

Evolution of p-factors in the probabilistic damage stability assessment of passenger ships

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ABSTRACT

The paper complements an earlier publication by the authors addressing the probability of survival in the IMO framework for damage stability assessment, the s-factor. The focus here is on the probability of occurrence of a certain damage scenario (breach), conditional on its dimensions and location (centre and port or starboard side), the p-factor. Pertinent assumptions and limitations are explained, following its evolution for specific application to passenger ships. Attempts to provide analytical descriptions of the damage breach distributions as tetrahedra shapes positioned along the ship length whilst accounting for changes in ship geometry, structural arrangements, and subdivision for consumption by the wider profession has led to misconceptions and misunderstandings of what exactly the p-factor is in the context of probabilistic damage stability calculations. This is evidenced by the fact that the same original damage breach distributions, derived in Project HARDER, based on largely cargo ships with age spread over the last three decades of the previous century, are still being used today for all ship types, including modern passenger ships. Filling this gap, a new database for passenger ships developed in the EC-funded Project FLARE, is briefly presented, leading to new damage breach distributions specifically for passenger ships. It is believed that this paper will throw considerable light in enhancing understanding on the p-factor, which has been cluttered with unnecessary complexity from the outset.

Keywords: *Ship damage stability, probabilistic and direct methods, damage breach distributions, p-factor.*

1. INTRODUCTION

The probabilistic assessment of ship survivability after an accident should be a comprehensive process estimating the conditional probability of losing ship stability in the wake of a casualty. Even though the definition of a probabilistic framework developed for the last 30 years, the actual regulations imposed by SOLAS 2009/2020, (SOLAS, 2009), incorporate just a few elements of the provided research output. The only cause of accident included in the SOLAS framework is collisions, totally neglecting other sources of hazards for ships as groundings that could be more frequent and dangerous for certain kinds of vessels as passenger ships. Furthermore, SOLAS provides a

classification of the safety level of a ship based on the evaluation of indices instead of promoting a direct approach for the estimation of flooding risk. In such a case, the resulting probabilistic assessment neglects relevant aspects for ship survivability as the operational area and operating environment, the structural arrangements, the breaches definition and distributions and the vessel type. More importantly, focusing on indices, as a substitute for direct assessment of flooding risk, deprives such assessment of the time element, hence crucial information on measures to affect improvements on the evolution of flooding leading to capsizing as well as evacuation arrangements and associated Risk Control Options (RCOs) affecting evacuation in such scenarios.

On the other hand, SOLAS regulation provides a clear logic to evaluate ship survivability through an Attained Subdivision Index (A-Index):

$$A = \sum_{j=1}^J \sum_{i=1}^I w_j \cdot p_i \cdot s_i \quad (1)$$

Where,

- j represents the loading condition under consideration.
- J represents the total number of loading conditions considered in the calculation of A, usually three draughts covering the operational draught range of the vessel.
- w_j represents a weighting factor applied to each initial draught.
- i represents each compartment or group of compartments under consideration for loading condition j .
- I is the total number of all feasible damage scenarios involving flooding of individual compartments or groups of adjacent compartments.
- p_i is the probability that, for loading condition j , only the compartment or group of compartments under consideration are flooded, disregarding any horizontal subdivision.
- s_i accounts for the conditional probability of survival following flooding of the compartment or group of compartments under consideration for loading condition j , weighted by the probability that the space above a horizontal subdivision may not be flooded.

The use of A-Index as a safety measure gives a fully decoupled approach for the determination of flooding probability (p-factor) and ship survivability (s-factor), as it was clear since the first studies of Wendel on probabilistic damage stability assessment (Wendel, 1960, 1968). This simple but efficient distinction between casualty occurrence (p) and its consequence (s) can be used to incorporate research outcomes of the last decades in the field of ship safety. The present work gives a detailed overview of the enhancements provided within the FLARE project concerning the definition of p-factors,

including relevant aspects of ship safety neglected or ignored by the current SOLAS regulation.

2. P-FACTOR DEFINITION

Whilst the s-factor relates to the probability of a ship surviving a given damage (breach) in each loading condition and environment (Vassalos and Mujeeb-Ahmed, 2021), the p-factor is used to define the probability of occurrence of a certain breach, in each one of the pertinent hazards (collision, side and bottom grounding) conditional on its dimensions and location (centre and port or starboard side). This entails the need of probabilistic information pertaining to each of these elements, which is provided by the marginal distributions of the breach dimensions and location. Breaches are defined as 3-dimensional objects (location, side, and vertical position along the ship length). Deriving from this, damage breaches are often thought of and described as cuboids, however, this is not always the case. In areas where there is curvature in the vessel waterline, i.e., outside of the parallel mid body, the damage breach ceases to be described as a cuboid. Instead, the penetration element of the damage breach follows the profile of the waterline corresponding to the draught being examined, offset by the penetration L_y . The p-factor is unaffected by this assumption as the dimensional properties of the damage remain the same. Instead, the geometrical properties of the breach are changed, see Figure 1. However, the spaces affected by the damage breach can vary.

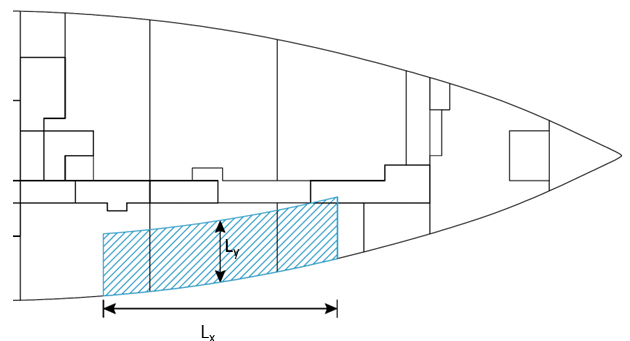


Figure 1: Breach definition along the ship side

All the clutter in the literature relates to how the p-factor is addressed in current SOLAS and how damage breaches are defined, concerning several pertinent characteristics:

- **SOLAS approach to defining and using the p-factor:**

“p-factor is the probability that, for a given loading condition, only the compartment or group of compartments under consideration are flooded, disregarding any horizontal subdivision”. SOLAS is still referring to “compartments” for collision damages only, using statistical data of breaches that relate to the last three decades of the previous century, the majority of which relate to cargo vessels; hence disregarding all related information of modern passenger ships, ship size, speed and structural arrangements, hence material and speed for the vessel under consideration. Moreover, the statistical database available in SOLAS includes allisions and contacts as part of the hazards. If there is no breach then there is no p-factor as its definition is conditional on heaving a breach of given dimensions, location, and position.

