

Ship stability-related effects on a critical distance of collision evasive action

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ABSTRACT

Numerous solutions have been developed to facilitate collision avoidance process and safety assessment at sea. These are based on proximity indicators, defined as an area around own ship that is to be kept clear from other vessels. One of such indicators is referred to as a ship's domain. Therein, a domain violation is recognized as an unsafe operation and needs to be avoided. However, the concept of a ship's domain does not originate from the collision avoidance field, rather it is rooted in the field of waterway's capacity assessment. Thus, the problem of transferability of the concept from one field to another emerges, resulting in the need for proper evaluation of the domain's characteristics that are suitable for the field of collision avoidance. Therein such features as ship's manoeuvrability and stability conditions seem to be indispensable since those affect the minimum area required for a ship to perform collision evasive manoeuvre.

The aim of the paper is three-fold. First, it sketches the minimum requirement for an area around own ship that needs to be kept free from other objects to ensure the safe passage of the ship. Second, it discusses the significance of stability-related effects on this area. Third, it is to provoke a discussion on the subject.

To this end extensive literature review is performed summarizing available domains, then to determine the safe area around own ship, a 6DoF ship motion model (*LaiDyn*) is adopted, along with encounter simulator.

Keywords: *ship collision avoidance, minimum distance to collision, ship operational stability.*

1. INTRODUCTION

From the operational viewpoint, a number of collision avoidance system (CAS) methods have been proposed, in line with developments in *e-Navigation*, [1]. However, the most widely used CAS is the *Automatic Radar Plotting Aid* (ARPA). This technology tracks several targets and displays proximity indicators, called CPA (*closest point of approach*) and TCPA (time to CPA), used for operational risk assessment. However, the passing distance does not translate into the required area for a safe and efficient evasive manoeuvre. Another type of proximity indicator stems from a concept of *ship domain*. Where *ship domain can be thought of as the sea area around the ship which a navigator would like to keep free, with respect to other ships and fixed objects*, see [2]. Nevertheless, the concept was initially developed for the purpose of waterway capacity evaluation and strategic risk assessment [2]–[4]. Despite that, it migrated to the field of

operational risk assessment and collision avoidance, as used by [5]–[8]. Another concept called *arena* has been introduced in [9], defined as *the area around the own ship which when infringed causes the mariner to consider whether to make a collision-evasive manoeuvre*. However, all those proximity indicators are subjective, referring to the comfort area defined by a navigator rather than a safety-critical area for a ship to perform evasive action. The difference between these two areas is substantial, and a navigator handling a ship should be aware of the safety area's dimension. It would be rather helpful when planning an evasive manoeuvre in an encounter, where the other, give-a-way vessel is not acting as supposed. This critical area depends on numerous factors, where the ship's dynamics is one of them. Interestingly, only a few studies take into account ship dynamics, as a factor determining the safe area for a given type of a manoeuvre, see for example [10]–[15]. However, those models face serious limitations, by considering one type of

manoeuvre for fixed rudder settings (turning circle at 20° rudder angle), one type of ship, fixed, presumably favourable stability condition. Adoption of maximum rudder angle for the collision evasive manoeuvre is not always advisable. Obviously, this results in the smallest turning radius for evasive action. However, it may lead to the development of significant roll angle, ultimately leading to an incident or even to ship capsizing, if the stability conditions are poor, see for example an accident of *m/s Hoegh Osaka* as described in [16]. In our earlier work [17], [18], a model determining the critical area for a Ro-Pax ship is presented, accounting for her dynamics, preselected stability conditions and simplified encounter conditions. The models stem from the concept of *Minimum Distance To Collision* (MDTC), as introduced in [19], [20]. Therefore in this paper, we discuss the minimum requirements for an area around own ship that needs to be kept free from other objects to ensure the safe passage of the ship in an encounter. Moreover, we discuss the significance of stability-related effects on this area. As a case study, we demonstrate the safe area for a container ship.

The structure of the paper is as follows: Section 2 introduces the concept of the safe area around own ship, Section 3 presents the methods adopted in the study and the developed model. In Section 4 the results are elaborated and discussed, while Section 5 concludes.

