Interpretation of results of numerical simulation

Arthur M. Reed, *David Taylor Model Basin, Carderock Division, Naval Surface Warfare Center*

ABSTRACT

Running a numerical simulation of motions in waves is in and of itself of little significance. The results of the simulation—the motion time histories must be processed to produce statistical quantities if they are to be of any practical use. Techniques for dealing with time histories of non-rare and rare events are presented. In the realm of nonrare statistics, the techniques are further divided into statistics for the linear and nonlinear motion regimes. The focus is on non-rare events, but predicting rare event statistics is discussed.

1. INTRODUCTION

The raw output from a time-domain simulation of motions in random seas is of little use, unless the simulation is lucky enough to encounter a rare event—a stability failure that results in the termination of the run. Thus, the simulations must be planned based on the expected outcomes from the simulations. This planning needs to establish objectives as to what will be achieved by performing the simulations.

Without belaboring the planning process, which is worthy of a paper of its own, it is assumed that the interest is in knowing the "aver-age" motion amplitudes, the maximum motions that a vessel would be expected to experience, whether a vessel will have exceeded a particular motion threshold in given operational period in a given sea state or if it could be expected to suffer a stability failure over its lifetime. These are different questions, which are approached using different statistical techniques. This paper will discuss the methods by which answers to both the non-rare and rare problems of seakeeping and ship stability are de-rived from the results of a time-domain simulation of motions in a random seaway. For the most part, the problem of setting of objectives and further planning will not be discussed. whether a vessel will have exceeded a particular motion threshold in given operational period in a given sea state or if it could be expected to suffer a stability failure over its lifetime. These are different questions, which are approached using different statistical techniques.

The first of these questions requires statistical analysis to determine the single significant amplitude (SSA) motion amplitude and the confidence intervals on the SSA motion—the non-rare problem. The other questions, relating to maximum motions and rare problem, will require either an extremely long computer simulation resulting, with a bit of luck in a stability failure, or reliance on statistical extrapolation.

This paper will discuss the methods by which answers to both the non-rare and rare problems of seakeeping and ship stability are de-rived from the results of a time-domain simulation of motions in a random seaway. The problem of setting of objectives and further planning will not be discussed.

2. THE NON-RARE PROBLEM

I n the case of simulations associated with a non-rare problem, either the "average" motions that a vessel will experience under a certain operational condition (loading condition, speed and heading) in a given sea state are computed, or the maximum motions that a vessel will experience in a given loading and operational condition in a specific sea state are determined. Either way, it is necessary to determine the "average" motions—the single significant amplitude (SSA) motions, so that further decisions can be made regarding the statistical approaches that will be employed.

The characterization of a vessels expected maximum motions in a given sea state and condition takes further statistical analysis relatively simple of quite complex, depending on whether the motions are in the linear or nonlinear regime.

