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A Method to Analyse Seakeeping Model Measurements in Time-Domain

K. Garme and J. Hua
Royal Institute of Technology, KTH
Stockholm, Sweden

ABSTRACT

The paper introduces a method to determine a wave equation from a measured wave system. The method is applied to wave measurements recorded in a seakeeping experiment. The obtained wave equations accurately repeat the wave gauge signals. The wave equation provides a link between a numerical and physical model test. This is verified for heave and pitch motions of the ship model in regular and irregular waves. Time domain simulation with the wave equation and initial conditions from the model test, gives motion time series in good agreement with the measurements.

KEY WORDS: Seakeeping, wave measurement, wave equation, ship motion

INTRODUCTION

Today, the linear seakeeping theory is well established for analysis of the linear dynamic responses of conventional ships in waves, such as wave-induced heave, pitch, sway and yaw motions and global wave loads. Numerical models for ship motions in waves, based on the linear strip approach, are frequently used design tools offering an alternative to model tests. However, the linear seakeeping theory is inadequate for analysing transient problems like slamming impact or dynamic instability –non-linear problems often associated with new ship concepts, high-speed ships or ships of unconventional hull shape.

The irregular character of ocean waves, and consequently of the ship responses, makes it practically impossible to determine the probabilistic characteristics of the non-linear dynamic problems from model measurements alone. The impressive test programme by Kan et al. (1993) illustrates this well. They did 1643 runs with two models in regular quartering waves to analyse capsizing mechanisms. Despite the large number of tests, however, risk assessment in irregular waves was left undone.

Model measurement together with mathematical modelling is a generally accepted approach to studying transient or time-dependent dynamics of ships in waves. Typically in time-consuming risk assessment, model experiments are often used to verify a mathematical model, which in turn will be used to calculate the risk probability.

However, comparing calculations and model measurements is not an easy task. In Salvesen et al. (1974), the linear strip theory is verified by comparison with model measurements, although some cases show large discrepancies between the theoretical calculations and the measurements. Such discrepancy could in general be due to the experimental accuracy, the theoretical model, or both. Another possible reason is the difficulty of connecting measured wave elevation and measured response. It is impossible to measure the incident wave system at the exact position of the hull and it is not obvious from a wave height measurement elsewhere what the waves exciting the responses looked like. In a regular wave there is a phase shift between a wave measurement at a point following the model and a point on the hull. In irregular waves the wave profile at the hull and at a measurement point will never look the same. In the frequency-domain this is of minor importance; in the time-domain, on the other hand, the wave-hull intersection is essential for the analysis. To enable comparison between measurements and time-domain simulation, the numerical model must expose the numerical ship to the same waves as the model in the test basin was exposed to.

This paper introduces a method to determine a wave equation, from measurements of a wave system, describing wave elevation and wave kinematics. The method is applied to wave measurements recorded in the seakeeping experiments on a ro-ro vessel, (Garme, 1997). The obtained wave equations accurately repeat the wave gauge signals. The wave equation as a link between a numerical and physical model is verified by comparing measured and simulated heave and pitch motions of the ro-ro model in regular and irregular waves. The time simulations were performed by the SMS code, (Hua & Palmquist, 1995), with the wave equation and initial conditions from the model test. The calculated time series were in good agreement with the measurements. Finally, modification and application of the method are discussed.

WAVE EQUATION DETERMINATION

Gravity waves of limited amplitude and slope are readily expressed by a Stokes expansion. In the following, the first-order wave component will be determined. From this solution the higher-order Stokes components can be calculated and added to the solution. According to the theoretical study by Nestegård & Stokka (1995), the second-order component is the predominant non-linear wave component. The obvious refinement to the wave equation if the analysed wave system has a non-linear character is to add the second-order Stokes wave component. If the Stokes expansion is valid in the seakeeping analysis the wave height measurement from one wave height meter would be sufficient to give the wave elevation everywhere in the basin at every moment of the test.