2. CONCEPT

Operational stability characteristics of selected vessels

Collision avoidance manoeuvres are usually planned with regard to the ship turning characteristics, as per wheelhouse poster. The stability issues are usually not considered, despite the effect of the ship's stability on her behaviour while exposed to the external force developed on the rudder and hull. Since the stability characteristics may affect the way the safe evasive action is conducted in an encounter, it is important for a bridge officer to be aware of its magnitude. Therefore, the feasible range of ship stability indicators for a given ship needs to be known, along with their effect on the size of the required minimum safe area for an evasive manoeuvre. A ship type that faces significantly different loading conditions in operation is container vessel. In Figure 1 ships'

metacentric height (GM) variations are shown for a set of such ships. The data is collected by the students of *Gdynia Maritime University* during their sea practices, and it covers a period of five years 2013-2018. The values of container vessels' GMs are spanning over 0.2- 4.5 m.

The series of ships operated by *Cosco* company is a good example of stability variations since the 300 m long vessels loaded down to their draft of around 10.5 - 11.6 m faces the GM ranging between 1.66 - 4.56 m. Thus, despite the same draft of the same ships, stability conditions govern her behaviour in the seas. The second characteristics of analysed ships is the area under the GZ curve calculated from zero up to an angle of heel 30°, as presented in Figure 2. Therein the significant spread of this parameter is seen. The stability of container vessels vary significantly in their daily operations, thus the behaviour of the ship and her response will vary. This should be accounted for in any research addressing the stability-related areas, e.g. manoeuvring and its derivatives such as collision avoidance. Therefore, a set of manoeuvring data shown on the bridge in the form of turning circles (relevant for ballast conditions and for fully loaded ones) are not enough to cover all practical loading conditions, and a better solution needs to be found, like a minimum safe area for an evasive manoeuvre.

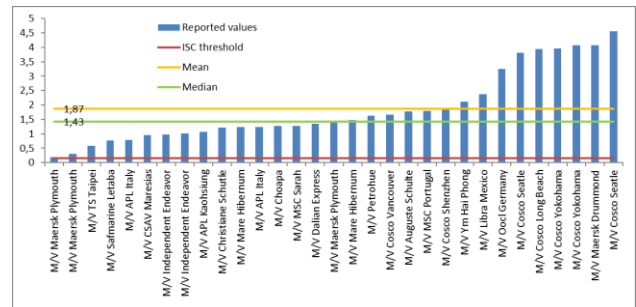


Figure 1: GM reported during routine operation of examined container vessels.

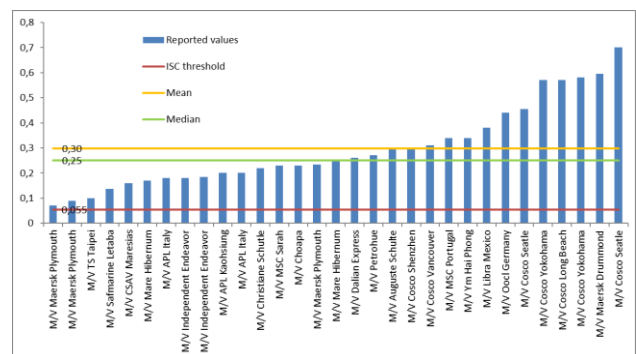


Figure 2: Area under GZ curve up to 30 degrees, reported during routine operation of examined container vessels.

Safe manoeuvring area definition

The minimum safe area for evasive manoeuvre is understood here as an area around own ship, which must be kept free from any objects that are on collision course with the own ship. The dimensions of the area are based on the ship's manoeuvring capabilities under given stability conditions, ensuring safe evasive actions. The latter denotes such manoeuvre, where the collision encounter is resolved and there is no harm to ship, crew or cargo. This means that the ship does not experience excessive roll angle or accelerations in the course of collision evasive action.

According to the assumptions of MDTC concept, as per [19]–[21], for each navigational scenario that can be interpreted as an arrangement of two vessels in the two-dimensional coordinate system only one MDTC exists. Safe area for the vessel can be obtained by computation of mentioned value for each navigational scenario and selected hydro-meteorological conditions. Because of utilization vessels' trajectories that include 6DoF motion model, stability issues and criteria can be directly incorporated into the subject of collision avoidance.

