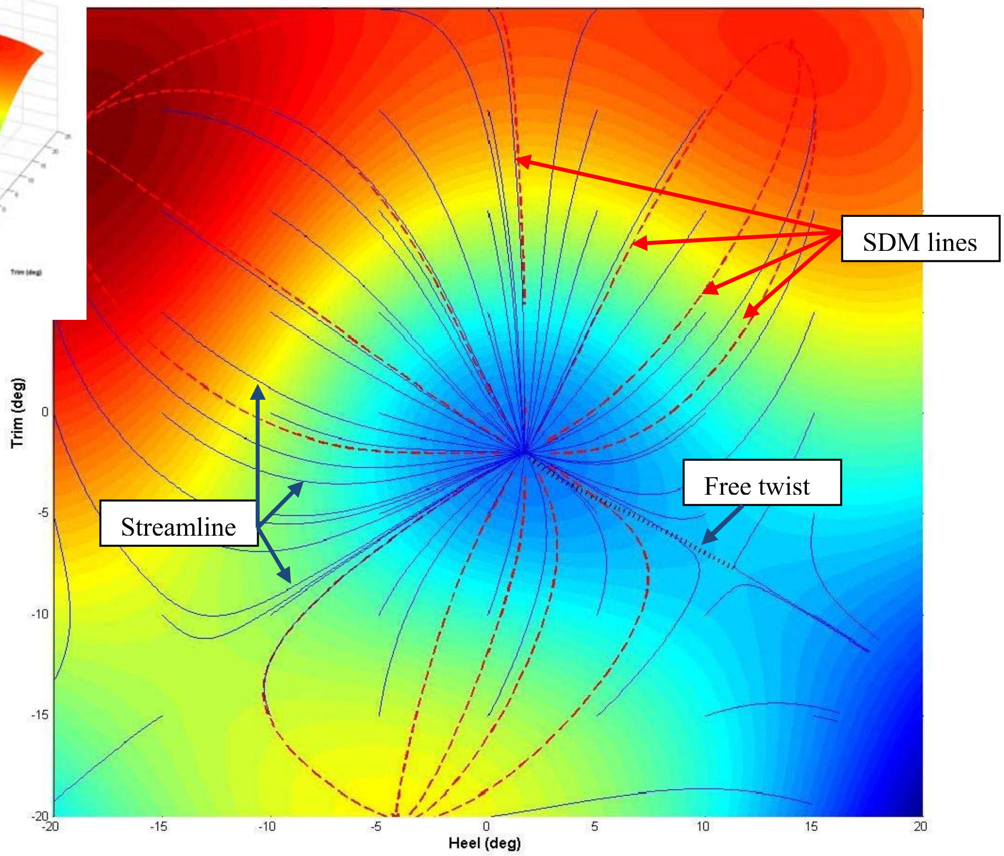


Figure 19 Stream lines, actual paths and free twist path, jack-up damaged, heel is around the x axis, trim around the inclined y axis.