- **Whether the distributions are marginal or conditional probabilities:**

Even though marginal distributions are supposed to be independent, attention should be paid to the damage penetration. The SOLAS framework implicitly assumes that for a collision damage breach the ratio between dimensionless penetration and dimensionless length cannot exceed 15. Therefore, an upper limit should be introduced, having as main consequence that damage length should be generated before damage penetration. Specific reference to this is made in the explanations provided for Figure 4 in the following.

- **Derivations of the breach distributions based on statistical or direct approaches:**

Crash analysis using verified numerical Finite Element codes, e.g., LS-Dyna or faster super-element codes, e.g., SHARP, as expanded upon later, are widely available, offering potential to address collision and grounding hazards for a specific ship in specified operational scenarios and environmental conditions. Yet, the profession continues to rely on statistical methods, using incomplete or in the case of passenger ships irrelevant statistical data, pertaining to cargo ships, for the definition of damage breaches.

- **Zonal or non-zonal approaches and definition of breaches in each approach:**

The reference of SOLAS to compartments, i.e., physical boundaries to be used in the integration of the probability distributions of breaches to derive the p-factors is still creating problems between the traditionalists and modern naval architecture. Former believe that the p-factor should be calculated with the help of the law of total probability, resulting from Kolmogorov axioms, as it is in the SOLAS Convention. Using MC sampling of the damage breach distributions is unable to calculate the true value of the A-index. As such, it is of no value for Naval Architects. This is the alienated view being referred to in the paper title. Notwithstanding the above, there are varying views on how to use the non-zonal approach with confusion being the standard situation.

- **Sampling methods for numerical simulations/calculations of ship survivability:**

Even though, there is some general guidance based on sampling error, there is no rigorous approach to define sample size for use in simulations/calculations, such number varying from 1,000 samples to 100,000, based on how closely the breach distributions are represented but without any reference to the reliability of data or the impact on damage stability calculation in using different sample sizes.

- **Crashworthiness considerations:**

The question of using crashworthy ship structures to positively affect (reduce) damage breach distributions is another element where confusion prevails, in terms of what exactly this is, how it can be used to improve damage stability, how it is calculated and how it is applied optimally. In particular, the fact that the probability term implicit in the p-factor does not change; only the condition pertaining to the damage size in a given location in the ship. As a result of this, even though the concept has been around for decades it has not found any real application in ship design in so far as damage stability is concerned.

Each of these aspects will be further addressed in the following sections.

SOLAS Damage Breach Distributions

The derivation of p-factors, currently in use in SOLAS, originates from the HARDER project (HARDER, 1999-2003), (GOALDS, 2009-2012), (Bulian and Francescutto, 2010), (Lützen, 2001)

during which collision damage statistics were processed to obtain probabilistic damage breach distributions, in terms of damage longitudinal position, longitudinal extent, transversal extent, the upper limit of vertical extent and side of damage (port/starboard). The mathematical integration of these distributions over box-shaped domains allows expressing the p-factors in the known analytical format of SOLAS on ship subdivision. The SOLAS underlying damage distributions have been obtained by pooling collision accidents of all types of ships available at the time, spanning the last 3 decades of the previous century. Moreover, the damage distributions do not explicitly consider the structural design, or crashworthiness of the ship. Practically, this implies that even if a ship is designed with a high crashworthiness level, no gain is to be expected in terms of safety in the framework of the current regulations. A second consequence is that SOLAS damage distributions embody an ‘average’ crashworthiness level of the historically damaged ships, which is not necessarily representative of a specific type of ship, or applicable to any type of ship., and, in particular passenger ships, especially the modern giants populating the current fleet.

More specifically, it is acknowledged that the collision statistics include in the main accidents involving cargo ships and tankers, Figure 2.

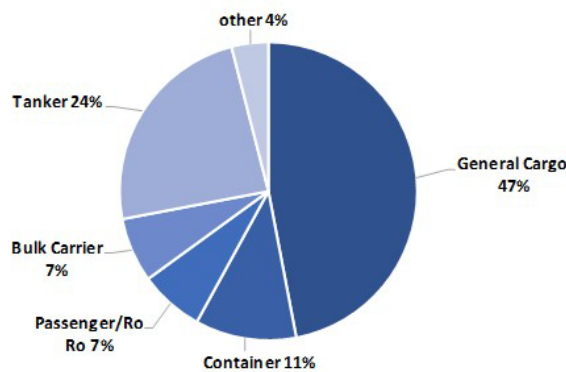


Figure 2: Ship-type breakdown in collision statistics, Project GOALDS, (2009-2012).

Geometrically, a collision-type damage is idealised in SOLAS as a box with two faces parallel to the waterplane, two faces parallel to the ship transversal plane and two faces following the hull longitudinal shape at the waterline. Furthermore, the damage box crosses the waterline as well as one side of the ship. In the general case, the damage is

modelled using the 6 geometrical parameters (L_x , L_y , Z_{UL} , Z_{LL} , damage side), illustrated in Figure 3.