In the first order wave equation, long crested waves are described by:

$$\zeta(x,y,t) = \sum_{n=1}^{N} a_n \cos(\omega_n \cdot t + k_n \cdot x) + \sum_{n=1}^{N} b_n \sin(\omega_n \cdot t + k_n \cdot x) \qquad (1)$$

Each sample of a measured wave-height time series, gives a ζ-value corresponding to a certain time instant, t, and position coordinates x and y. The wave height meter follows the model's plane motion and consequently, x and y are functions of time. If frequencies, ω_n, and wave numbers, k_n, are known, an equation for each sample leads to an over-determined system of equations. The coefficients a_n and b_n are then easily computed by a least squares method. For a time series of N samples described by n cosine and n sine functions the problem is formulated as:

$$\bar{\zeta} = A \cdot \begin{pmatrix} \bar{a} \\ \bar{b} \end{pmatrix} \qquad (2)$$

where

$$\bar{\zeta} = \begin{pmatrix} \zeta(x_1, y_1, t_1) \\ \zeta(x_2, y_2, t_2) \\ \cdot \\ \zeta(x_N, y_N, t_N) \end{pmatrix}$$

is an N x 1 vector of measurement values,

$$A = \begin{pmatrix} \cos(\omega_1 t_1 + k_1 x_1) & \cos(\omega_2 t_1 + k_2 x_1) & \cdots & \cos(\omega_n t_1 + k_n x_1) & \sin(\omega_1 t_1 + k_1 x_1) & \sin(\omega_2 t_1 + k_2 x_1) & \cdots & \sin(\omega_n t_1 + k_n x_1) \\ \cos(\omega_1 t_2 + k_1 x_2) & \cos(\omega_2 t_2 + k_2 x_2) & \cdots & \cos(\omega_n t_2 + k_n x_2) & \sin(\omega_1 t_2 + k_1 x_2) & \sin(\omega_2 t_2 + k_2 x_2) & \cdots & \sin(\omega_n t_2 + k_n x_2) \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \cos(\omega_1 t_N + k_1 x_N) & \cos(\omega_2 t_N + k_2 x_N) & \cdots & \cos(\omega_n t_N + k_n x_N) & \sin(\omega_1 t_N + k_1 x_N) & \vdots & \cdots & \sin(\omega_n t_N + k_n x_N) \end{pmatrix}$$

is an N x $2n$ matrix of the trigonometric functions and

$$\begin{pmatrix} \bar{a} \\ \bar{b} \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ \cdot \\ a_n \\ b_1 \\ b_2 \\ \cdot \\ b_n \end{pmatrix}$$

is a $2n$ x 1 vector of the unknown coefficients. Multiplying equation (2), from the left, by the transpose of matrix A, turns the over-determined system of equations into the corresponding least squares problem,

$$A^T \bar{\zeta} = A^T A \begin{pmatrix} \bar{a} \\ \bar{b} \end{pmatrix} \qquad (3)$$

The method to solve the system (3) depends on its condition. If the system is well conditioned, Gaussian elimination is applicable; otherwise, a more sophisticated method is needed. In the next section where the method is applied, the system was sufficiently solved by Gaussian elimination.

The frequencies and wave numbers might be determined from an analysis of a wave height meter fixed in the basin, the wave generator motion or simply from the known input parameters to the wave generator.

COMPARISON OF CALCULATED AND MEASURED WAVES

The method of wave equation determination was developed in connection with the seakeeping model test series, (Garme, 1997), where the objective was to perform motion measurements tailored for comparison to calculations in the time-domain. The waves in the wave basin were measured by means of two wave height meters, both following the model at a distance where slight influence from model-created waves was expected. Tests were performed in regular, irregular, and crossing waves. Crossing refers to wave systems propagating towards each other, intersecting at right angles. Each irregular wave was represented by three regular wave components with frequencies chosen in the vicinity of model heave, roll and pitch natural frequencies. The three-component wave is justified by its simplicity. The sea environment is irregular and still assumed possible to express analytically from a single wave-height measurement. With the waves in an analytical form a numerical ship could be exposed to the same waves as the model during the test and the calculated and measured time-series could be compared.