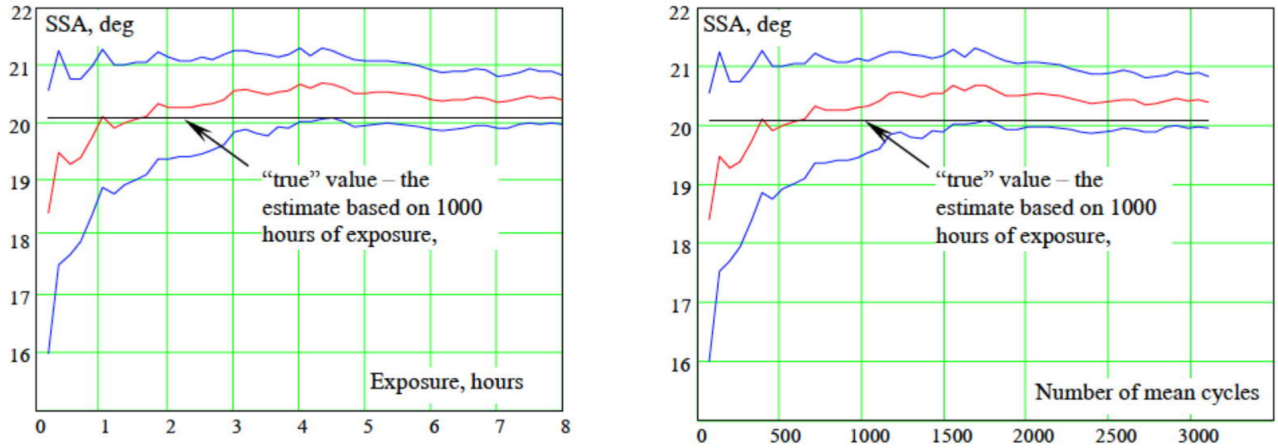

Figure 1: SSA Convergence of predicted roll motion as a function of run length based on synthetic data generated by LS3DoF; left-hand figure as function of time, right-hand figure as a function of cycles. (Courtesy of Vadim Belenky)

The process begins with the computation of the vessels motions for a minimal period of time, typically 3 h[1]. The length of time necessary to characterize the motions with a reasonable certainty is discussed in Reed (2019). Reed (2019) shows that at least 1000–1200 motion responses are necessary — many more responses than recommended in some other references that state that as few as 50 wave encounters are adequate.

For a seaway with a modal period of around 10 s, 1000 wave encounters requires around 3 h of data. However, it should be noted that a vessel does not respond to every wave encounter in every mode of motion, so that in fact it could require 25–30-percent longer than the 3 h to achieve the ideal 1000–1200 responses. Figure 1 shows the convergence of the SSA for roll as both a function of time and number of wave encounters, using synthetic data generated using LS6DoF (K. M. Weems and Belenky 2015). Based on this data, it might even be concluded that 6 h of data and 2500 wave encounters are required for convergence.

The motion computations can be a single run of 3 hs duration, or could be an ensemble of several shorter runs totaling 3 h, say 9 20 min runs. If a single run is employed, then care must be taken to ensure that the autocovariance function of the incident wave train remains well behaved throughout the entire length of the simulation, without any repeats—this requires a great number of Fourier series terms if the seaway is represented by a series with random phases, which is the most common way of generating a seaway for simulations. On the other hand, if a number of shorter runs is used, to ensure that the runs are statistically independent, unique wave seeds must be used to initialize seaway for each run.

To compute the SSA motions and confidence intervals for the motions of interest, the variance and variance of the variance of the motion time histories are computed (Belenky, Pipiras, and K. Weems 2015; ITTC 2017; Pipiras et al. 2018). Given the variance and the variance of the variance, the standard deviations of the motions are calculated as the square root of the variance and the SSA is twice the standard deviation. The confidence intervals follow in a similar manner, based on the confidence intervals of the variance.

If the only requirement is to predict the "average" motions, the SSA of the motions, that a vessel will experience while operating at a condition in a given sea state, this completes the process. This process must be repeated for every speed, heading to the seas, loading condition and seaway—significant wave height and modal period.

When it is necessary to predict the maxi-mum motions that a vessel will experience in a given condition in a particular seaway or to determine whether a vessel will exceed a particular motion limit or criteria, then additional statistical analysis is required. Computationally and statistically both of these questions are answered in a similar manner. Assuring, with a reasonable confidence, that the vessel does not exceed an operational limit only

[1] Unless otherwise noted, all times will be full-scale durations.

requires comparing the expected maximum motions against the requirement to see if that limit will be exceeded.

The statistics used to predict the maximum expected motions depend on the magnitude of the motions that are expected and the vessels hull form. The magnitude of the motions and the hull form determines whether the statistics are being analyzed in the linear motion regime or the nonlinear motions regime, and thus the statistical models that are required.

The process in the linear regime

If the motions are in the linear region then the problem is simple, while if the motions are in the nonlinear regime, then statistical extrapolation must be employed. Significantly greater simulated time is required for predictions in the nonlinear regime. For roll, the motion which this paper will focus on, linearity depends on the GZ curve, linearity applies as long as the initial range of the GZ curve relatively constant slope—for virtually all vessels, it can be reason-ably assumed that the motions are linear through 25º or 30º. This is where the expected motion amplitude comes into play, if the vessels motions will not exceed the linear response regime then it should not be necessary to simulate more than the 3 h of motions used to determine the SSA motions.

