Time domain realization of extreme responses of a bilinear oscillator

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ABSTRACT

In this paper, we present a method to observe the extreme response of a nonlinear dynamic system in the time domain. The goal of the research is to provide short-time-window environments for a ship in a seaway such that different dynamical extreme events can be simulated. Although much work focuses on a means to determine the probability of an extreme, this work seeks to observe the extreme in the time domain such that causal relationships can be uncovered and the design can be improved. Previous work has shown how the Design-Loads Generator (DLG) method works for different ship processes such as heave, slamming, and roll, but due to the complicated nature of these processes, and the lack of truth about the probability distribution of the process, there are still open questions about the accuracy of the method, particularly with regard to application to nonlinear systems. In this paper we study a very simple problem that has nonlinear behavior but is simple enough that the distribution of extreme response can be obtained to evaluate the DLG method. Specifically, a bilinear oscillator under Gaussian band-limited white noise force is studied. The results from the proposed method are compared with the long-time Monte Carlo Simulation. As part of this study the sensitivity of extreme response distribution to the initial conditions, and the length of time around the extreme that should be simulated is analyzed for this problem.

Keywords: *Bilinear Oscillator, Extreme Response, Design Loads Generator, Neural Networks.*

1. INTRODUCTION

Naval architects are interested in extreme ship responses when designing marine vessels. Extreme responses like large roll, large vertical bending moment, and etc. can bring failure to the vessel during its operation. Though the wave or wind loading can be regarded as Gaussian stochastic processes, the response might not be as simple as Gaussian due to nonlinearity of the dynamic systems. It is fundamental to accurately predict and simulate extreme responses of nonlinear dynamic systems.

To capture the extreme response associated to a long exposure time window which lasts years or decades, a brute-force long-time Monte Carlo Simulation (MCS) using high-fidelity numerical tools is not possible. For example, the Navier-Stokes equation solver in computational fluid dynamics (CFD) can take days to complete a simulation window of minutes, not to mention that the large number of deterministic simulations that are required in order to describe the distribution clearly. One advantage of being able to model the extreme response in time domain is that it allows for observation of the events and environment that leads to an extreme such that the design can be improved. Also, high-fidelity tools that are expensive should only be used when necessary, which is namely when the dynamical system exhibits strong nonlinearity. For example, rms motion for a ship can be estimated by linear or weakly nonlinear potential-flow methods, but the largest motion excursions should be studied using full-scale Reynolds number computational fluid dynamics simulations.

In this paper, short-time MCS are prescribed for a bilinear oscillator as a surrogate for a ship dynamical response. The bilinear oscillator model is chosen since it is simple enough to efficiently evaluate the response using long-time MCS, and it can be significantly nonlinear to produce a fully non-Gaussian response. In addition, many marine behaviors, like offshore mooring system, can be suitable modeled as bilinear oscillators (Thompson, 1983). The current paper demonstrates the DLG process and discusses the effect of the nonlinearity, the sensitivity of the initial condition, and the computational cost of the current method when modeling extreme responses in the time domain.

2. TECHNICAL APPROACH

The Design-Loads Generator has been used to predict extreme ship responses (Alford et. al., 2009, 2011, Kim et. al., 2012, Xu, et. al., 2019). A very brief summary of the method is presented here for a generic dynamical system.

Given the design time window length $T_L$which is too long to directly simulate, the largest response during the time, denoted as $M_X(T_L)$, is a random variable. They are many studies to determine the distribution of $M_X(T_L)$. A generalized extreme value distribution (GEVD) can fit well to asymptotic behavior of distributions belonging to Fréchet, Weibull or Gumbel families. With Poisson assumption, a Peaks-over-Threshold (POT) model is developed to determine the extreme value distribution (Smith, R.L., 1987). The sub-asymptotic behavior can be fitted to the tail of extreme values by a parametric model (Naess A, Gaidai O. 2008). In the current paper, an affordable medium-long time window, denoted by T_m, is simulated to collect local maximas and spectrum of oscillator's response. The local maximas are then used to extrapolate the extreme value distribution from window T_m to window T_L.

Once the extreme value distribution for $T_L$and response energy spectrum are achieved, the Design Loads Generator (DLG) method (Kim, 2012) is applied to generate short-time response waveforms around the extremes. The DLG method, which is an Acceptance-Rejection based filter algorithm, is able to generate phases such that the resultant extremes follow the extrapolated distribution of $M_X(T_L)$.

$$M_X(T_L) = \sum_{i=1}^{N} A_{Xi} cos\phi_i \tag{1}$$

where $M_X(T_L)$ is the extreme response random variable, N is the number of Fourier components, A_{Xi} are the response ith Fourier amplitudes for frequency η_i, and ϕ_i are the response random phases generated by the DLG that correspond to an extreme from the distribution at a focusing time ($\tau = \tau^*$).

