Study on short-term prediction of roll in beam sea

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ABSTRACT

The formula to determine the roll angle for structural strength assessment in ClassNK's Technical Rule and Guidance gives an value based upon maximum roll amplitude at probability Q=10^{-8} on long-term prediction of roll amplitude. The long-term prediction is obtained from combining short-term prediction of roll amplitude and a probability of occurrence of short-term irregular sea in long term. In the current rule, non-linearity of roll is included as some correction coefficients obtained from model experiments and empirical knowledge at the time of development. However, the type of vessels has increased after the time of development, and the coefficients are not always suitable for the newest vessels. However, the type of vessels has increased after the time of development, and the coefficients are not always suitable for the newest vessels. The purpose of this study is to propose a rational short-term prediction method considering nonlinearity of roll. In this paper, applicability of a non-Gaussian PDF (Probability Density Function) for PDF of roll angle is investigated.

Keywords: *Short-term Prediction, Roll, Non-Gaussian Distribution.*

1. INTRODUCTION

The current formula to determine the roll angle for structural strength assessment in ClassNK's Technical Rule and Guidance gives an value based upon maximum roll amplitude at probability Q=10^{-8} on long term prediction of roll amplitude. The probability Q is defined as the number of encounter waves, which is roughly corresponding to 25year of designed life of a ship divided by 10s of average encounter wave period. The long-term prediction is obtained from combining short-term prediction of roll amplitude and a probability of occurrence of short-term irregular sea in long term. And the short-term prediction is the energy spectrum method based on the principle of linear superposition, which uses roll response function at small wave height and wave spectrum of short-term irregular waves. Additionally, non-linearity of roll is included as some correction coefficients obtained from model experiments and empirical knowledge at the time of development. However, the type of vessels has increased after the time of development, and the coefficients are not always suitable for the newest vessels.

Therefore, the fundamental revision is required, which is not only revision of correction coefficients to apply the present formula to all type vessels in recent years, but also proposal of rational new method to be able to apply to the vessel which will be further diversified in the future.

The purpose of this study is to propose a rational short-term prediction method including non-linearity of roll. In this paper, it is considered to apply a non-Gaussian PDF (: Probability Density Function) to PDF of roll angle. Roll measurement tests in irregular beam waves for scale models of PCC and LNG carrier are carried out to obtain probability density of roll, and the measured results are compared with Gaussian PDF and a non-Gaussian PDF to investigate its applicability.

2. PROBABILITY DENSITY FUNCTION OF ROLL

Gaussian distribution

Gaussian PDF is given as:

$$p_1(\phi) = \frac{1}{2\pi\sigma}\exp\left\{-\frac{\phi^2}{2\sigma^2}\right\} \quad (1)$$

where ϕ is roll angle (: time history data) [rad] and σ is standard deviation of roll angle. The standard deviation of roll angle σ is obtained from time history data of roll angle in irregular wave. If roll is linear, standard deviation can be obtained using Eq. (2) according to energy spectrum method[1)] based on the assumption of linear superposition.

$$\sigma^2 = \int_0^\infty [f(\omega)]^2[A(\omega)]^2 d\omega \quad (2)$$

where $f(\omega)$ is wave spectrum and $A(\omega)$ is frequency response function of roll for small wave height.

Non-Gaussian distribution

If roll can be expressed by a one degree of freedom motion equation, a non-linear roll equation can be given as

$$\ddot{\phi} + \alpha\dot{\phi} + \beta\dot{\phi}\left|\dot{\phi}\right| + \frac{W}{I_{xx}}GZ(\phi) = M_{wave}(t) \quad (3)$$
$$GZ(\phi) = GM\phi + GZ_2\phi^2 + GZ_3\phi^3 + GZ_4\phi^4 + GZ_5\phi^5$$

where t is time, a is linear damping coefficient, β is quadratic damping coefficient, W is ship weight, I_{xx} is moment of inertia of roll (including added component), GM is metacentric height, GZ_i is ith component of GZ polynomial fit and $M_{wave}(t)$ is time history of wave excitation moment.