Wave equations were calculated for all wave systems with results in close agreement with measurements. Long crested waves are described by equation (1) and crossing waves by equation (4).

$$\zeta(x,y,t) = \sum_{n=1}^{N} a_n \cos(\omega_{xn} \cdot t + k_{xn} \cdot x) + \sum_{n=1}^{N} b_n \sin(\omega_{xn} \cdot t + k_{xn} \cdot x) + \sum_{m=1}^{M} c_m \cos(\omega_{ym} \cdot t + k_{ym} \cdot y) + \sum_{m=1}^{M} d_m \sin(\omega_{ym} \cdot t + k_{ym} \cdot y) \qquad (4)$$

Wave equations were determined with data from one wave height meter at a time. The equations could accurately recreate not only the measured signal they were based upon, but also, generally, the measurements from the other wave height meter, see Fig. 2, 4, 5 and 6. In a few cases the wave equation based on information from one of the measurement points did not sufficiently repeat the measurement recorded at the other, see Fig. 3. Model generated waves are believed to be the explanation, despite the ambition to put the wave height meters where they were not influenced by the model. No attempt has been made to prove this but the explanation is supported by considering test no.30 and no.32 of Garme (1997), which are tests in regular head seas with wave height meters principally located as in Fig. 1.

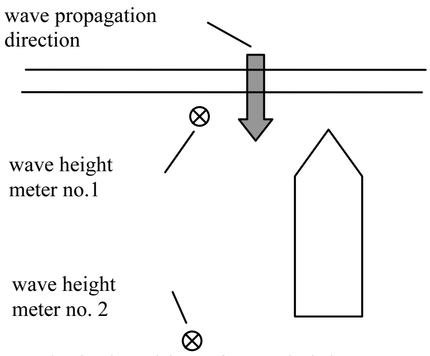

Fig. 1 Principal position of wave height meters.

In test no. 30 the model was heading at a speed of 1.4 m/s. The waves were close to harmonic, characterised by a frequency of 4.4 rad/s and an amplitude of 0.03 m. A wave equation based on the fore wave height meter accurately repeated both wave height recordings, see Fig. 2. In test 32 the model speed was less, 0.96 m/s, wave frequency 3.25 rad/s and amplitude 0.087 m. From Fig. 3 it is seen that a wave equation based on the fore gauge underestimates the aft signal. The differences in model speed, wave frequency and amplitude explain why the aft wave height meter was influenced by the model-created waves in test 32 but not in test 30. In test 32 the frequency of encounter is in the vicinity of pitch and heave natural frequencies and close to twice the natural frequency of roll motion. This in combination with larger wave amplitude results in major motion responses (see Table 1), and consequently the wave generation becomes much greater in test 32 than in test 30. The lower speed in test 32 also allows the radiated waves time to reach the aft wave height gauge.

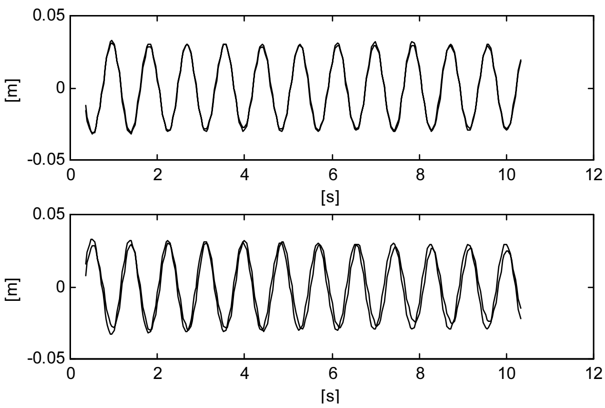

Fig. 2 Test no. 30. Regular long-crested waves measured by two wave height meters following the model in head seas. The wave equation coefficients are determined from the measurement in the upper graph.