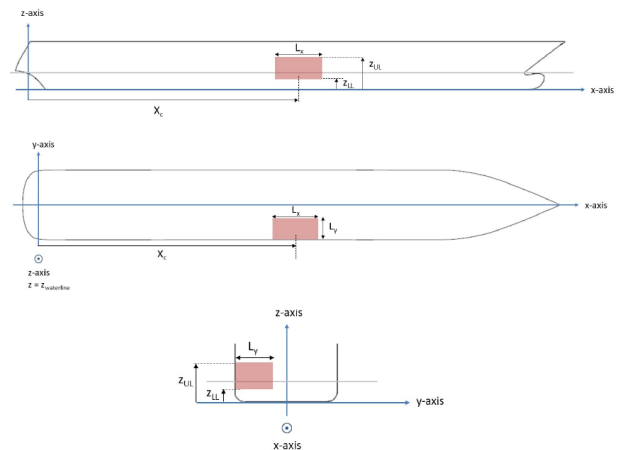


Figure 3: Geometric properties of a damage breach

From a probabilistic point of view, the SOLAS underlying damage breach distributions associated with each potential damage parameter are exemplified in Figure 4. This figure provides an overview of the geometrical model of a collision breach, together with the independent marginal cumulative distributions of the breach characteristics in non-dimensional form. Starboard and portside damages are equiprobable. The damage is defined as potential, meaning that it could extend also outside the vessel limits. This aspect requires particular attention concerning the positioning of the damage at the ship extremities, keeping consistency with the analytical formulation of zonal p factors. In case the potential damage is fully contained within the ship length L_s , L_x corresponds to the damage centre. If the damage partially extends outside the vessel, then the location of X_c should be changed as described in (Bulian and Francescutto, 2010). Even though marginal distributions are supposed to be independent, attention should be paid to the damage penetration L_y . The SOLAS framework implicitly assumes that for a collision damage breach the ratio between dimensionless penetration and dimensionless length cannot exceed 15. Therefore, an upper limit $L_{y\max} = 15 \cdot B \cdot L_s / L_x$ should be introduced, having as main consequence that damage length should be generated before damage penetration. As a last remark, the internal limit of the damage follows the waterline at $z \leq T$ shifted by L_y , then the collision damage is not always box-shaped.

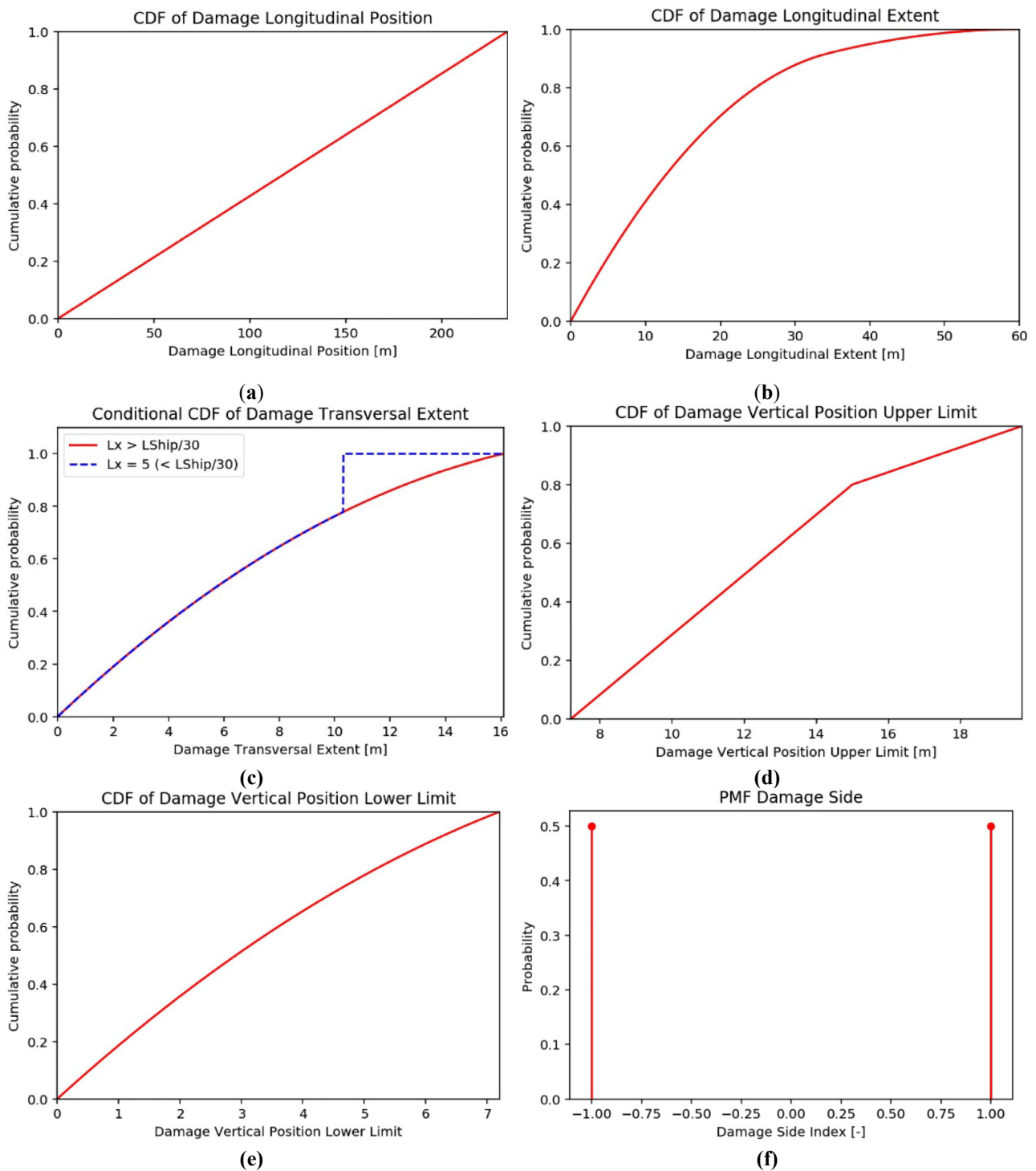


Figure 4: (a) Damage centre longitudinal position cumulative distribution function; (b) Damage longitudinal extent cumulative distribution function; (c) Damage transversal extent conditional cumulative distribution function; (d) Damage vertical position upper limit cumulative distribution function; (e) Damage vertical position lower limit cumulative distribution function; (f) Damage side index probability mass function.

Deriving p -factors using Zonal and Non-Zonal Damage Breach Distributions

In the zonal approach to probabilistic damage stability, currently adopted by IMO, collision damage cases are defined as three-dimensional cuboids, as outlined previously. These are determined following discretisation of the vessel

subdivision into zones, which can be conducted either in line with physical subdivision boundaries or “virtual” boundaries. Damage probabilities (p -factors) are then derived for each of these zonal damages, and combinations thereof, using damage statistics in the form of marginal distributions, as provided in Figure 5. Damage breach p -factors are then generated by integrating the joint probability

function of non-dimensional damage location and non-dimensional damage length $f(\bar{x}, \bar{y})$ with respect to each damage zone and combination of zones. The resultant probability then accounts for the occurrence of all damage cases that would fall within the range of either a single zone or a combination of zones.