For motions in the linear regime the maximum expected motions ae purely a function of the standard deviation (σ) of the motions, and the only decision is whether to use σ or to be conservative and use a "σ" based on the upper confidence limit for the motions. The key here is that ship motions are assumed to be Gaussian and for narrow banded seas, the motions are equally or even more narrow banded due to the ship being a well-tuned filter for those modes of motion for which there is a restoring force. Thus the extremes of the process are Rayleigh, and for linear statistics the extremes of the Rayleigh distribution are directly related to the standard deviation of the motions (Ochi and Motter 1973; Ochi 1998).

For a given number of responses, there are available tables that give the expected extreme motions with a 95-percent confidence limit, i.e., 95-percent of the responses will be less than this limit (SNAME 1989, p. 91). The 95-percent non-exceedance maximum amplitudes, $\hat{y}_n$, are:

$$n = 100 \quad \hat{y}_n = 3.90\sqrt{m_0}$$

$$n = 1000 \quad \hat{y}_n = 4.45\sqrt{m_0}$$

where n is the number of cycles over which the limit is to apply and m_0 is the variance of the motions ($\sqrt{m_0}$ is the standard deviation). For motion limits, n = 1000 is a good choice, as most storms only last about 3 h, which corresponds to approximately 1000 wave encounters. SNAME (1989) provides no source for the above $\hat{y}_n$ limits, but equation (6.19) of Ochi (1998) provides a generalized formula for computing the limit:

$$\hat{y}_n = \sqrt{2\ln(n/\alpha)}\,\sqrt{m_0} \qquad (1)$$

where α if the fraction of cycles that are to exceed the limit, and m_0 is as before. In the table above α is 0.05 (= 1 − 0.95).

Equation (1) is sufficient to assess the expected motions of a vessel based on its motion time history. However, it can also be used to determine whether the vessel meets a limiting criteria, and to determine the acceptable SSA motions for a vessel to satisfy a criteria.

As a *totally fictitious* example, if there were a requirement that a cruise ship not exceed 25º of roll in a storm, the formula $\hat{y}_n = 4.45\sqrt{m_0}$ could be inverted to determine that the SSA based on the computed motions should not exceed 11.2º (11.2º = 25º/2.225, where 2.225 = 4.45/2).

Based on the above, it obvious that it is easy to assess the interpret the results of a simulation when the motions are in the linear regime. How-ever, when the motions are extreme, and thus outside the linear regime the interpretation be-comes more complex and requires the simulation of longer time histories.

The process in the nonlinear regime

In the event of needing to characterize non-rare motions in the nonlinear regime, requires the development of the statistical distribution of the motions that have been predicted so that tail of the distribution can be evaluated to determine the probability of a certain motion level being exceeded. This is accomplished by fitting an appropriate statistical distribution to a histogram of the predicted motions, which in turn requires sufficient data for the histogram to represent the tail with sufficient fidelity.

There is not a good definition of what is enough data. The American Petroleum Institute (API) (API 2005) in their guidance for model testing states that to characterize ship motions, 3 h of data should be collected, and that to characterize extremes that at least five times more data is required. Extending the API guidance for model testing to simulations and assuming that motions in the nonlinear regime are extreme motions, that would say that a minimum of 15 h of motion data is required. K. M. Weems, Belenky, and K. J. Spyrou (2018) have used 50 h of data for their studies on statistical extrapolation (obtained as 100 1/2 h data sets). However, they have not performed any convergence studies to determine minimum data requirement—they obtain satisfactory results with 50 h of data for their cases. So it appears that somewhere between 15 and 50 h of motions must be simulated for statistical extrapolation, *for each condition that includes nonlinear motions*. Yet other researchers have used 100 h of data (Glotzer et al. 2017)

As stated above, the statistical extrapolation process requires fitting a statistical distribution to a histogram of the time-history data from the simulation. Knowledge of the appropriate statistical distribution affects the amount of date required, as it influences the number of parameters that need to be determined to define the distribution for extrapolation. If the motions are in the linear range, then the normal distribution is appropriate and only one parameter needs to be determined, the standard deviation (as has been described above, statistical extrapolation is not required if the data is Gaussian). Figure 2 shows a histogram with a distribution fit and illustrates statistical extrapolation.