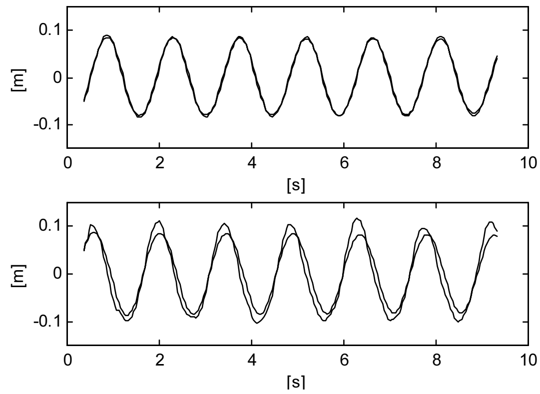

Fig. 3 Test no. 32. Regular long-crested waves measured by two wave height meters following the model in head seas. The wave equation coefficients are determined from the measurement in the upper graph.

Table 1 Response amplitudes calculated as $\sqrt{2}$ times the standard deviation of the measurement.

Test no.	#30	#32
Model speed [m/s]	1.4	0.9
Heave amplitude [m]	0.006	0.057
Pitch amplitude [deg]	0.361	5.1
Roll amplitude [deg]	0.161	3.7

Fig. 4 and 5 show examples of measured and corresponding calculated long-crested three-component irregular waves and Fig. 6 on crossing three-component waves. Each figure contains two graphs, one for each measurement point. The wave equation is based on the fore wave height meter.

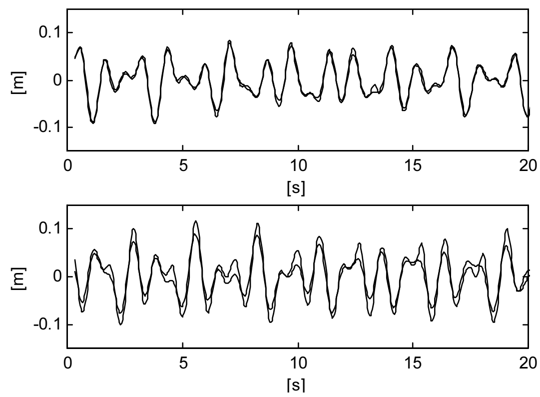

Fig. 4 Test no. 66. Irregular, three-component, waves measured by two wave height meters following the model in head seas. The wave equation coefficients are determined from the measurement in the upper graph.

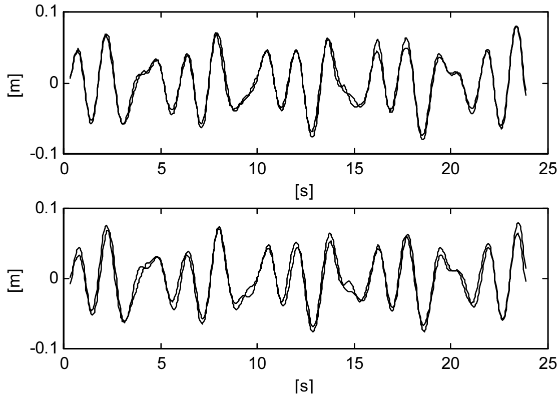

Fig. 5 Test no. 74. Irregular three-component waves measured by two wave height meters following the model in athwartships seas. The wave equation coefficients are determined from the measurement in the upper graph.

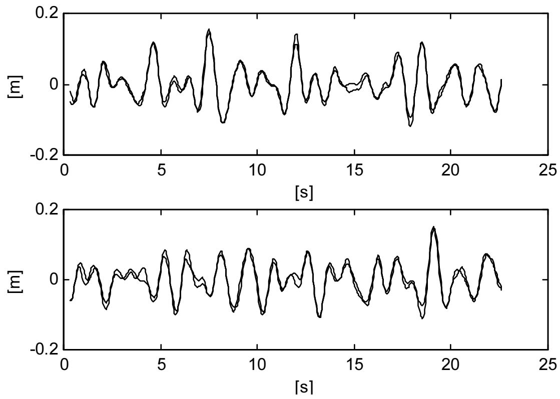

Fig. 6 Test no. 113. Crossing waves, consisting of two irregular three-component wave systems. Wave measurements were made by two wave-height meters following the model. The wave equation coefficients are determined from the measurement in the upper graph.