As non-dimensional damage location and non-dimensional damage length are considered independent parameters, their joint probability density function can be expressed as shown in Equation 2.

$$f(\bar{x}, \bar{y}) = a(\bar{x})b(\bar{y}) \quad (2)$$

The respective p-factor for a given damaged zone or combination of zones can then be calculated through the integration of the underlying probability functions for length and location as follows:

$$p(\bar{x}_1, \bar{x}_2) = \int b(\bar{y}) \int a(\bar{x}) d\bar{x} d\bar{y} \quad (3)$$

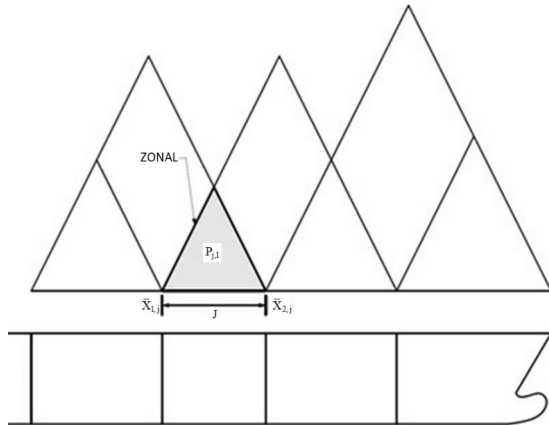


Figure 5: 1-Zone damage example using the zonal method

Once the probability of damaging a given zone or combination of zones is known, the zonal approach then requires two additional reduction factors r and v to account for the probability of differing degrees of damage penetration and height, respectively. The purpose of these factors is to weigh the damage probability in a manner reflective of the underlying damage distributions. Therefore, the final p-factor for a given zonal damage described by location, length, penetration, and height is given by Equation 4.

$$p_i = p(\bar{x}_1, \bar{x}_2) \cdot r(\bar{x}_1, \bar{x}_2, b_i) \cdot v(H_d) \quad (4)$$

In contrast, the non-zonal approach works by sampling pertinent damage probability distributions to produce a multitude of damage breaches, characterised by size and location. For this purpose,

Monte Carlo sampling is generally adopted to create a damage sample from the marginal damage distributions previously described. The process utilises inverse transform sampling, which involves inverting the cumulative distribution function (CDF) of a given random variable, say F_X , to produce F_X^{-1} . Random numbers, u , are then generated from a uniform distribution in the interval $[0, 1]$ and are inputted into the inverse CDF to solve for variable x , see Figure 6. This creates a sample population of the random variable being addressed that is representative of the underlying distribution.

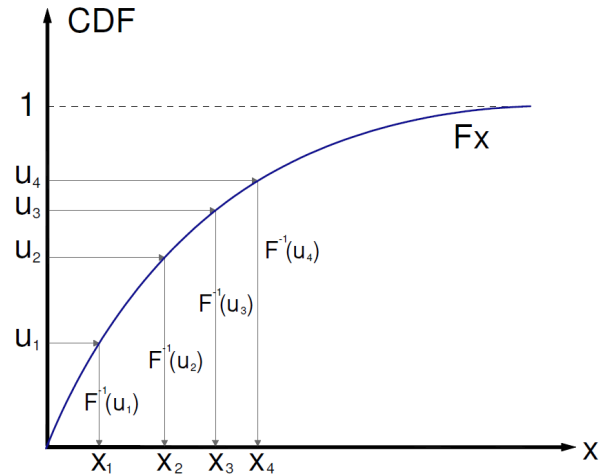


Figure 6: Inverse transform sampling

By applying the above process to each of the damage distributions, shown in Figure 4, damage breaches can be constructed by combining the output from each sampled distribution. For a given vessel, each of these breaches will lead to a certain combination of rooms having been compromised. Inevitably, a number of damage breaches will lead to the same rooms being affected, thus creating a smaller subset of distinct damage cases for use in the Attained Index calculation. The frequency of repeated cases is then used to determine the damage probability (p-factor), corresponding to n/N , where n is the number of breaches damaging the same compartment (thus referring to a damage case) and N is the total number of breaches generated (sample size). Figure 7 below provides an illustrative example of the difference between zonal and non-zonal approaches. Here, on the left, the traditional zonal approach can be observed, where the probability of damaging the single zone highlighted is determined by integrating the joint probability function of damage location and damage length with respect to the extremities of the zone and its location. In contrast, on the right-hand side the non-zonal

approach is demonstrated, where individual sampled damage scenarios appear as unique points all affecting the same single-zone domain. Here, the damage probability for this one zone damage would be determined as n/N , which if a damage sample size of 1,000 scenarios is assumed, would lead to a p-factor of $6/1000=0.006$. If a sufficient damage sample is taken, the zonal and non-zonal p-factors will converge to the same value.

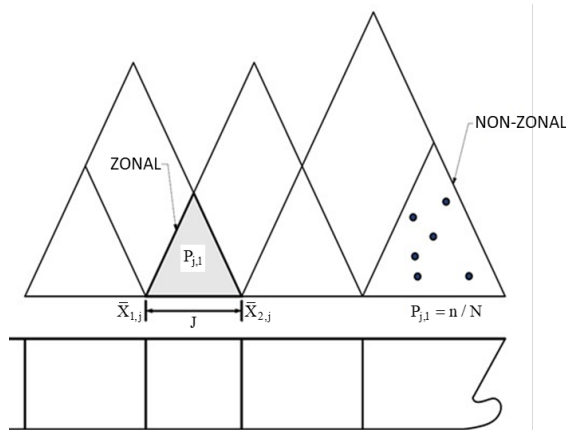


Figure 7: Zonal Vs Non-Zonal 1-Compartment Damage

The determination of p-factor is detailed in the HARDER project (HARDER, 1999-2003), Lützen (2001, 2002), Pawlowski (2004), and amended in SOLAS2009 probabilistic framework, IMO (2006), catering for collision hazards only. However, collisions are not the only possible hazard constituting the flooding risk for a ship, especially for passenger ships. For the latter, lack of due consideration for grounding (side and bottom) hazards at IMO level over the past few decades, catering for these through deterministic requirements, has shifted the flooding risk focus with side and bottom groundings constituting now the majority of the flooding risk for passenger ships. Figure 8 is indicative of the current situation with flooding hazards for passenger ships.