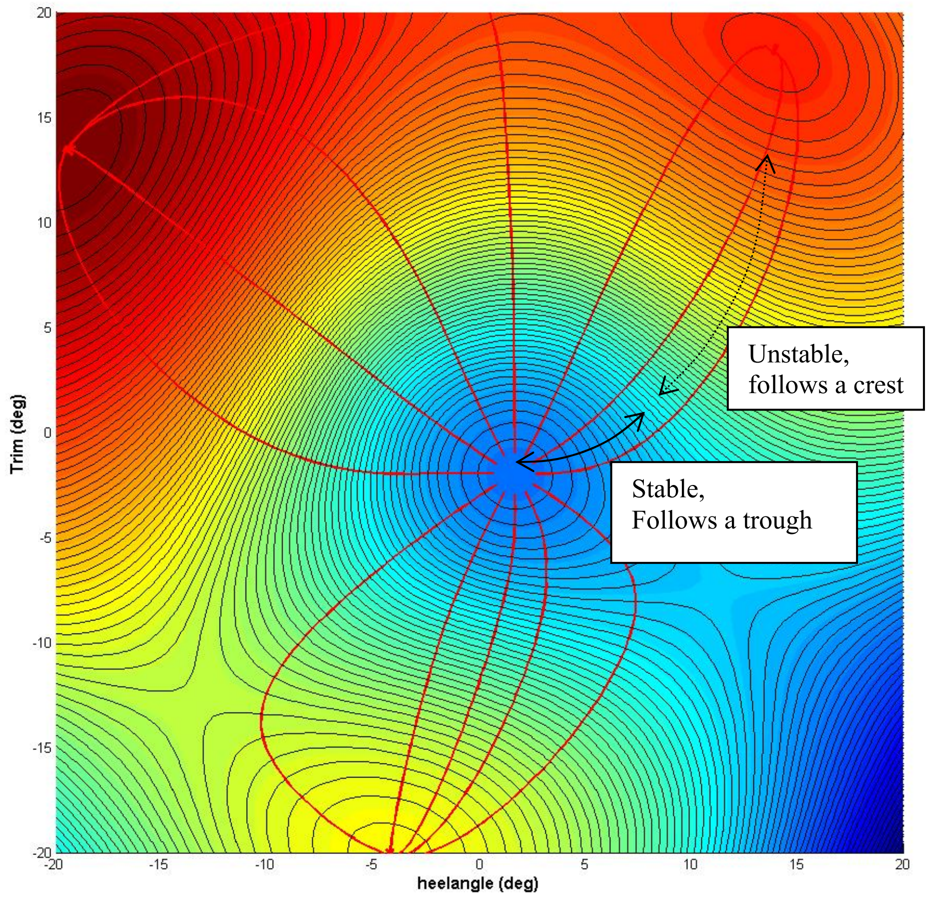


Figure 20 Iso energy contours of figure 18, indicating stability of the position

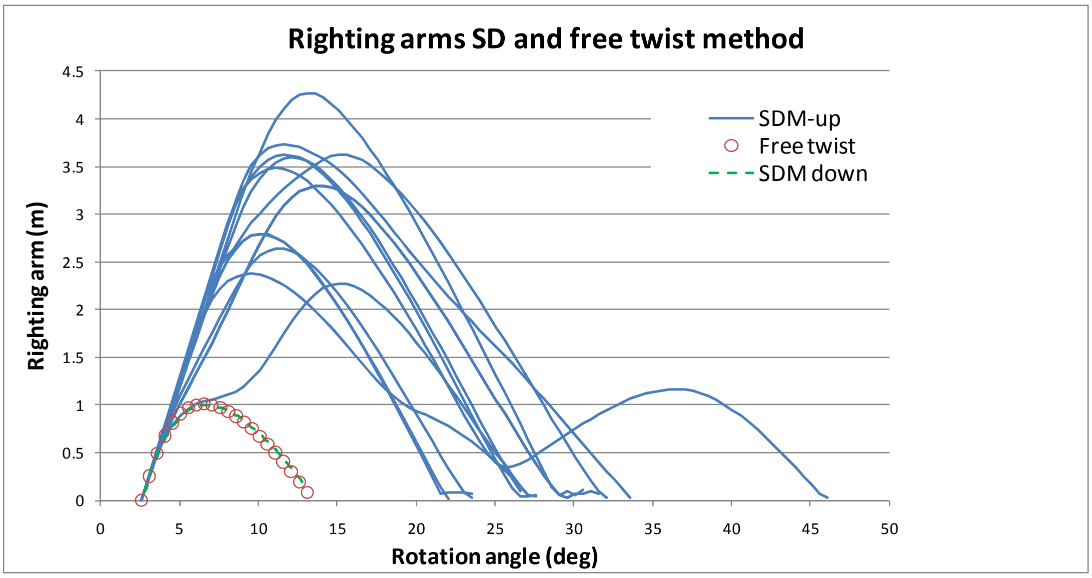


Figure 21 Righting arm curves, SDM and free twist