In the examples, each wave component in the wave equation is represented by three frequencies: the frequency the wave-maker was programmed to generate, one frequency slightly above this value, and one just below. The purpose of the latter two was to catch inexactness in the wave generation. Having chosen the frequencies, the component wave number was determined from the potential solution of linear gravity waves at finite depth, equation (5):

$$k = \frac{\omega^2}{g\tanh(kh)} \qquad (5)$$

where h is the basin water depth, ω the wave frequency and g the constant acceleration of gravity.

Choosing the wave frequencies in this way, based on the wave generator input, is of course far from optimal. A better way would be to develop a routine, which on the basis of approximate frequencies (for instance the wave generator input), could determine frequencies and eventually the coefficients that give the best fit to measurement. In this context it should be mentioned that the length of a measured time series should also be considered when calculating the wave equation. The wave system generated by a wave maker is more or less time dependent. Recordings over a long period of time might therefore be less well reproduced by a sum of sine and cosine functions. On the other hand, for a complicated wave system, a sufficiently long time series is necessary to catch the wave pattern.

Wave systems with strongly dispersive or dissipative behaviour demand time-dependent wave equation coefficients. A possible way to treat such waves would be to cut the measurement into sequences and determine an equation with constant coefficients for each sequence. It might also be necessary to include the second-order terms as mentioned in the beginning of the previous section.

A wave equation can be determined from one wave height measurement alone. But, since a gravity wave will lose in amplitude and not be absolutely stable in frequency during its propagation it is important to do the wave height recording in the vicinity of the model, and thereby avoid measuring a slightly different wave system than that which the model experiences.

The method presented here, to construct a wave equation from a single wave height recording is also believed to work for more complicated wave systems; however, to gain accurate results a routine for choosing the component frequencies would be necessary together with a method for solving large over-determined systems.

COMPARISON OF CALCULATED AND MEASURED SHIP MOTIONS

To demonstrate the usefulness of the determined wave equation, the SMS-code (Hua & Palmquist, 1995), is used for time-domain simulation of the ship motions in heading waves, with the wave equations and the initial motion conditions from the model measurement (Garme, 1997). The results are in good agreement with the corresponding measured motion.

The model test of an 11300 tonne ro-ro ship, carried out at SSPA Maritime Dynamic Laboratory (MDL) presented in detail by Garme (1997), serves with time series of wave and ship motion measurements. The model test series, aimed primarily at verifying different mathematical models of ship motion, touches on a number of issues on ship dynamics, such as loss of stability in quartering waves, roll motion in heading waves and linear and non-linear motions.

The code SMS is based on the non-linear strip approach. In the actual simulations, surge and yaw motion was denied, which is equivalent to a physical model test with constant forward speed and straight motion path. The Froude-Krylov forces are calculated over the momentary wetted hull surface. The linear potential theory is applied for the calculation of the radiation and diffraction forces. Consequently these forces are calculated on the mean wetted hull surface with the assumption that the relative motion of the hull surface to the wave surface is small.

First, the heave and pitch motion of the ro-ro ship in a heading near-regular wave is simulated with the wave equation from the model measurement. The wave amplitude is about 2.89 m and wave frequency 0.558 rad/s (Test No. 32 in Garme (1997)). The ship speed is 8.51 knots.