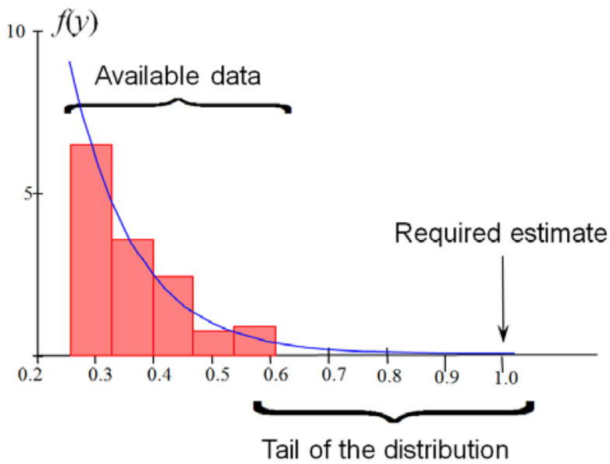

Figure 2: Tail of histogram fit with a GPD, showing extrapolation. (Campbell et al. 2014)

When the motions data is from the non-linear range, then usually the most general of distributions, the generalize Pareto distribution (GPD) (Pickands 1975; R. L. Smith 1987) must be employed. The probability density function (*pdf*) of the GPD is defined as:

$$f_{(\xi,\mu,\sigma)}(x) = \frac{1}{\sigma}\left(1+\frac{\xi(x-\mu)}{\sigma}\right)^{\left(-\frac{1}{\xi}-1\right)}$$

for $x \geq \mu$ when $\xi \geq 0$, and $\mu \leq x \leq \mu - \sigma/\xi$ when $\xi < 0$; where ξ is the shape, μ is the threshold (also called the location in the literature) where GPD starts to be applicable and σ is the scale. For $\xi = 0$ the GPD is the exponential distribution. If the tail of the distribution is above the exponential distribution the distribution has a "heavy tail; $\xi > 0$ and is defined for all $z \geq 0$. However, if the tail of the distribution lies below the exponential distribution the distribution has a "light tail; $\xi < 0$ and $0 \leq z \leq -1/\xi$. Figure 3 illustrates heavy and light tails relative to the exponential distribution.

The threshold is more of a parameter for the GPD, than a value describing the character of the distribution. The GPD is used in particular to fit the tail of distribution and is not appropriate for approximating an entire distribution over its whole range of support. Therefore, the choice of the threshold is not particularly critical to the fit of the distribution. If the threshold is chosen too small, portions of the underling distribution that are inappropriate to the describing the tail of the distribution will be included, and if too large a threshold is chosen, useful data for defining the tail will be excluded. Thus, several choices for the threshold should be used and the smallest one that does not appear to affect the details of the tail chosen, as it should minimize uncertainty.

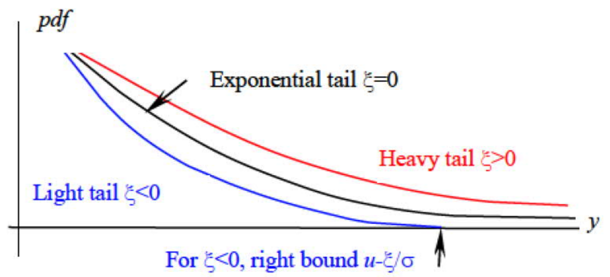

Figure 3: Heavy and light tails of a distribution relative to an exponential distribution. (Belenky, K. Weems, Pipiras, et al. 2018)