Maki (2016) and Maki et al. (2018) apply the method which is proposed by Sakata et al. (1979 and 1980) and Kimura et al. (1980, 1995, 1998 and 2000) to roll motion problem in irregular waves. In this frame work, solution of FPK (: Fokker-Planck-Kolmogorov) equation for external force as white-noise is approximately utilized. Kimura et al.(1995) reports that the form of PDF is strongly affected by the potential of the system for the case of non-white excitation. Therefore, they approximate the actual PDF for colored noise by the PDF for white noise. The non-Gaussian PDF of roll angle and roll angular velocity is described as:

$$p_2(\phi,\dot{\phi}) = C\exp\left[-d\left\{\alpha H(\phi,\dot{\phi}) + \frac{8\beta}{9\pi}\left(2H(\phi,\dot{\phi})\right)^{\frac{3}{2}}\right\}\right] \quad (4).$$

The coefficients C and d in Eq.(4) are determined by Eqs.(5)-(6). Eq.(5) means the normalization condition of the PDF whereas Eq.(6) does the condition for variance.

$$\int_{-\infty}^{\infty} d\dot{\phi} \int_{\phi_{VN}}^{\phi_{VP}} p_2(\phi,\dot{\phi};d)d\phi = 1.0 \quad (5)$$

$$\int_{-\infty}^{\infty} d\dot{\phi} \int_{\phi_{VN}}^{\phi_{VP}} \phi^2 p_2(\phi,\dot{\phi};d)d\phi = E[\phi^2] \quad (6)$$

$$H(\phi,\dot{\phi}) = \frac{1}{2}\dot{\phi}^2 + \frac{\omega_0^2}{GM}\int_0^\phi GZ(\phi)d\phi \quad (7)$$

where ϕ_{VP} and ϕ_{VN} indicate two vanishing angles of roll restoring moment. H in Eq.(7) is dynamic energy at certain roll angle and roll angular velocity. In this research, integrations described by Eqs.(5)-(6) are conducted by using double exponential formula.

In order to obtain the PDF of Eq.(4), variance of roll angle, damping coefficients and restoring coefficients are necessary. In this study, the following three approaches are considered, however, and only first one of them is adopted. First one is that variance of roll angle and damping coefficients are obtained from model tests, and restoring coefficients are calculated. Second one is that coefficients of roll motion equation Eq. (3) are obtained theoretically (e.g. a strip method, Ikeda's roll damping prediction method and restoring calculation of) and variance of roll angle is obtained from solving Eq. (4) with Monte-Carlo Simulation. Third one is that all coefficients of Eq.(4) are obtained from the least square fit for measured probability density of roll angle.

3. SUBJECT SHIPS

Principal particulars of model ships

Subject ships are typical large PCC and LNG carrier in recent years. Figure 1 shows the body plans of the ships, and Table 1 shows their principle particulars of the subject ships. Height of the center of gravity KG and natural roll period T_n are obtained from an inclining test and a free roll decay test, respectively.

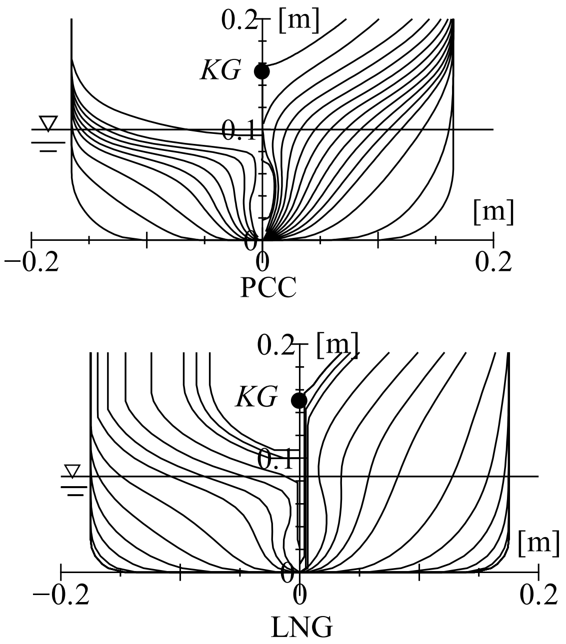

Figure 1: Body plan of models of PCC and LNG carrier.