After the phases that lead to extreme response ϕ_i are generated, the corresponding extreme response waveforms are determined, each with time length, T_s. Since the dynamic system is often nonlinear and the explicit ODE is not available in many cases, a neural network is used to infer the system input (external force in this case) that leads to each waveform (Xu et. al., 2018). The neural net can be trained using system input and output from the medium-length (T_m) simulation results.

A bilinear oscillator is used as the nonlinear dynamic system to illustrate the method to be introduced. The nonlinearity comes with the different stiffnesses under different response regions. More specifically, when the displacement of the oscillator is larger than or equal to zero, the stiffness coefficient is k_1, and when the displacement of the oscillator is smaller than zero, the stiffness coefficient is k_2. The governing dimensional ordinary differential equation (ODE) is written as follows.

$$mx'' + cx' + \begin{cases} k_1 \\ k_2 \end{cases} x = \sum_{i=1}^{N} a_i \cos(\omega_{fi} t + \varphi_i) \tag{2}$$

where m is the mass of the oscillator, c is the damping coefficient, x, x', x'' are the displacement, velocity, and acceleration of the oscillator respectively. The external driving force is represented as Fourier series with ω_{fi} as the frequencies, a_i, φ_i as the corresponding amplitudes and phases of the ith component.

A dimensionless form of the equation is determined by first defining a characteristic period and the corresponding frequency as:

$$T = \pi\sqrt{m/k_1} + \pi\sqrt{m/k_2}$$
$$\omega = \frac{2\pi}{T} = \sqrt{K/m} \tag{3}$$

where $K = \frac{4k_1k_2}{(\sqrt{k_1}+\sqrt{k_2})^2}$. The discretized Fourier frequencies and time are be non-dimensionalized as:

$$\eta_i = \omega_{fi}/\omega, \tau = t\omega \tag{4}$$

Furthermore, the dimensionless displacement and the force amplitude are defined as:

$$X = x/(\sum_{i=1}^{N} a_i /K), A_i = a_i/\sum_{i=1}^{N} a_i \tag{5}$$

Finally, the dimensionless ODE is written as:

$$\ddot{X} + 2\zeta\dot{X} + \begin{cases}(1+\sqrt{\alpha})^2/(4\alpha) \\ (1+\sqrt{\alpha})^2/4\end{cases} X = \sum_{i=1}^{N} A_i \cos\left(\eta_i \tau + \varphi_i\right) \tag{6}$$

where $\zeta = c/(2m\omega)$ is the damping ratio, and $\alpha = k_2/k_1$is the stiffness ratio. Without losing generality, $\alpha \geq 1$ is assumed, which means the negative half region has larger stiffness.

The external force is a Gaussian process with a band-limited white noise energy spectrum,

$$S(\eta) = \begin{cases} S_0, & 0 \leq \eta \leq 1 \\ 0, & otherwise \end{cases} \tag{7}$$

The dimensionless frequencies are determined as $\eta_i = \frac{1}{N-1} i$, and the corresponding dimensionless amplitudes are $A_i = 1/N, (i = 0,1,\cdots, N-1)$. The random phases φ_i are independently and uniformly distributed from $-\pi$ to π.

In this paper, the results at different level of nonlinearity (different values of α) from the proposed method are compared with the long-time MCS results. The sensitivity of the extreme distribution to various initial conditions are discussed. The required length of time used in short-time (T_s) simulations will also be analyzed in the workshop.

3. RESULTS

The oscillator's responses are all simulated using MATLAB ode45. One realization of such responses under random external force is shown in Fig. 1.

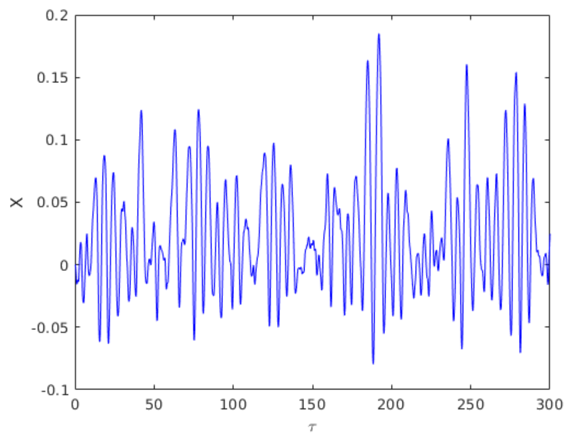

Figure 1: One realization of oscillator's response under random external force ($\alpha = 5, \zeta = 0.1, (X_0, \dot{X}_0) = (0, 0)$)

The response is smaller in the negative region as expected due to the larger stiffness in this regime. Fig. 2 shows a 2000-realization ensemble of extreme waveforms with extreme time shifted to $\tau - \tau^* = 0$, where $\tau^*$is the time when extreme response occurs.