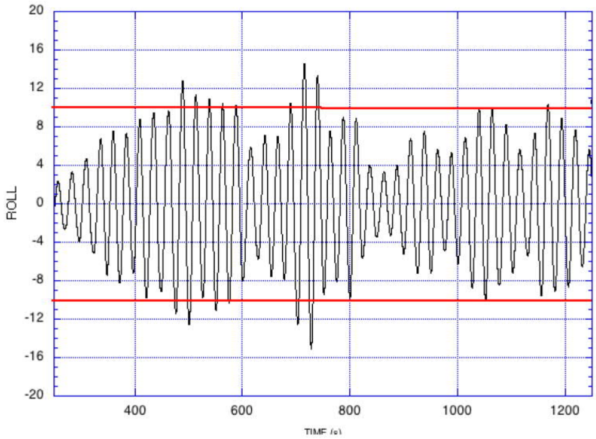

Figure 4: Peaks over threshold (POT) for a Roll Time History, heavy horizontal lines are the threshold.

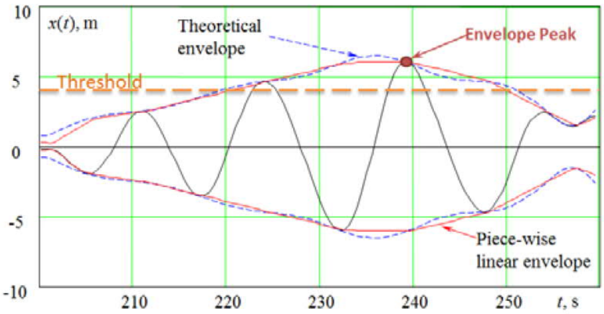

Figure 5: Envelope peaks over threshold (EPOT) for a Heave Time History (Campbell et al. 2014)

The scale and shape parameters are the ones that need to be fitted to define the tail of the GPD distribution. The need to accurately deter-mine these parameters will have a significant influence on the length of the simulation that must be run—the amount of data required to fit these parameters with reasonable accuracy.

There are a number of papers that describe fitting a GPD to ship motions time history data. As it is necessary to only fit the tail of the histogram, these papers apply either one of two methods to exclude the majority of the data, the data that makes up the peak of the histogram. These methods are peaks over threshold (POT) and envelop peaks over threshold (EPOT)[2]. In the EPOT approach, an envelope is constructed connecting the peaks and reflected troughs motion time history. The envelope can be deter-mined by taking the Hilbert transform of the time history (), or by brute force connecting the peaks and reflected troughts with straight lines—either is satisfactory for the purpose of determining the peaks above the threshold. Figure 4 shows an example of a POT using ±10º as the threshold, and Figure 5 shows an EPOT, for a different roll time history, again using a 10◦ threshold. Either the POT or EPOT approach is acceptable, though one must use a statistically independent set of peaks, so the clustering that results from the POT is less ideal than the EPOT approach, as one must eliminate clustered (adjacent) peaks, selecting only the maximum from the cluster, when using the POT approach. All the papers mentioned in the following discussion use EPOT.

The earliest of the papers fitting a GPD to the data is Campbell et al. (2014). T. C. Smith and Zuzick (2015) (and T. C. Smith 2019) per-form a formal validation of statistical extrapolation methods for predicting the tail of the distributions for roll, pitch and vertical and lateral acceleration. They employ two methods to determine the confidence intervals of their fit distribution, one that assumes a normal distribution for the distribution of the scale and shape parameters, and the other follows the method used by Campbell et al. (2014), except that they use the logarithm of the scale parameter to ensure that it remains positive. More recently Belenky, Glozter, et al. (2016) have used the GPD to study the nature of the tail of the extreme roll distribution.

As the tail of the roll distribution is fat (Belenky, K. Weems, Vladas Pipiras, et al. 2018), it is possible to make use of that fact to simplify the statistical extrapolation of roll by using a power law—Pareto distribution (PD) to fit the tail rather than the GPD. The *pdf* of the PD is defined as:

$$f_{(x_m,\alpha)}(x) = \frac{\alpha x_m^{\alpha})}{x^{1+\alpha}}$$

for $x \geq x_m$, where x_m is the threshold and α is the exponent (equivalent to 1/ξ in the GPD). The threshold of the PD serves the same function as the threshold of the GPD, so the PD only has one parameter, the exponent, that defines the tail, reducing the length of record (amount of data) needed to define the distribution, and rigor-ous methods for determining the exponent (Beir-lant et al. 2004; Dupuis and Victoria-Feser 2006; Mager 2015).