Table 1: Principal particulars of the models.

name of ship	PCC	LNG
scale	1/97.5	1/140
overall length: L_{OA} [m]	2.054	2.095
breadth: B [m]	0.330	0.35
depth: D [m]	0.351	0.193
draught: d [m]	0.100	0.084
ship weight: W [kgf]	36.68	41.22
height of the center of gravity: KG [m]	0.152	0.150
metacentric height GM [m]	0.0126	0.0118
natural roll period: T_n [s]	1.96	2.19
position of bilge keels	s.s.3.4 - s.s.5.6.	s.s.3.65- s.s.6.45
initial trim [m]: $d_a - d_f$	0	0
LCG [m] from midship (+ aft)	0.0615	- 0.0193

Characteristics of roll restoring

Figure 2 shows calculated *GZ*-curves of the models. In the calculation, *GZ* is obtained under the equilibrium condition of vertical force and trim moment for each heel angle. This figure also shows the linear restoring lever *GM* of the *GZ*-curve. This figure shows that *GZ*-curve of PCC is linear up to 22 degree of heel angle and *GZ*-curve of LNG career is linear up to 10 degree of heel angle.

Eq. (8) and Eq. (9) show the fifth order polynomials for *GZ*-curves (-30< φ <30) of PCC and LNG carrier whose coefficients are decided by the least squares method.

$$GZ(\phi) = 0.0126\phi + 0.00310\phi^3 + 0.00727\phi^5 \quad (8)$$

$$GZ(\phi) = 0.0118\phi + 0.04099\phi^3 - 0.06807\phi^5 \quad (9)$$

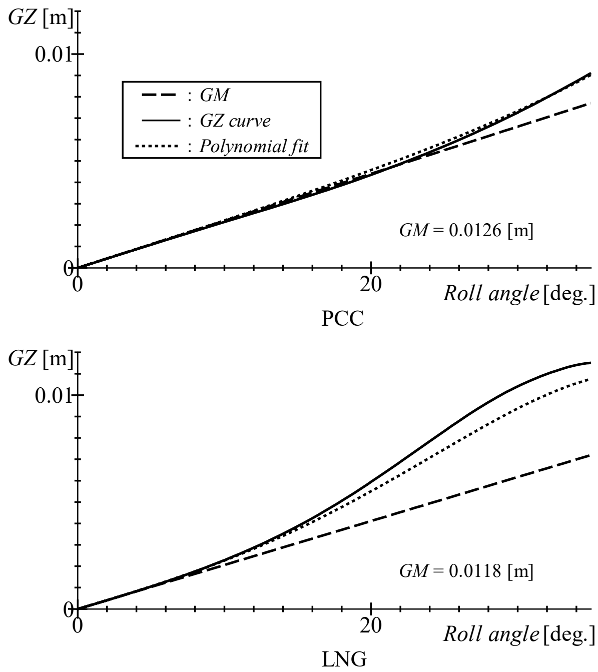

Figure 2: Calculated GZ-curve of these models.

Characteristics of roll damping

In order to obtain roll damping coefficients, free decay test is conducted. Roll, heave, pitch and sway of model are free. Measurement device is shown in Figure 5. By constraining the roll axis of the measurement devise, four initial heel angles (5, 10, 15 and 20deg.) are given. After releasing the constrain instantly, roll decay motion is measured with a potentiometer.

Using the measured results, the figure whose vertical and horizontal axis are roll peak angle φ_n and its occurrence time t_n is obtained as shown in Figure 3. And the curve in Figure 3 is fitted by a polynomial by the least squares method. From the polynomial, φ_n at t_n is re-obtained, and $\Delta\varphi_n$ and φ'_n of extinction curves shown Fig.4 are obtained by Eq. (10).

$$\varphi'_n = \frac{\varphi_n + \varphi_{n+1}}{2}, \quad \Delta\varphi = \varphi_n - \varphi_{n+1} \quad (10)$$

where sign of φ is degree. In order to obtain roll damping coefficients of Eq. (3), extinction curve is express as the Froude's expression of Eq. (11).

$$\Delta\varphi = a\,\varphi'_n + b\,\varphi'^2_n \quad (11)$$

The relation between extinction coefficients and roll damping coefficients is Eq. (12).

$$\alpha = \frac{4a}{T_\phi}, \quad \beta = \frac{3b}{4} \cdot \frac{180}{\pi} \qquad (12)$$

where, T_ϕ is natural roll period.

Roll damping coefficients of PCC and LNG carrier are obtained from Fig.4 as α=0.254, β=0.486 for PCC and α=0.281, β=0.374 for LNG carrier.