Determination of MDTC values for many scenarios and cases allows defining the general area where the last moment of evasive manoeuvre execution by the vessel is still feasible. To this end, a wide range of ships' headings and weather conditions needs to be accounted for resulting in a considerable number of combinations. The projection of the MDTC values in the function of the wave direction and relative bearing between the vessels creates around the ship the safe manoeuvring area. Due to the time-consumption of proposed calculations for a large sample of input data, the computer application is developed, called ships encounter simulator.

3. METHODS

6DoF ship's motion model

The study focuses on effects resulting from coupling between the ship manoeuvrability and her stability response to external forces due to seas. Thus, the most convenient approach is not to separate these both characteristics and rather consider them as a complex response to all relevant forces like seas, wind and rudder action. To

comprise them, the state-of-the-art 6DoF ship motion model called *LaiDyn* is utilized, which is developed as a hybrid non-linear simulation model for a ship being considered a rigid body in the time domain, [22], [23]. Hence the assumption of small amplitude oscillatory motions are adopted [24], the radiation and diffraction forces are calculated according to the linear approach. The non-linear part, for example, hydrostatics (hull shape), wave force, manoeuvring, and propulsion, were taken into consideration [25]. Especially the two latter ones are crucial for our research since rudder action and propulsion during the ship turn to remain core issues for collision avoidance problems.

The model is validated in two-fold. First in the course of the towing tank model tests conducted at Aalto University [26], [27], second through the numerous external benchmark studies [28], [29]. The results of those are found satisfactory for the purpose of this research, where *LaiDyn* produces ship's trajectories for varying hydro-meteorological conditions defined by significant waves (H_s) and angle of wave's attack on the ship's hull.

Ships encounter simulator

Subsequently, the trajectories generated by *LaiDyn* are fed into an encounter simulator, where different navigational scenarios are modelled, resulting in the mutual arrangement of the vessels and their angular positions (ships' headings and bearings), as well as the hydro-meteorological conditions considered. The general principle of encounter simulator's operation, as presented in Figure 3, is based on causing the collision between the vessels which are figures on a 2D coordinate system that estimate projections of ships' hulls at the given angle. Afterwards, ships are successively moving apart in the straight line by given time step. For each iteration, according to the simulation case and navigational scenario, into the position of the particular vessel, the trajectory is loaded and set. The simulator validates the realisation of the evasive manoeuvre and if the collision between the ships still exists, the next backward step is executed. The loop is processed as long, as the set trajectories cause the collision. At the first position where tracks allow for the safe passage (collision-avoidance is successful), the application is breaking the loop and computes the safety parameters of ships' encounter. These include MDTC value, positions of the ships and the relative

bearings to the target, for manoeuvre execution moment.

Before the trajectories are set into the simulator, tracks are filtered out according to the adopted safety-related criteria. In the research presented here, a criterion of rolling angle is taken into account. The input files are screened for the threshold for roll angle exceedance. In case its value, as computed by *LaiDyn*, exceeds the threshold, the trajectory is considered unsafe and is rejected from the dataset to be processed.