[2] Note that this threshold in not the same threshold that is used in the definition of the GPD.

Glotzer et al. (2017) have evaluated a number of methods for fitting the confidence interval (particularly the upper bound) for the exceedance probability in the GPD framework: the normal method, the lognormal method, the boundary method, the bootstrap method, the profile (likelihood) method, and the quantile method. Glotzer, et al. use the maxim likelihood method for estimators ξ and σ, employing both direct and quantile methods. They conclude that the quantile method based on profile likelihood works best, and the bootstrap method the poor-est. They also find that the normal and lognormal methods are slightly anticonservative.

In an effort to reduce uncertainty, Glotzer et al. (2017) examine using knowledge of the expected motion responses to further refine the fit. In particular, they take advantage of the fact that if the roll exceeds a certain limit, that capsize will result, and the fact that pitch is typically limited to 12º–15º, based on the shape of the longitudinal GZ curve. These limit dictate that the shape parameter of the GPD will be negative, and determine its value. Resulting in the need to fit only a single parameter, the scale parameter, σ.

3. THE RARE PROBLEM

It should be recognized that the simulation of a single stability failure is of little statistical significance—what if the vessel were to experience the 1-in-100,000 wave in the first few minutes of the simulation? And, in the case of predicting stability failures such as capsize in the dead-ship condition, one is seldom lucky enough to predict a failure in a reasonable length simulation. Proving that this is a truly rare stability failure in a random seaway, would require the simulation of many thousands of additional hours of motion histories. Therefore, another approach to predicting the occurrence of actual rare events.

The split-time method appears to be the most feasible way of assessing stability failure. However, it must be noted that the split-time method does not rely on a single time history of motions, but rather relies on repeated perturbations of a motion time history to identify up-crossings at high enough rates so as to result in a stability failure. This requires a custom modification of a motion simulation code and thus is in reality beyond the scope of the effort defined by the title of this paper—interpretation of the results of a numerical simulation.

The split-time method was first reported in Belenky, K. M. Weems, and Lin (2007) and Belenky, K. M. Weems, and Lin (2008), where roll at zero speed in beam seas was analyzed. The essential idea behind the split-time method is that of breaking the motion responses into nonrare and a rare portions. The motions are predicted in the usual manner until the predicted motion amplitude exceeds a pre-established threshold. At the point the simulation is halted and the state recorded. Then the motion predictions are continued for a a few cycles to oscillating about its upright equilibrium position or proceeds to a stability failure. The motion predictions are then repeated from the state where the threshold was exceeded with the roll rate at the moment of exceedance perturbed upward or downward to identify the critical roll rate at up-crossing that defines the boundary between stable motion equilibrium and stability failure. Figure 6 illustrates this process.

A series of the “distances” of the roll rate from that dividing rate is used as a metric to de-fine the exceedance rate, accumulated over an extended period of time—50 to 100 h, is fitted with a GPD to determine the exceedance rate. This process must be repeated multiple times to assure that the results are statistically consistent.

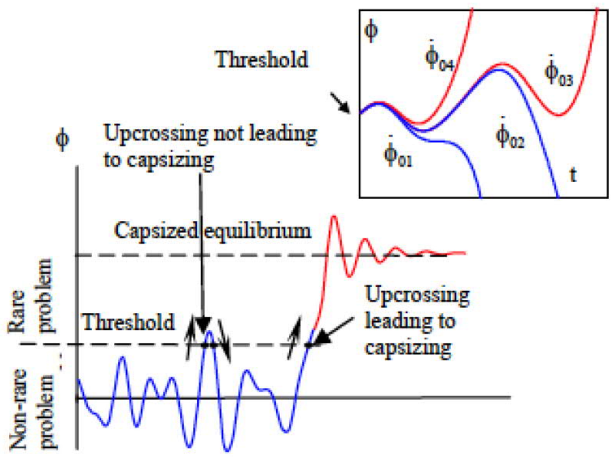

Figure 6: Split-time method at zero speed in beam seas, showing extrapolations with different roll rates at the threshold—threshold is constant. (Belenky, K. Spyrou, et al. 2012)