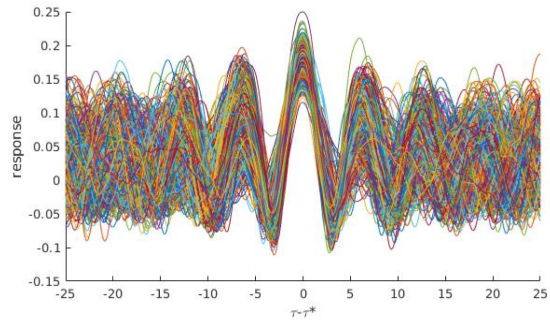

Figure 2: A 2000-realization ensemble of extreme waveforms with extreme time shifted to $-\tau^* = 0, (X, \dot{X}) = (0, 0)$.

Fig. 3 plots the same data from Figure 2 in the form of a histogram that shows how the distribution of response evolves before, at, and after the extreme occurrence.

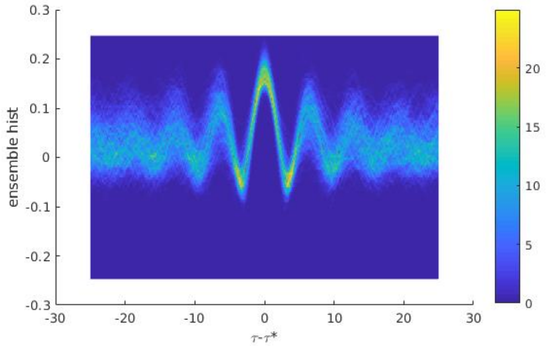

Figure 3: Waveforms have smaller variance when extreme happens.

The histogram of extreme values at the focusing time ($\tau - \tau^* = 0$) is plotted in Fig. 4. To measure the sensitivity of extreme response values to different initial conditions, a grid scan of initial conditions $(X, \dot{X})$is conducted and their histograms are plotted together with same transparency (=0.05) in Fig. 5. The blurriness in the edge qualitatively shows the dependency on the initial condition. (Larger dependency for more blurry edge). A quantitative measure for the dependency can be defined as the quantity

$$D = \frac{1}{B}\sum_{j=1}^{B} \sigma_j \tag{8}$$

where B is the number of histogram bins, and σ_j is the standard deviation among all histogram's counts in bin j. The dependency measure will be compared at different levels of nonlinearity.

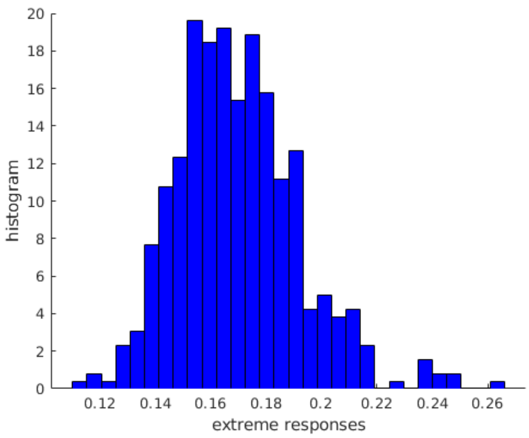

Figure 4: Extreme value histogram for the initial condition $(\boldsymbol{X}, \dot{\boldsymbol{X}}) = (\mathbf{0}, \mathbf{0})$.

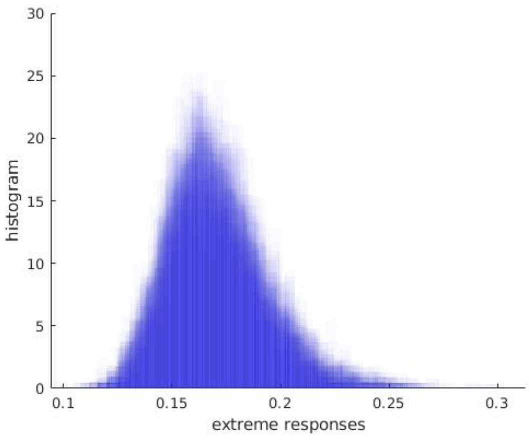

Figure 5: An overlap of histograms corresponding to different initial conditions.

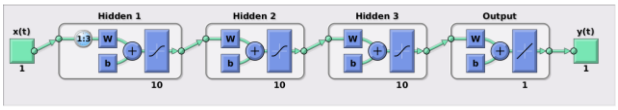

Figure 6: Neural Network architecture to infer the driving force time series. (3 hidden layer, each with 10 neurons)

The response spectrum and extreme response distribution will be compared at different nonlinearity. The distribution of waveforms at extreme time will be compared between long-time MCS and DLG-generated waveforms. A neural network shown in Fig. 6 is trained to infer the external forces producing each waveform. Finally, the simulated responses under inferred forces are compared with long-time MCS results.

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