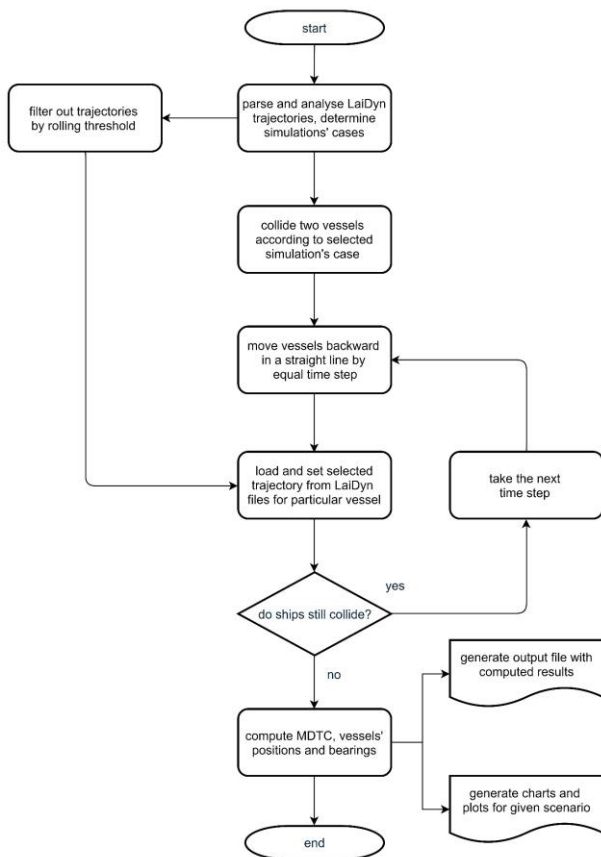


Figure 3: Flowchart of ships encounter simulator.

Simulations' cases and ship's model characteristic

The simulation cases analysed here attempt to cover two aspects. First is the evaluation of the influence of waves in the course of ships' encounter on the MDTC, where a fixed rudder angle is applied. Therein the MDTC values are calculated for the following navigational scenario:

- Own Ship (OS) proceeds to the North (heading = 000°) and executes the manoeuvre by set the rudder to 20° on the starboard side.
- Target Ship (TS) proceeds the course 225° and she keeps her course and speed.

Second, is the influence of the ship's stability (especially vertical centre of gravity resultant roll motion) on the safe rudder angle and resulting manoeuvring area. To determine the MDTC, which corresponds to the last moment for execution of evasive manoeuvre in the function of the vessel's relative bearing, the following scenarios are considered:

- Own Ship (OS) proceeds to the North (heading = 000°) and executes the manoeuvre by set the rudder to a maximum allowable value resulting from rolling threshold separately for port and starboard side for two considered VCGs.
- Target Ship (TS) proceeds on different starting courses from 000° up to 315° for each 45° interval. For each heading vessel keeps her course and speed.

Characteristic of the analysed container vessel's model is presented in Table 1, while waves parameters used in simulation cases are tabulated in 2. Projection of estimated vessels' hulls on the plotting sheet for the scenario of waves' direction impact is depicted in Figure 4.

Table 1: Characteristic of a used ship model.

LOA [m]	Beam [m]	VCG [m]	Draft [m]	Mass [t]	Speed [kts]
262.0	40.0	14.92	12.3	76027.6	20.0
262.0	40.0	17.91	12.3	76027.6	20.0

Table 2: Waves parameters used in the study.

Waves parameters			
Significant height [m]	Period [s]	Angle interval [°]	Number of angles
3.0	7.0	45.0	8
7.1	10.9	45.0	8
13.0	14.7	45.0	8

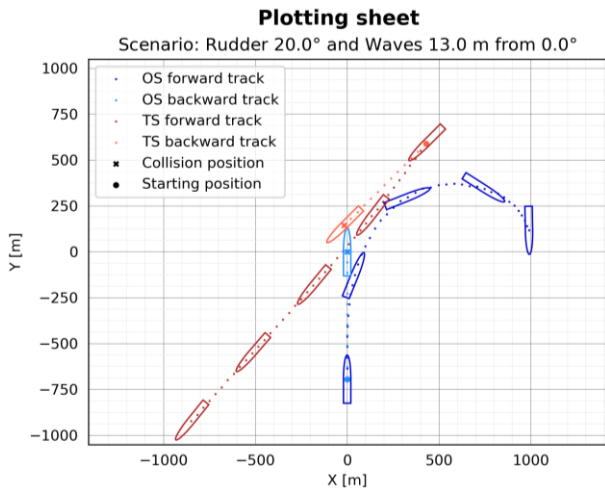


Figure 4: Scatter plot from the simulator for one of the considered navigational scenarios.

Roll angle filtering

Due to various heights and relative directions of waves with respect to ship’s hull, her rolling magnitude differs. Thus, depending on the wave’s parameters, trajectories that result in roll angle lower than the threshold are allowed, those where the threshold is exceeded are rejected. Depending on the wave parameters, the same manoeuvre where the same rudder angle is applied result in different roll angles in the course of the manoeuvre. The threshold value for roll angle, as a stability indicator for a ship, is arbitrary taken as 20°.