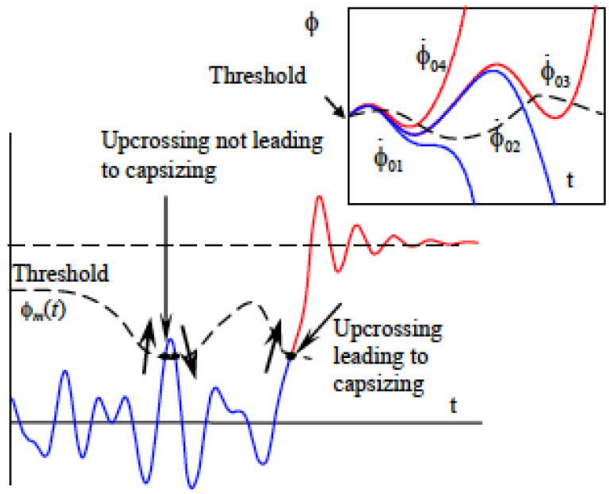

Figure 7: Split-time method at speed in bow quartering seas, showing extrapolations with different roll rates at the threshold—threshold varies with time (as a function of attitude on waves). (Belenky, K. Spyrou, et al. 2012)

The problem described above is idealized, in that the righting-arm curve of a vessel in beam seas is essentially constant—like that of a vessel in calm water. In bow or stern quartering seas, the righting-arm curve becomes time varying, complicating the problem even further. The extension of the split-time method to an unsteady righting arm curve is illustrated in Figure 7.

Belenky, K. M. Weems, Lin, and K. Spyrou (2010) extended their split-time method model to forward speed in bow quartering seas to deal with this more complicated problem, of a time varying righting-arm curve and began to discuss the application of the split-time method to surf riding and broaching, Belenky, K. Spyrou, et al. (2012) further extended their bow quartering seas and surf-riding analyses. Belenky, Pipiras, and K. M. Weems (2013) extended split-time method to pure loss of stability in waves, which requires a rigorous assessment of the instantaneous roll restoring force in waves. All of the above work is summarized in Belenky, K. Weems, and Lin (2016).

K. M. Weems and Belenky (2018) and K. M. Weems, Belenky, and K. J. Spyrou (2018) present a validation of the split-time method using a simplified model for predicting the motions, this simplified model allows the simulation of hundreds of thousands to millions of full-scale hours of motions in extreme seas in a few days. Each of these extended runs produces a few hundred stability failures, allowing the calculation of exceedance rates against which the results of the split-time method exceedance rates can be compared. Belenky, K. Weems, Pipiras, et al. (2018) use this data to study the tail of the distributions of the metric used to determine the critical roll rate.

4. SUMMARY

The characterization of ship motions in the linear and nonlinear regimes is described. In the linear regime, the extremes can be easily characterized using the standard deviation of the motions. In the nonlinear regime, an extended simulation length is required for a reasonable pre-diction of the tail of the statistical distribution to be determined—this tail in turn can be evaluated to provide estimates of the probability of extreme motions. The Generalized Pareto Distribution and Pareto Distribution are used for these fits. To facilitate the fitting of the tail to a histogram of the motion data a peaks over threshold (POT) or preferably an envelope peaks over threshold (EPOT) technique is employed to eliminate the smaller motions from the histogram.

It is not reasonable to directly observe stability failures using a time domain ship motion simulation tool. Therefore advanced techniques such as the split-time method must be utilized. A high-level over view of the split-time method is provided with many references to the implementation of the method. Even with the use of the split-time method, the prediction of exceedance rates for stability failures is not trivial.

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