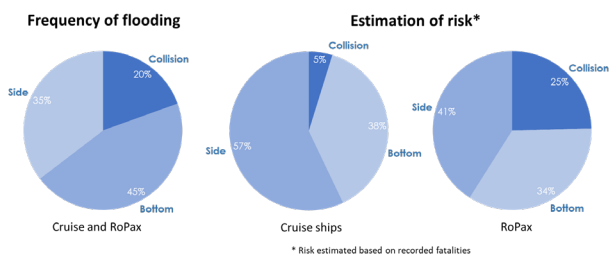


Figure 8: Recent statistics on the flooding risk of passenger ships, Project FLARE, (2019-2022).

Whilst SOLAS is still very relevant and the requirements for collision are still valid, there is a

growing need for revision by adopting a more holistic regulatory framework accounting suitably for all pertinent hazards. Figure 8 from Project FLARE is indicative of the current situation with flooding hazards for passenger ships. The alarm for this state of affairs has been raised in the past, (Papanikolaou et al., 2004; Papanikolaou and Eliopoulou, 2008).

Notwithstanding this, research on the topic of grounding hazards has been persistent and systematic, with significant contributions at IMO level from Projects SAFEDOR (2005-2009), GOALDS (2009-2012), EMSA III (2013-2015) and eSAFE (2017-2019) but IMO rejected to include this in pertinent regulations. Specific developments include an accident database addressing all hazards, (Mujeeb-Ahmed et al., 2021a) and leading to new damage breach distributions, (Mujeeb-Ahmed et al., 2021b). Directly related to p-factors determination in probabilistic damage stability calculations, a non-zonal approach for breach generation has been developed, e.g., (Zaraphonitis et al., 2015; Bulian et al., 2016), as well as calculations of all pertinent indices and their combination, based on the current IMO framework and accounting consistently for all hazards, (Zaraphonitis et al., 2017; Bulian et al., 2020).

Sampling Breach Distributions for Damage Stability Assessment

When generating scenarios by sampling probability distributions, it is important to ensure that the sample is a fair and accurate representation of the underlying distributions. The magnitude of the error incurred here is predominantly a function of the sample size and as such, it is of great importance to ensure that a statistically valid sample is considered. However, as the sample size increases, so does the calculation time and computational cost, so one must seek to strike a balance between these two competing objectives. In order to make this determination, two approaches could be pursued; one using a commonly adopted engineering approach and a more rigorous mathematical approach in the sampling process. In this respect, there are two points to consider. The first relates to how accurately the sampling process represents a given distribution in which case a rigorous approach will produce netter results. The second relates to the uncertainty in the determination of the statistical distributions being derived from limited accident

data (the marginal distributions expanded upon in earlier). In the latter case, given the fact that accident data is limited and unlikely, unwarranted accuracy in the sampling process might not bear improved accuracy in the calculation of damage stability and survivability (Attained Index of Subdivision from static calculations and Survivability Index from time-domain simulations), using suitable numerical models, (Vassalos and Paterson, 2021).

Adopting a pragmatic approach, the Standard Error (SE) of the mean may be used to ascertain sample quality and is a measure of the accuracy in which the sample mean \bar{x} reflects the actual distribution mean μ , calculated in accordance with Equation 5. Two examples of distributions are considered here. One addressing a single parameter (SOLAS distribution of sea states, characterised by H_s) and the second, flooding risk aggregation, in this case represented by the time it takes a specific ship to capsize in pertinent critical flooding scenarios, CDF of time to capsize, (TTC). By assessing the magnitude of the Standard Error as a function of sample size (N), the relationship between these two parameters can be derived, as shown in Figure 9, for the first case. Here, it can be observed that there are diminishing returns in error reduction for sample sizes greater than 750 samples. Similar tendencies were identified when assessing other parameters in this way, with a variation ± 50 samples found across all cases. This would indicate an optimal sample size of 700-800 samples, in this particular case. However, the sampling process itself, provides a subset of all probable cases with proportional representation of various extents but fails to capture all possible scenarios. This is particularly true in the case of low probability events (the rail-end of such distributions), which are often poorly represented within small samples. To provide an example, if one were to compare a random damage sample to zonal damages, the ratio of 2-compartment to 4-compartment damages would most likely be the same in each case, however, the sample would only consider a fraction of all probable 2 and 4-compartment cases. As such, by increasing sample size a greater number of these “black swan” events would be captured, even though the error may remain for the most part unchanged.

$$SE = \frac{\sigma}{\sqrt{n}} \tag{5}$$

Where,

σ = sample standard deviation

n = number of samples

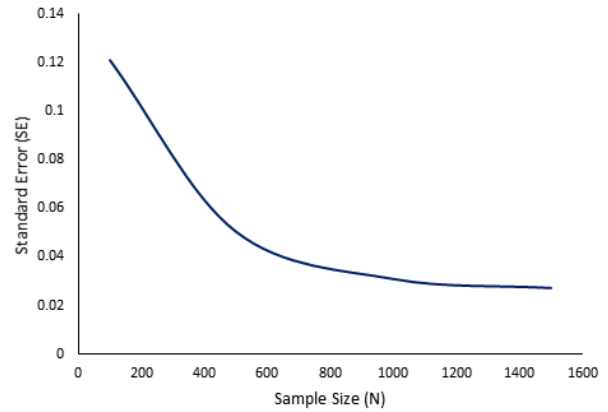


Figure 9: Standard Error (SE) relative to sample size (N) for H_s

In addition to considering the Standard Error, confidence intervals are normally derived for each sample in order to illustrate the range of confidence across the sample CDF. For this purpose, the Dvoretzky–Kiefer–Wolfowitz inequality, (Dvoretzky, 1956), is being utilised, which allows different rates in violation to be identified across the range of the distribution, see Equations 6 and 7. An example of how this error varies relative to sample size is also provided in Figure 10.