Totally, the 1872 input files from *LaiDyn* describing the ship’s trajectories are generated. Because the realisation of waves for given parameters in the software is stochastic, each case is obtained three times. The trajectories where the rolling threshold is exceeded are considered unsafe from the operational perspective thus rejected from the database. The result of filtering is depicted in Figure 5. Therein the total number is shown however, the percentage of rejection differs across values of VCG. For VCG = 17.92 only 3 cases are rejected, whereas for VCG = 14.92 in 174 cases the threshold is exceeded. The rejected cases are observed for one out of three wave heights, which is $H_s = 13\text{ m}$ – see Table 2. The number of rejected trajectories depends on the relative direction of the wave to the ship, as depicted in Figure 6.

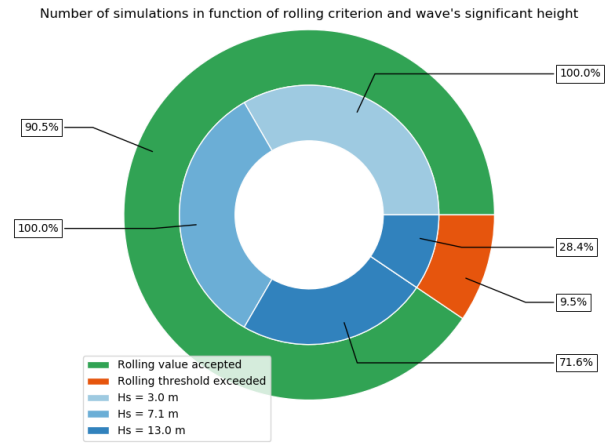


Figure 5: Percentage of rejected simulations according to the exceeding of the rolling threshold for two analysed values of VCG.

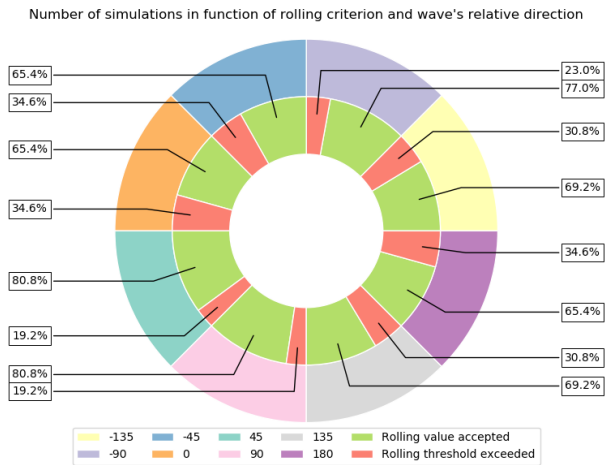


Figure 6: Percentage of rejected simulations according to the roll angle for wave’s directions and $H_s = 13\text{ m}$.

4. RESULTS AND DISCUSSION

The effect of ship stability on MDTC at fixed rudder angle

Here we determined the MDTC for a single scenario, taking into account the relative angle of the given wave ($H_s = 13\text{ m}$) and two VCG values. The realisation of a turn at 20° rudder angle to starboard side is not possible for all wave’s direction without an exceedance of the rolling threshold, see Figure 7. Therein the outer scale refers to the relative angle of the wave, the inner scale denotes the MDTC value, and the VCG values are color-coded. The MDTC required for safe evasive manoeuvre (where the roll angle is below the threshold), heavily depends on the relative direction of wave and ship’s stability. For a ship with VCG = 17.9 m all relative directions of waves are feasible for collision-evasive action, however in some cases, longer MDTC is required (for the head-on seas), whereas in other cases the

short MDTC suffices (for a wave coming from the relative direction of -45°). However, for the same ship with $VCG = 14.9$ m, only a tiny sector is feasible, for an evasive manoeuvre where the roll angle is not exceeded, which is the grey area in Figure 7.

Therefore the ship stability and loading conditions are safety critical factors for the collision-avoidance process.