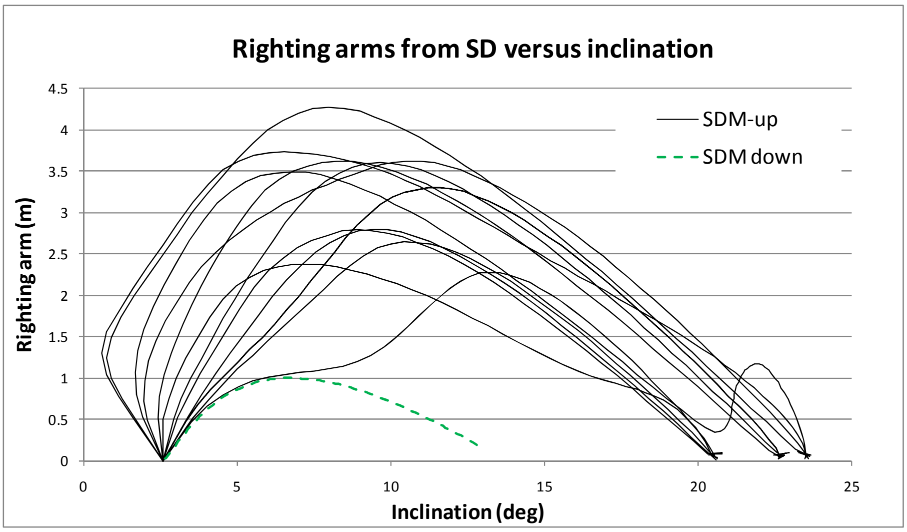


Figure 22 Righting arm curves, SDM and free twist

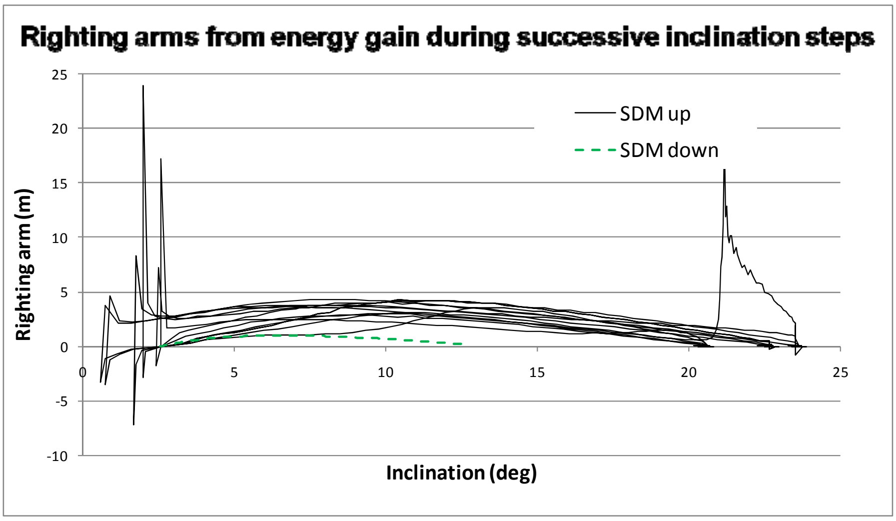


Figure 23 Righting arm curves, SDM and free twist

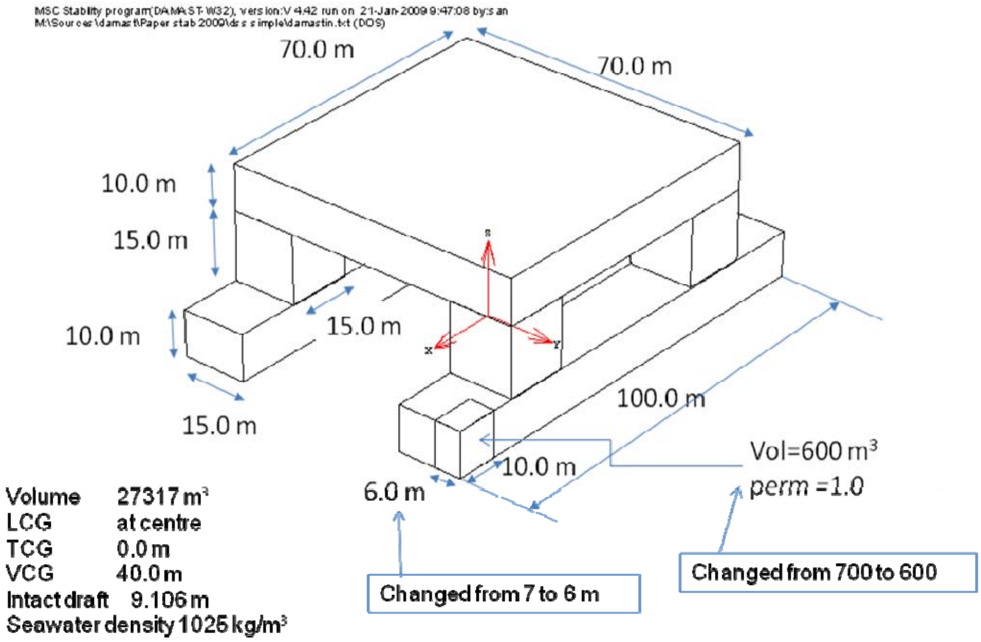