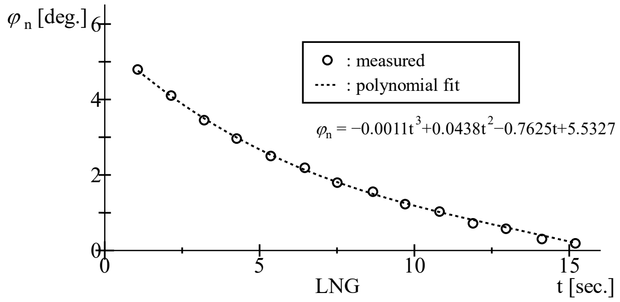

Figure 3: Peak angle of roll obtained by free decay test measured by potentiometer with 4 degree of free model (□0 = 5 deg.).

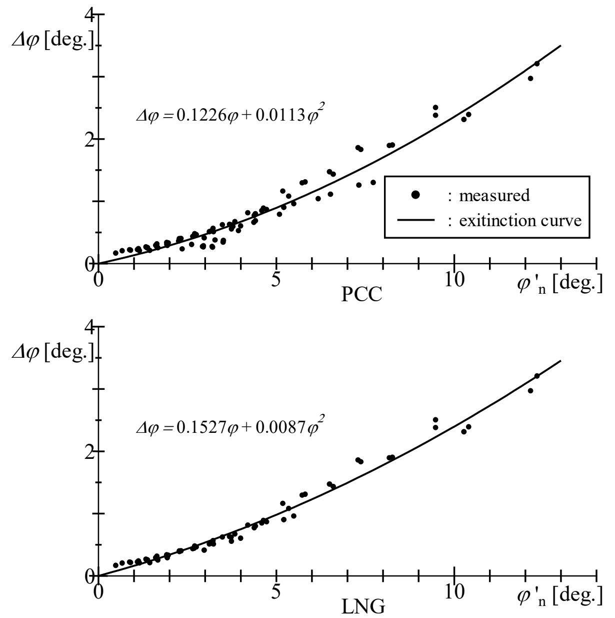

Figure 4: Extinction curves obtained by the data on Fig. 3.

4. ROLL MEASURMENT IN BEAM WAVES

Measuring device and coordinate system

Figure 5 shows a schematic view of experiment and its coordinate system. In this model experiment, surge and yaw are fixed whereas roll, sway (and drift), heave and pitch are free. Wave height is measured with a servo type wave height meter attached to model basin. Data is collected with 100Hz of sampling frequency. The carriage is pushed according to the drifting speed in order to avoid the sub-carriage hit both ends.

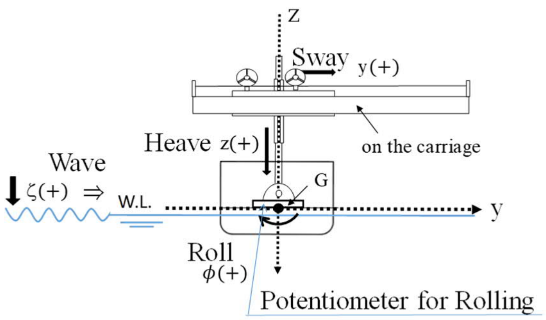

Figure 5: Schematic view of the motion measurement with fixed surge and yaw from the behind of hull.

Roll measurement in irregular beam waves

The wave spectrum of the long wavelength irregular wave of IACSRec.34 (: ISSC spectrum) shown as Eq.(13) is used.

$$S(\omega) = \frac{H_{1/3}^2}{4\pi}\left(\frac{2\pi}{T_Z}\right)\omega^{-5}\exp\left[-\frac{1}{\pi}\left(\frac{2\pi}{T_Z}\right)^4\omega^{-4}\right] \qquad (13)$$

where $H_{1/3}$ is significant wave height and T_z is average zero up-crossing wave period. In this paper, it is adopted that peak period of the wave spectrum T_p is natural roll period T_n to cause large roll amplitude. The relation between peak period T_p and T_z is given as Eq. (14).

$$T_Z = T_p\left(\frac{4}{5\pi}\right)^{0.25} \qquad (14)$$

Therefore, T_z of PCC and LNG carrier are 1.392s and 1.561s, respectively.