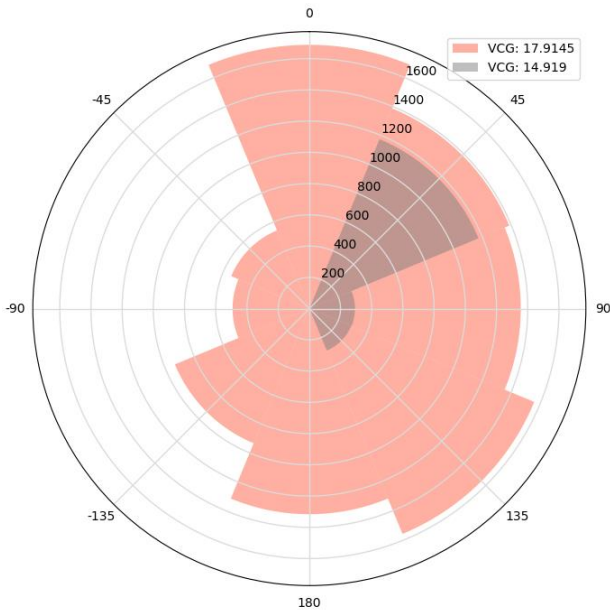


Figure 7: MDTC obtained in the simulations for different wave's directions with $H_s = 13$ m.

The effect of ship stability on MDTC at safe rudder angle

Subsequently, the MDTC is obtained for the maximum allowable values of rudder angle for two VCGs values. The following parameters are considered: the wave height (13 m) and relative direction (-45°), eight relative bearing values to the target, eight starting courses of the target.

For each scenario, the admissible rudder angle is determined, as per the rolling criteria. For $VCG = 14.92$ m, the maximum rudder angle for the port side is determined at 5° , while for starboard side it is 15° . For $VCG = 17.915$ m, it is possible to make a turn to both sides with full rudder angle of 30° without exceeding the roll threshold. The results are depicted in Figure 9 and 10. Therein two distinct safe manoeuvring areas are shown, that are related to the direction of the ship's turn. The green area denotes a situation where the vessel turns to starboard, whereas the red area means the port side turn. It is

evident, that the turn to starboard (green area) should be executed earlier than the turn to port (red area). Obviously, the shape of the areas is governed by the starting relative bearing to the target.

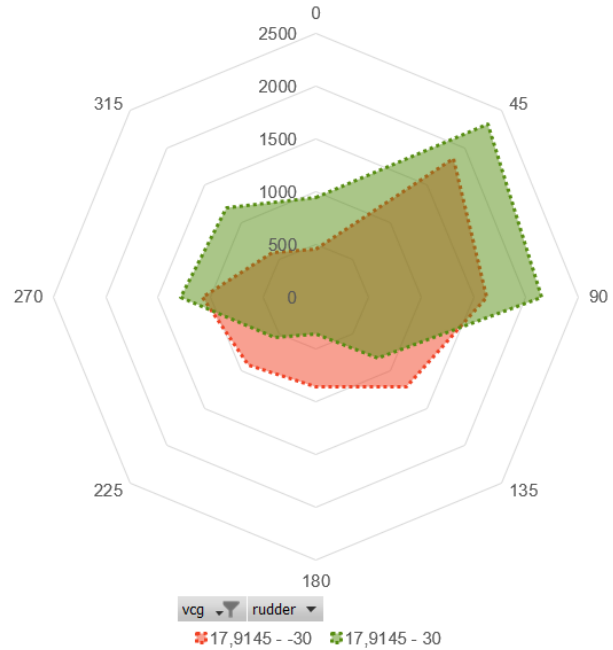


Figure 8: MDTC obtained for different bearings and turning sides for $VCG=17.9$ m. The outer scale refers to the starting relative bearing to the target, while the inner scale denotes the MDTC.