Figure 24 Shape of the semi submersible

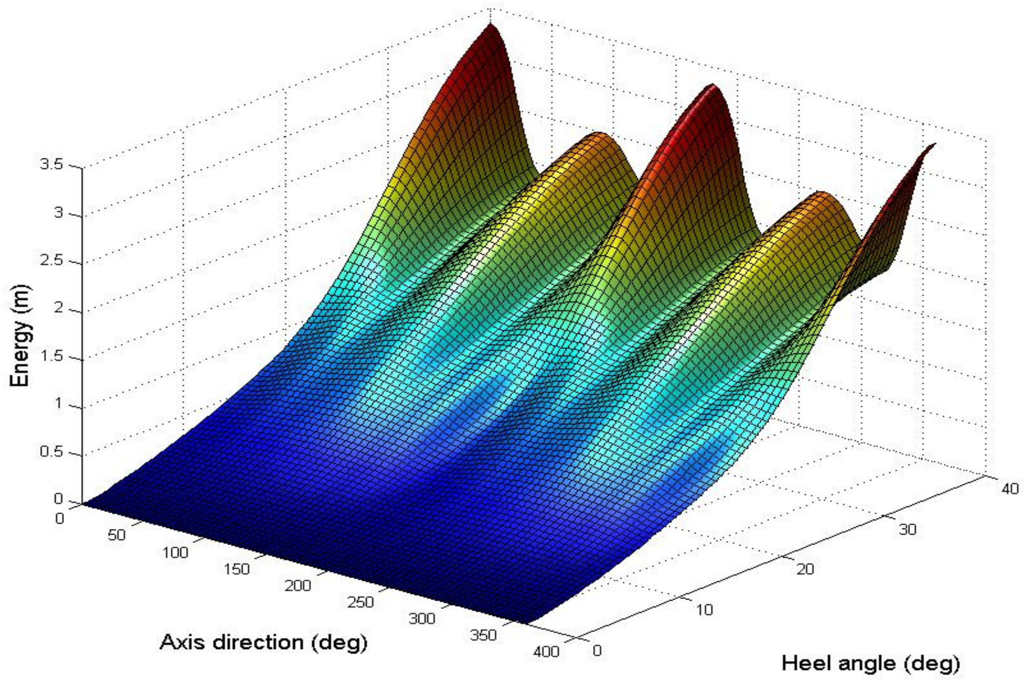


Figure 25 Energy surface semi submersible intact

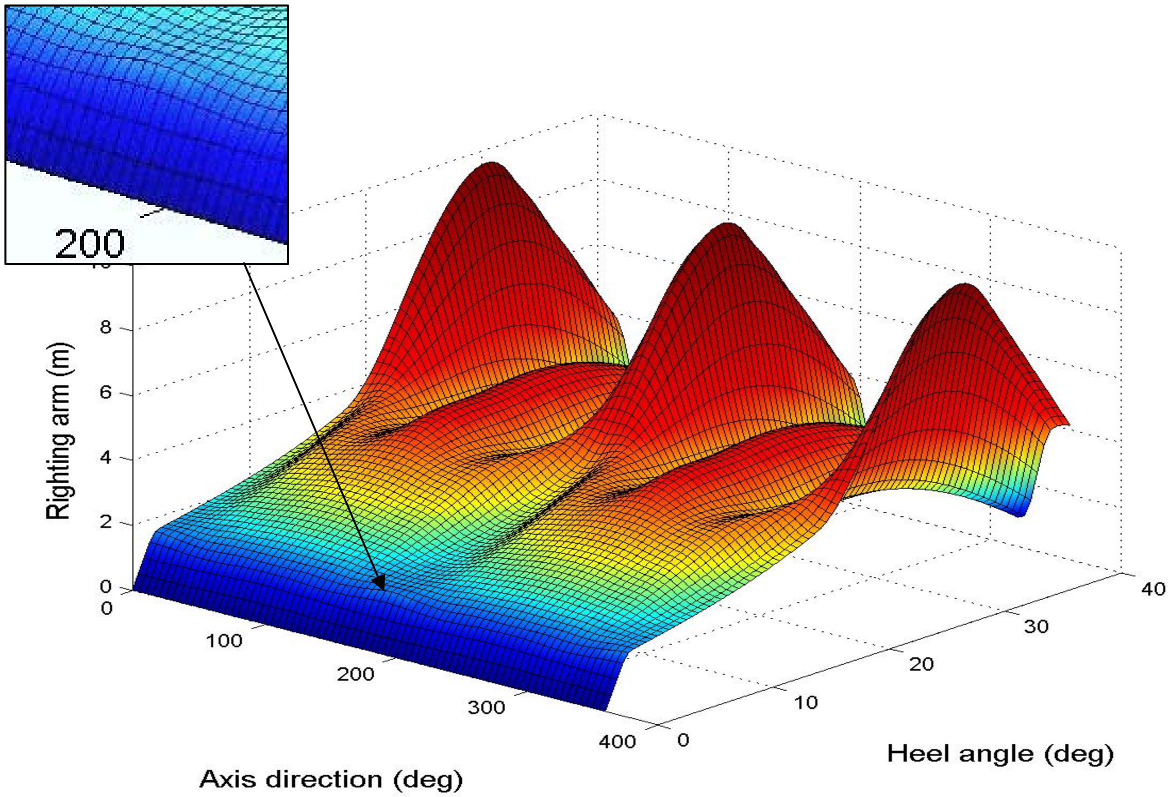