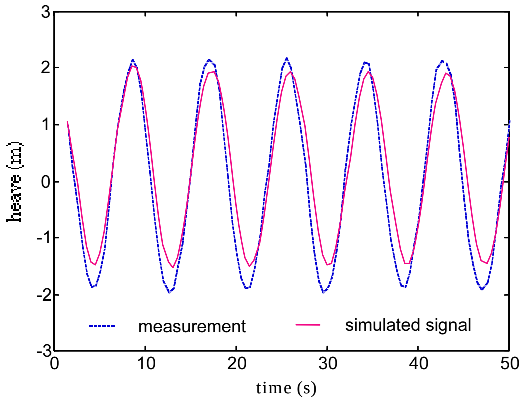

Fig. 7aTime history of heave motion in regular waves (test 32) and the corresponding computer simulated signal.

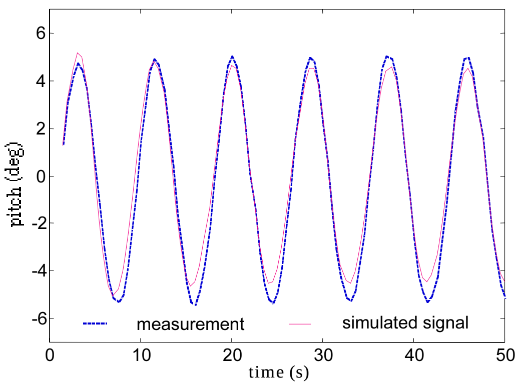

Fig. 7bTime history of pitch motion in regular waves (test 32) and the corresponding computer simulated signal.

Fig. 7 shows the simulated and model-measured heave and pitch motions as functions of time. Generally, the simulated amplitudes of the heave and pitch motion are lower than the measured. The phase relationships of the simulated motions are in very good agreement with the model measurement. It should be pointed out that the model measurement was carried out with a self-propelled model while the computer simulation corresponds to a towed model. Nevertheless, the comparison provides good insight into the reliability of the mathematical model, and the possibilities of modifying it.

An irregular wave is represented by three regular waves in the model measurement. The primary purpose is to validate the superposition theory, which is the basis of linear seakeeping theory. Fig. 8 shows the simulated time history of heave and pitch motions in such an irregular, heading wave in comparison with the model measured. The ship speed is 14.84 knots in full-scale (Test No. 66 in Garme (1997)). The results confirm the potential of the wave equation as a tool in seakeeping analysis. The deviation from simulated and recorded time series originates from measurement errors, approximations in the numerical code, discrepancies between the ship model and the digitalised hull and from discrepancies between the wave equation and the waves in the model test.

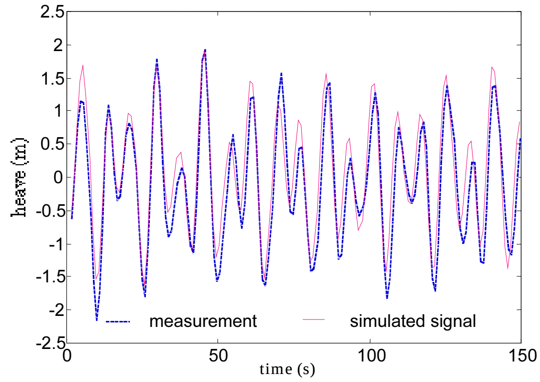

Fig. 8a Time history of heave motion in irregular waves from test 66 and the corresponding computer simulated signal.

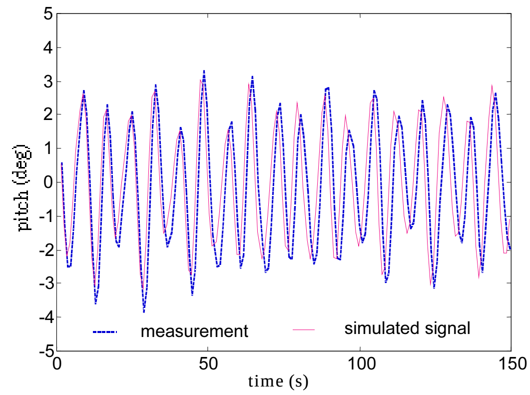

Fig. 8b Time history of pitch motion in an irregular wave from test 66 and the corresponding computer simulated signal.