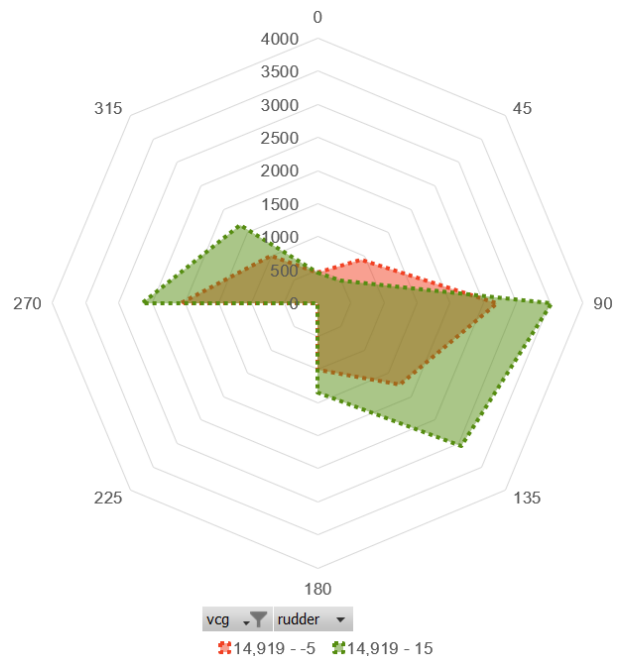


Figure 9: MDTC obtained for different bearings and turning sides for $VCG=14.9$ m. The outer scale refers to the starting relative bearing to the target, while the inner scale denotes the MDTC.

Discussion

As presented, the ship’s stability may have a crucial impact on the moment of evasive manoeuvre’s execution, especially for the rough seas. The MDTC values differ significantly in the relation of the vessel’s loading condition because VCG affects the obtained roll angles. It also varies according to hydro-meteorological conditions, especially the height and direction of the wave’s angle of attack on a ship that makes her leeway. The study opens numerous questions to be considered in the course of future works, as follows, see also Table 3.

1. The rolling threshold value for a particular vessel is taken arbitrarily. In the future different approach should be considered. Adjusting the threshold could be obtained e.g. in an empirical way by performing a very large number of simulations. Also introducing the second stability criterion into the simulation software, for instance by computing the accelerations could affect in complementary hybrid-approach to the presented issue. Thus, raised the problem of ship’s rolling during anti-collision should be continued and considered in the future works to determine the most realistic approach to stability issues in encounter situation of two vessels.

2. Roll angle values for particular cases exceed the set threshold but not the maximum rudder commands. It affects the rejection of trajectories for the same wave’s parameters. For instance, for rudder angle 20° rolling exceeds the threshold, but for 25° or 30°, it does not. This is due to non-linear effects in ship motion during the turning and the time of exposure to the exciting moment plays its role. It means that the largest value of the rolling results mainly from the waves impact, not from the list that is generated during the vessel’s turning.

3. The stochastic implementation of waves in *LaiDyn* trajectories should be refactored. Currently, three simulations for each case are generated and included in ships encounter simulator. In the next researches, this number could increase to improve the probability of waves’ parameters modelling. It seems to be significant because presented results indicate a big impact of waves in the problem of ships’ anti-collision.

Table 3: Issues for future consideration.

Issue to be addressed	Advantages compared to present solutions	Related challenges
Roll threshold adjusting	The threshold can be tuned according to the ship’s type and characteristic	Lack of a commonly accepted method for the threshold setting
Accelerations as a complementary criterion for stability incidents	To adjust the acceleration threshold to the cargo lashing system fitted on-board To ensure proper conditions for work on-board	Threshold value not commonly accepted Possible high-frequency oscillations producing high accelerations
Stochastics description of waves’ parameters.	To comprise a realistic sea wave thanks to the determination of such number of cases that will include stochastics in waves implementation	Lack of a clearly accepted number of samples Time-consumption during generation of many numbers of the same simulation cases

5. CONCLUSIONS

The aim of this paper is to discuss the minimum requirements for an area around own ship that needs to be kept free from other objects to ensure the safe passage of the ship in an encounter, accounting for the significance of stability-related effects. This has been achieved by developing an encounter simulator and a model based on the set of trajectories obtained from the 6DoF ship motion model. The safe manoeuvring area is demonstrated for a sample container ship.

Preliminary results presented in this paper indicates that the problem of vessels’ collision avoidance is much more complex than contemporary considered and it requires a multi-criteria approach. The moment of execution, as well as the type of evasive manoeuvre depends not only on COLREGs (*International Regulations for Preventing Collisions at Sea*), but also on loading condition of the ship, relative position of encountering ships, parameters of the target, and environmental parameters like height or direction of waves.

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