Figure 26 Surface plot of the righting arms

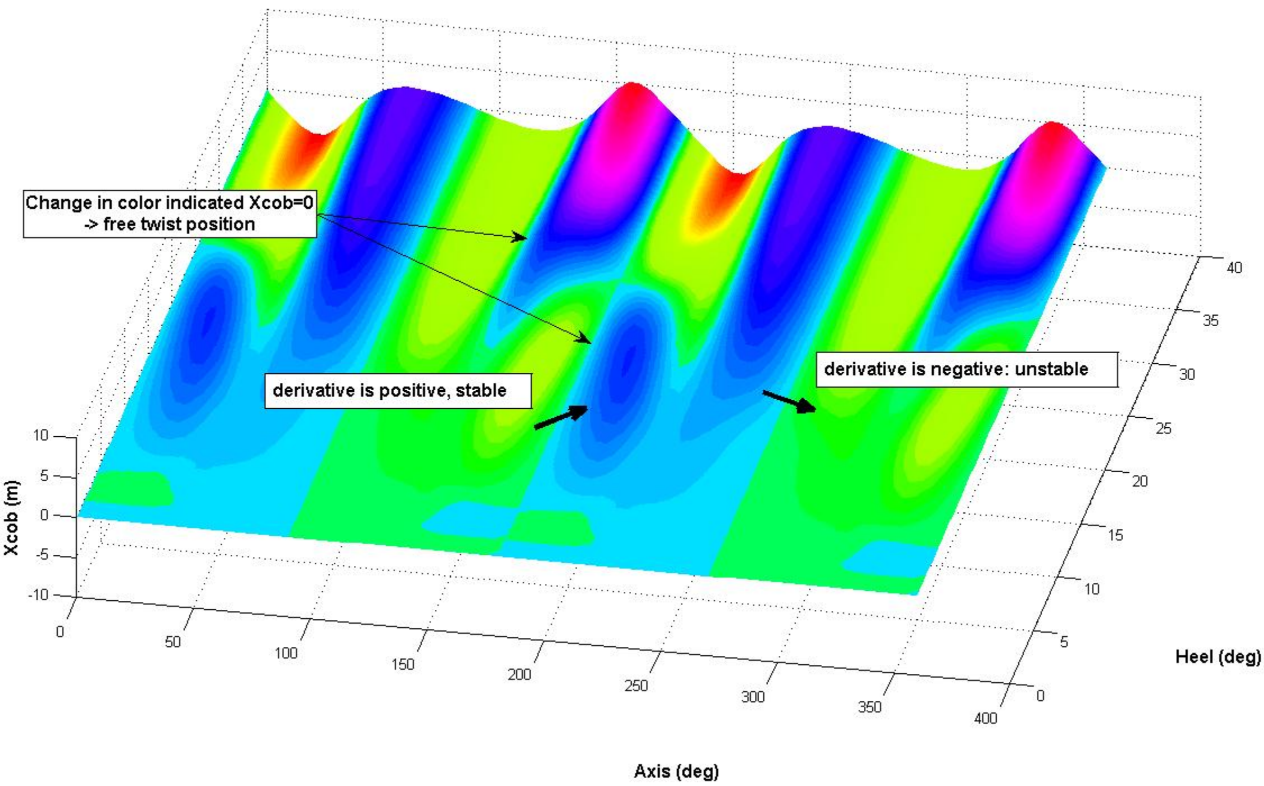


Figure 27 Surface plot of the trimming arms

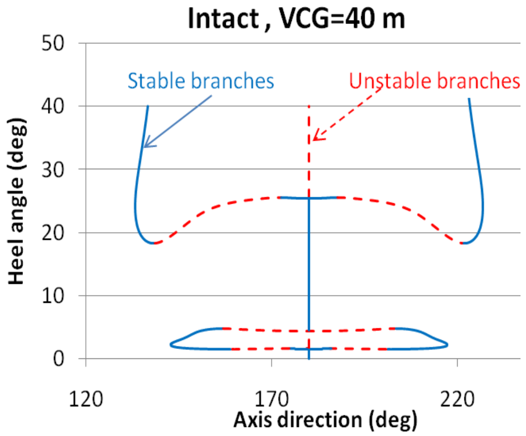


Figure 28 Stable and unstable paths

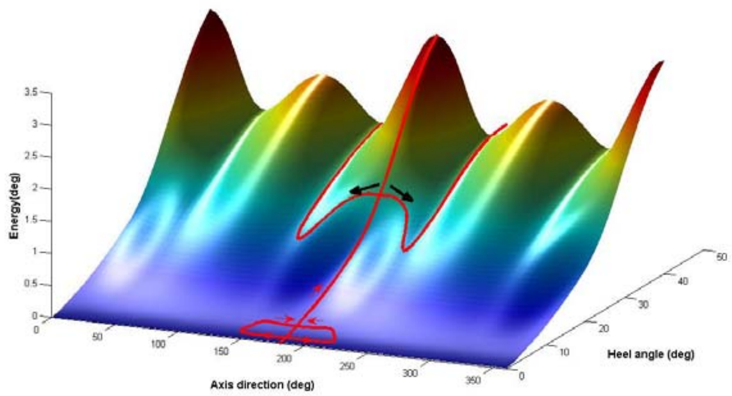