The formulas of significant wave height for strength assessment in ClassNK's Technical Rule and Guidance is given as

$$H_{1/3} = 0.85 \times \left\{10.75 - \left(\frac{300 - L}{100}\right)^{1.5}\right\} \times \sqrt{\frac{L + \lambda - 25}{L}} \qquad (15)$$

where L is overall length of ship and λ is wavelength obtained by using natural roll period. From Eq.(15), the measuring conditions of the significant wave height of PCC and LNG carrier are 16.089 cm and 13.996 cm. The number of encounter waves is at least 700 waves each case.

Results

Fig.7 shows the PDF of roll angle. In this figure, measured result, Gaussian PDF of Eq.(1) and non-Gaussian PDF of Eq.(4) are shown. It is noted that non-Gaussian PDF shows the integral value of Eq.(4) for roll angular velocity.

Comparing these results, it is clear that the difference of them is negligible up to about ϕ=10deg regardless type of ship. On the other hands, in the range over ϕ=10deg, non-Gaussian PDF is smaller than Gaussian PDF and the measured results is similar to Gaussian PDF, however, the measured results for PCC shows asymmetry and the measured results for LNG carrier become lager than Gaussian PDF around ϕ=20deg.

Non-Gaussian PDF includes the non-linearity of roll damping and roll restoring, therefore, non-Gaussian PDF becomes smaller than Gaussian PDF at larger roll angle. However, non-Gaussian PDF include the effects of asymmetry of time average value of roll angle, it is difficult to see the effects on the results. On the other hand, non-Gaussian PDF include the effects of asymmetry of time average value of roll angle, however, it is difficult to see the effects on the results.

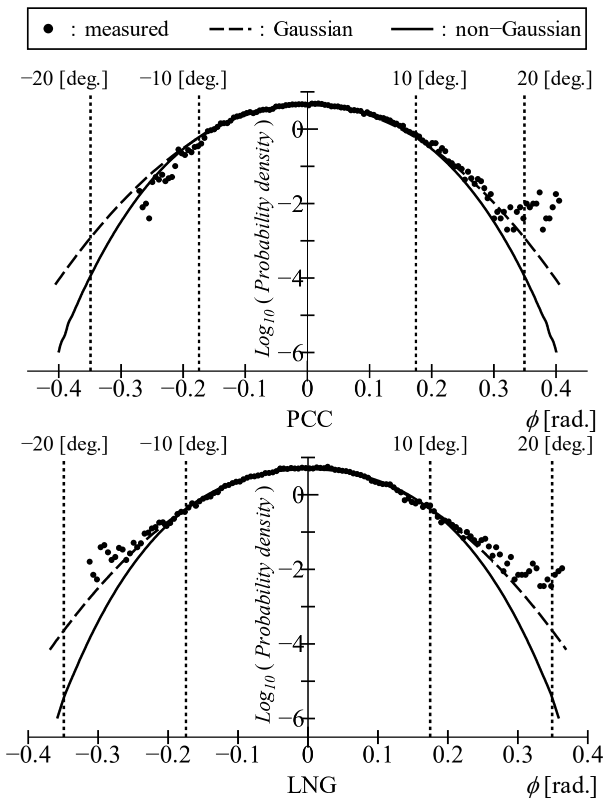

Figure 7: Probability density of roll angle.

In this paper, it is assumed that roll can be expressed by a one degree of freedom motion equation, non-Gaussian PDF is obtained. In order to make the reasons of discrepancies clear, the coupling motion effects on roll in irregular waves may need to be investigated.

5. CONCLUTIONS

In order to propose a rational short-term prediction method including non-linearity of roll, a non-Gaussian PDF is investigated and compared with measured results and Gaussian PDF, and the following conclusions are obtained.

1. The non-Gaussian joint PDF of roll angle and roll rate is utilized for the analysis. Here, this PDF is for the one degree of freedom roll equation with non-liner damping and restoring.
2. It is confirmed that the non-Gaussian PDF indicates the non-linear effects of roll equation by comparing with the Gaussian PDF.
3. The non-Gaussian PDF is compared with measured results, however, there is difference between them.

One of the reasons of the difference may be that the actual roll cannot be represented by the one degree of freedom equation due to coupling effects from sway to roll. In the near future, it will be investigated what equation is suitable for roll motion analysis.

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