$$F_n(x) - \varepsilon \leq F(x) \leq F_n(x) + \varepsilon \tag{6}$$

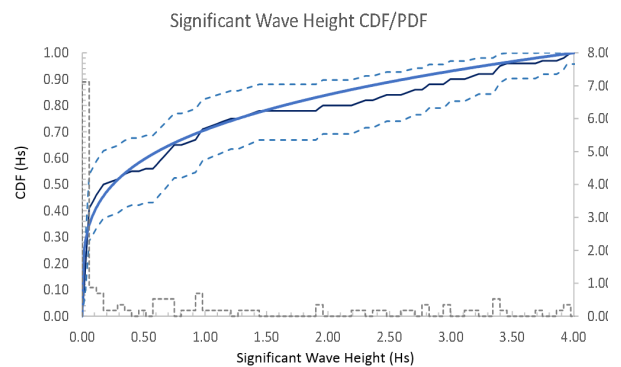
$$\varepsilon = \sqrt{\frac{\ln \frac{2}{\alpha}}{2n}} \tag{7}$$

Where,

$F(x)$ = the true sample CDF

$F_n(x)$ = lower and upper bounds

$1-\alpha$ = level of confidence, i.e., $\alpha=0.05$ for 95% confidence



(a)

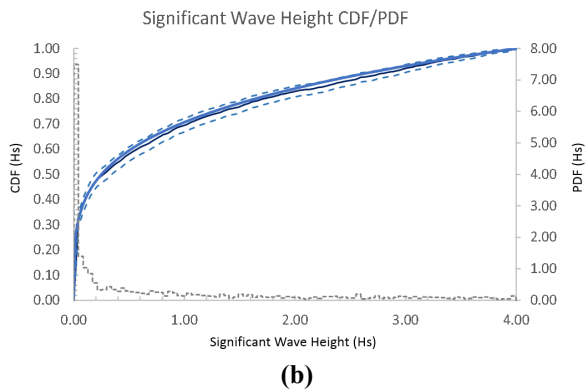


Figure 10: Confidence intervals: (a) 100 samples; (b) 1,500 samples

Considering the sampling process from a more mathematical perspective, studies and applications in computer science suggest that Latin Hypercube (LH), Quasi-Monte Carlo (QMC) or Randomised Quasi-Monte Carlo (RQMC) methods ensure a faster convergence rate than the traditional Monte Carlo approach when addressing complex functions, (Cools and Nuyens, 2014). Considering this in the particular case of application to damaged ship stability/survivability, a preliminary study, limited to Cruise RoPax bottom groundings, has been carried out for the non-zonal approach implementing a RQMC sampling method on a reference barge, (Mauro et al., 2021). Traditionally, the application of Monte Carlo sampling of pertinent distributions in assessing ship survivability is well documented, (Vassalos and Paterson, 2021). However, such a method introduces randomness in the process, leading to a dispersion of the attained survivability index within multiple sets of generated damages. To this end, recent work in Mauro et al. (2021) investigates sampling methods alternative to Monte Carlo, based on Latin Hypercube and Randomised Quasi-Monte Carlo processes. The sampling methods application for collisions, side and bottom groundings on a reference barge available in the literature for benchmark purposes shows that the Randomised Quasi-Monte Carlo method based on multidimensional Sobol sequences grants a lower dispersion of the final survivability index data within samples of equivalent size. The application on a sample Cruise ship of Monte Carlo and Randomised Quasi-Monte Carlo methods highlights the possibility to reduce the number of damages necessary to evaluate the survivability index within an engineering consistent confidence interval. The sampling process of damages within the SOLAS probabilistic framework has been analysed,

proposing three alternative sampling processes useful to reduce uncertainties and A-index variability whilst adopting a non-zonal approach. More specifically, the performance of LH and RQMC sampling with standard MC approach is addressed. The test case for collision, bottom and side grounding damages on a simple reference barge, highlights how the RQMC method based on multi-dimensional Sobol sequences (SMPL-3) gives more benefits than other procedures in the reduction of variability for partial and total A-indices calculations. A detailed analysis on the evaluated p-factors highlights that the reduction of variability in A-Index is strongly related to the reduction of the p values evaluated per each unique damage case among multiple repetitions. Moreover, SMPL-3 method is capable to detect a higher number of unique damage cases compared to other methods. Therefore, it could significantly reduce the number of samples to be generated to achieve a target confidence level on the results. The benefits provided by SMPL-3 have been further highlighted testing the sampling process on a complex internal layout, more granular than traditional geometries used for static calculations. Comparing results with traditional MC sampling, it has been found that the SMPL-3 method grants the same Confidence Interval (CI) on the final A-index using approximately 1/3 of the total breach samples. However, to clearly identify a suitable lower limit for the sample size needed for damage stability assessment, a more extensive study on a wider number of ships with different size is needed. Nevertheless, the results on the reference barge and on the sample cruise ship indicate that the adoption of SMPL-3 method could be very effective with different internal layouts and size. The same procedure can be extended also for dynamic analysis, where the benefits in terms of calculation reduction could be even higher than for static calculations.

Structural Crashworthiness

General Considerations

Structural design has traditionally been exploited as a means of managing safety, related to accidental loads and breaches of hulls. In the 20th century, nuclear-powered ships faced a clear danger if the reactor were to be physically damaged, e.g., by a ship-to-ship collision. This led to Woisin, (Woisin,

1979) describing some reconfiguration of the hull that would result in a higher tolerance in the collision energy of the side structures prior to undergoing breaching. These first investigations served the purpose of, not only creating more crashworthy side structure designs, but also in capturing the mechanics of ship-to-ship collisions. From that period, the work of Minorsky, (Minorsky, 1959) should be noted, which established the proportional relationship between the capacity to absorb collision energy and the volume of the structure involved in deformation. McDermott (McDermott and R.G. Kline, 1974) showed that the key element for ship structures to have an extended capacity to absorb energy is to allow the structure to undergo large membrane tension. Based on his conclusion, substantial work followed with Pedersen and Zhang (2000), attempting to estimate collision energy and loads based on the Minorsky empirical formula, while Amdahl (1982), Lützen (2001), Wierzbicki and Abramowicz (1983), and Kitamura (1997, 2001), developed analytical methods using an upper-bound theorem, referred to super-element solutions, the latter addressing both collisions and groundings. Deriving from these findings, a series of novel designs of both side and bottom structures have been and are still being investigated, Lehmann and Peschmann (2002), Ludolph and Boon (2000), Graaf et al. (2004), Naar et al. (2002), Klanac (2011) and Klanac et al. (2005). What all these studies have in common is that their conceptual developments are focused on the definition of the topology of a novel crashworthy structure, such as shown here in Figure 11.