Figure 29 Energy plot for a semi submersible in transit

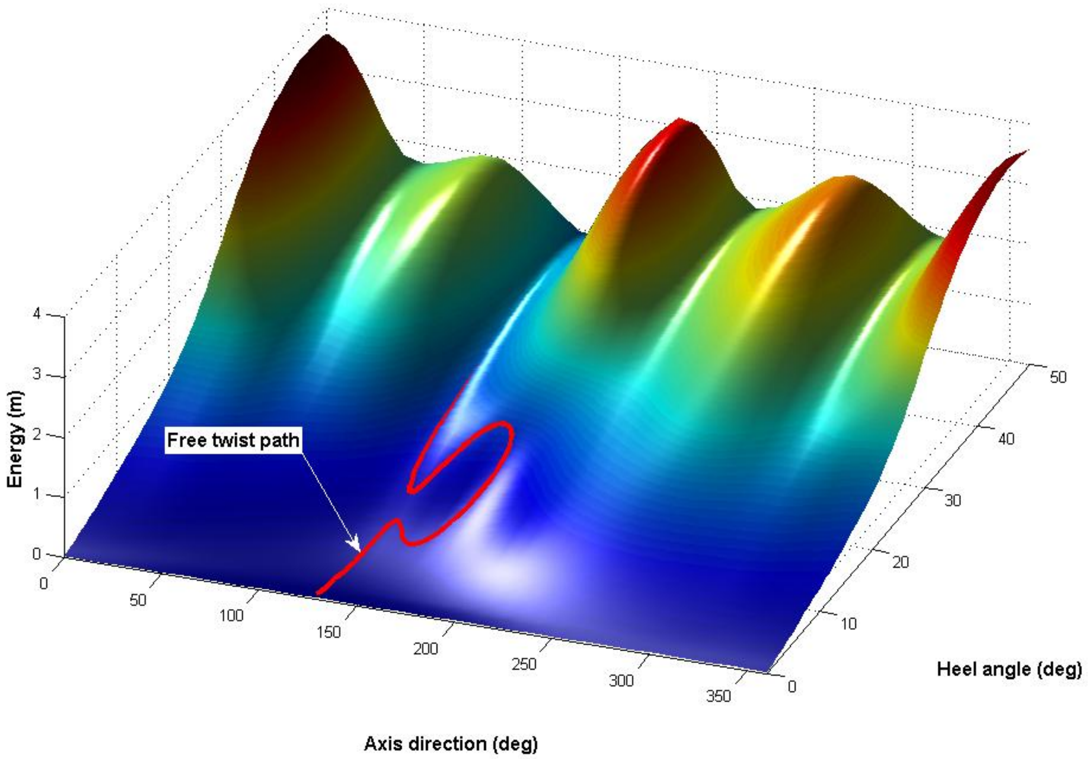


Figure 30 Energy plot damaged

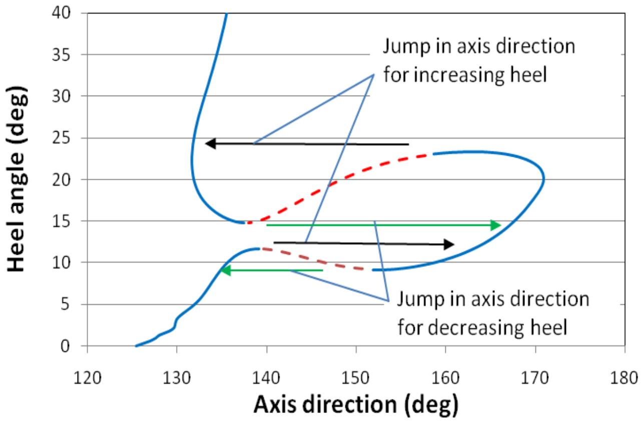


Figure 31 Relation between heel angle and axis direction, free twist

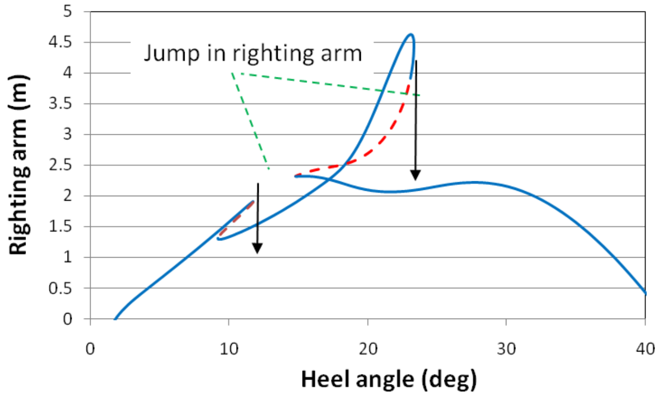


Figure 32 Relation between heel angle and axis direction, free twist

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