APPLICATIONS

Considering the wave-induced motions as primary response, secondary responses would be, for instance, added resistance, global wave loads, relative motion and relative velocity which are due to the combined effects of the surrounding water wave motion and the primary responses. Some dynamic instability problems, such as parametrically excited roll motion and broaching can also be categorised as secondary response.

As is known, it is impossible to exactly describe seakeeping problems by means of mathematical models due to the complexity of the hydrodynamic mechanisms. In practice, it is always preferable to have a model for analysis which is as simple as possible, while still being physically relevant. The simplified model has to be verified either by model or full-scale measurements before application. For the primary responses, satisfactory mathematical models and numerical methods exist for conventional ships and for some types of high-speed vessels. However, quantitative verifications are in general not sufficient as far as the secondary responses are concerned. This is due to the practical difficulties in accurately measuring and analysing the relevant responses.

Consider for instance slamming pressure, an important design parameter in hull structural design. High local pressure develops as a ship bow leaves the wave surface and re-enters with great relative velocity. The magnitude of the slamming pressure is not only proportional to the square of the relative velocity, but is also sensitive to the hull geometry and wave surface slope. There are many different models for predicting slamming pressure. However, uncertainty arises when applying a method in practical design, since it is incompletely verified. The wave equation approach described in this paper, is a key to connecting a numerical simulation model to model measurements. In the slamming case, the wave equation could give the water particle velocity and the wave shape at the moment of impact, useful as input to a numerical model or for comparison with a simulated event. Then the discrepancy between measurements and calculations could be studied as a geometrical effect on slamming pressure.

During the past years, a large number of theoretical studies have been carried out on ship dynamic stability problems, see Proceedings STAB´94 (1994) and Proceedings STAB´97 (1997). Nevertheless, comparative studies with model and full-scale measurements are rare, resulting in lack of reliably verified theoretical models. Here again it is believed that the wave equation approach enables quantitative comparison between calculations and measurements. Actually, some dynamic stability problems are very sensitive to the wave motion around the hull, for instance, roll motion of ro-ro ships in following or quartering waves. The problem is mainly governed by time-varying quasi-hydrostatic moments in terms of wave excitation and restoring moment. These two moment components are easily calculated in a quasi-hydrostatic manner. So with a time sequence of roll motion from model measurement and the corresponding wave motion obtained by the wave equation, the relationship between roll and wave motion can be determined through the calculated quasi-hydrostatic roll moments based on the wave equation.

Another possible application is to study model-generated waves, by comparing wave equations based on measurements of the undisturbed incident waves with measurements where model-generated waves have perturbed the wave pattern. This could be interesting in the analysis of stability loss in following and quartering waves, in which model-generated waves can be so large relative to the incident waves that we can no longer ignore their influence on the stability.

SUMMARY AND CONCLUSIONS

This paper proposes a method to transfer wave elevation measurement to an analytical time-domain expression describing the model test wave environment. This gives the necessary information to study time-dependent interaction between the hull and the incident waves. The method is a tool to facilitate comparison of numerically simulated time series of ship dynamic behaviour in waves with model measurements. It is also believed that model generated waves (radiation and diffraction waves) could be studied within the scope of this method.

The method and its application as a link between model tests and numerical simulations, has been verified through seakeeping tests on a 1:35 scale ro-ro vessel (Garme, 1997). The measurement-based wave equation could accurately recreate the wave elevation measured at an independent measurement point. The wave equation was used as a time-domain wave model in the code SMS, (Hua & Palmquist 1995), to simulate the ship motion response. The calculated and measured time series of the heave and pitch motion were in good agreement.

The sea conditions in the model test were regular, irregular and crossing waves. The irregular and crossing wave systems were composed of three regular wave components. The wave equation method is believed to work on more complicated wave systems although the simple least squares determination of its coefficients has to be improved.

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