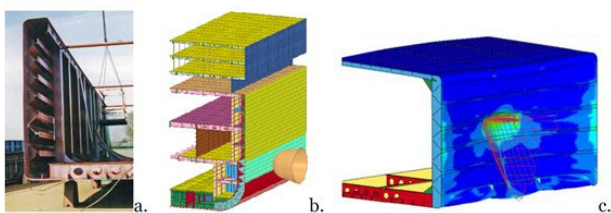


Figure 11: Concepts of crashworthy structures: (a) Longitudinal structure on-board an inland waterway gas carrier, Ludolph and Boon (2000); (b) Transverse structure on board a RoPax vessel, Ehlers et al. (2008); (c) Corrugated structure on board an inland waterway, Ehlers et al. (2008).

Based on these estimation methodologies, many studies have been conducted focusing on protecting certain regions of interest against external forces, such as offshore structures in Storheim and Amdahl (2014), Mujeeb-Ahmed et al. (2020), an LNG tanker

in Wang and H. C. Yu, (2008). More recently, Paik, (2007, 2020) and Wilson (2018), proposed advanced techniques for finite element modelling to simulate structural crashworthiness with increased accuracy in collisions and groundings. Most of these studies conclude that the crashworthiness of ships can be controlled effectively with conventional double-bottom and double-sided structures. Concerning the latter, a detailed methodological approach has been presented in Conti et al. (2021), with application on a Cruise ship operating in the Finland archipelago, which is further elaborated later.

Impact of crashworthiness on p-factors

As mentioned earlier, the damage probability distributions utilised within SOLAS are based on accident statistics without taking explicitly into account the structural design, or crashworthiness of the ship. This implies that even if a ship is designed with a high crashworthiness level, no gain is to be expected in terms of safety in the framework of the current regulations. In principle, SOLAS damage distributions embody an ‘average’ crashworthiness level of the historically damaged ships, which is not representative of a specific type of ship, for example modern passenger ships. However, in the same way, these distributions can also be formed on the basis of the crash analysis conducted on an area within the vessel having structural protection. This would yield local damage distributions (p-factors) to be used instead of the standard SOLAS assumptions in case of damages involving such protected spaces. The impact of this consideration is demonstrated heuristically in Figure 12 and expanded further in Section 4.

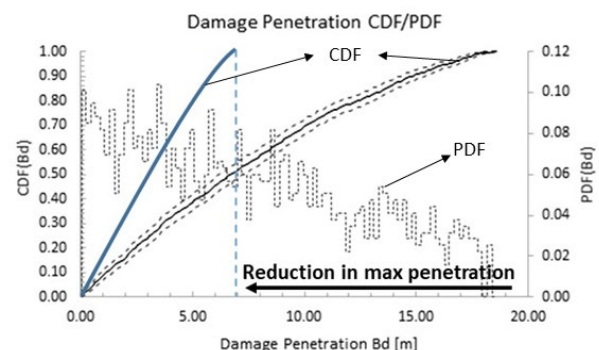


Figure 12: Impact of a crashworthy ship structural section (blue line) on the damage breach penetration distribution for a typical ship structure (black line)

3. STATISTICAL APPROACH TO P-FACTOR DETERMINATION

Probabilistic distributions of ship collision and grounding breaches is an essential part in the design of crashworthy ships. As indicated earlier, current SOLAS damage distributions for collision are developed based on all ship types. However, large differences in ship design, operation, and safety regulations, may render such assumptions invalid. Moreover, the number of accidents leading to flooding of large passenger ships are rare, which poses a statistical challenge to obtain desired and accurate distributions. Over the years, there has been continuous research effort toward the development of damage breach distributions, through various EU-funded projects such as HARDER (1999-2003), SAFEDOR (2005-2009), GOALDS (2009-2012), EMSA III, (2013-2015) and eSAFE, (2017-2019). However, what is currently adopted by SOLAS regulations still pertains only to the earliest of these projects, namely Project HARDER. To address this gap, a concerted effort in the EU Project FLARE, focused on devising new damage breach distributions, specifically for large passenger ships, whilst addressing both collision and grounding accidents. To this end, use is made of a newly developed accident database undertaken within this project, leading to the development of pertinent damage distributions for damage length, height, penetration, and damage location.

Overview of the FLARE accident database

This section provides a brief discussion of the flooding database developed in FLARE, Mujeeb-Ahmed (2021a, 2021b). Figure 13 illustrates the distribution of flooding cases for different types of accidents, spanning the period 1999-2020 for Cruise and RoPAX ships, extracted from IHS Sea-web. The record shows that the number of hull/machinery damages and grounding dominate, followed by collision. This study focuses mainly on ship flooding due to the initiating events developing external to the ship, namely collision and grounding, disregarding contact where only a few flooding events (8) are registered.

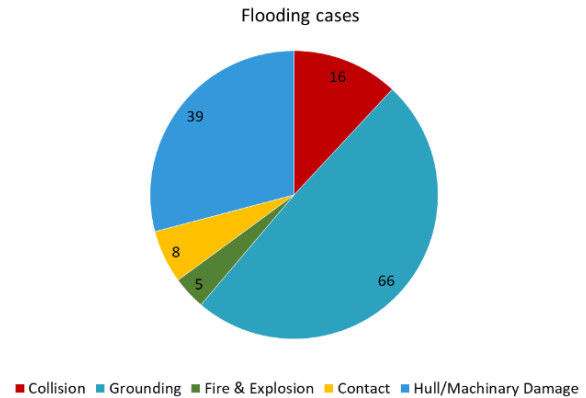


Figure 13: Number of flooding cases for different accidents registered over the last 20 years for Cruise and RoPax ships.

Data Filters

Focusing on the scope of the database development, the following filters are employed to extract the casualty and fleet at-risk data:

- Accident period: 1999-01-01 to 2020-10-31 (last 20 years)
- Accident type: collision and grounding
- Ship size: GT \geq 3500
- Ship length (overall): \geq 80 m
- Ship type: Cruise, RoPAX, Pure passenger, and RoPAX (Rail)
- Location: worldwide
- Class type: IACS and non-IACS (for the fleet at risk)

Keeping in mind the 1995 SOLAS Conference and scope of the FLARE project, worldwide accidents during the last 20 years have been investigated. The identification of different accidents into collision and grounding are in line with the definition of accident types mentioned in IMO MSC/Circ. 953, i.e., Collision: striking or being struck by another ship (regardless of whether underway, anchored, or moored); stranding (or grounding): being aground, or hitting/touching shore or sea bottom or underwater objects (wrecks, etc.). To filter large passenger ships from the database, a lower threshold value of 3,500 GT is selected, representing an average value based on a simple comparison of Cruise and RoPAX ships having an overall length of 100 m. It is, essentially, a compromise between having enough data in the database for meaningful statistical analysis while focusing on large passenger ships. For the same reason, the filter for the ship-built year in the accident period has not been applied in this study.

Data Sources

Figure 14 summarizes various sources from which the data is collected. The FLARE database is built mainly on five sources, supplemented by data from ship operators and other public sources, namely:

- Sea-web (by IHS Markit), IHS, (Sea-web, 2021)
- IMO GISIS (Global Integrated Shipping Information System), (IMO GISIS, 2021).
- EMSA EMCIP (European Marine Casualty Information Platform), (EMSA EMCIP, 2021).

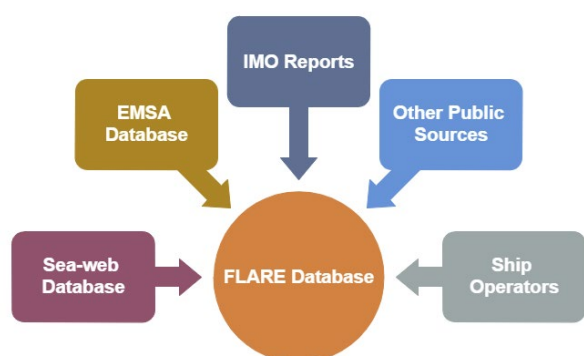


Figure 14: Main information sources of the FLARE database

Initially, all the collision and grounding accidents were thoroughly examined based on the different accident categories defined in the Sea-web, whilst cross-checking accident data with available accident reports and other online sources.

Data Taxonomy

A well-structured taxonomy has been defined to ensure the data is captured and organised in a meaningful manner. The newly updated taxonomy results are evolved from the Sea-web, EMSA EMCIP, and IMO GISIS databases with the addition of fields related to the natural light at the time of the accident, more explicit details on the weather conditions, damage component, and location.

Probabilistic Modelling of Breach Distributions

Based on the developed accident database, a detailed statistical analysis was undertaken to derive breach distributions for pertinent ship types and hazards. Most of the breach information mentioned in the database contains qualitative descriptions, for example relating to the breach as the hole, gash, tear, crack, above/below the waterline, etc., with no real quantitative measures of the damage opening. Table 1 shows the number of samples provided for the different damage locations in collision and

grounding accidents whilst Table 2 indicates the total number of breach data (quantitative measures) available in the database for collision, side, and bottom grounding. The figures clearly indicate that the recorded number of cases is scarce, especially for damage penetration, where such information has been registered in only one case (bottom grounding).

Table 1: Number of accident cases providing qualitative measures of hull damage positions

Damage Position	Collision	Grounding
Bow	56	7
Stern	4	8
Port	48	12
Starboard	84	46
Above the waterline	66	3
Below the waterline	19	85

Table 2: Number of accident cases providing quantitative measures of hull breaches

Damage extents	Collision	Side grounding	Bottom grounding
Length (L)	32	14	12
Width (W)	10	10	5
Penetration (D)	0	0	1

Figure 15 to Figure 17 show the percentage of accidents in the longitudinal, transverse, and vertical damage positions of the ship hull related to collision accidents. For both RoPAX and Cruise ships, a similar trend is observed for all the damage positions. Along the length (longitudinal) position of the ship, the bow of the ship dominates, which includes 42% RoPAX and 52.6% Cruise. The majority of the collisions occurred above the waterline (84.6% RoPAX and 77.8% Cruise). The collisions at the starboard side (52.9% RoPAX and 54.5% Cruise) of the ship marginally dominate the port side.

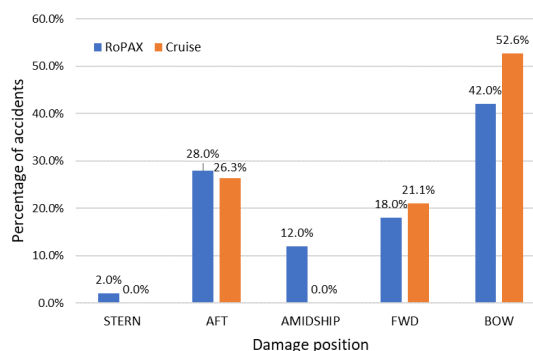


Figure 15: Longitudinal distribution of damage breaches

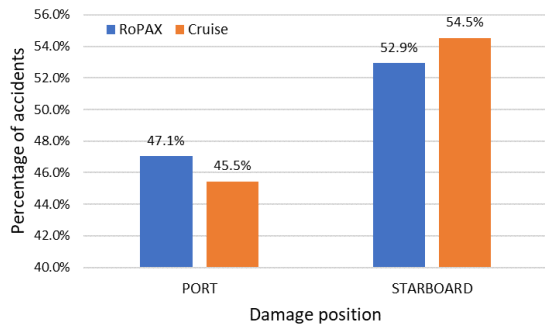


Figure 16: Transverse distribution of damage breaches

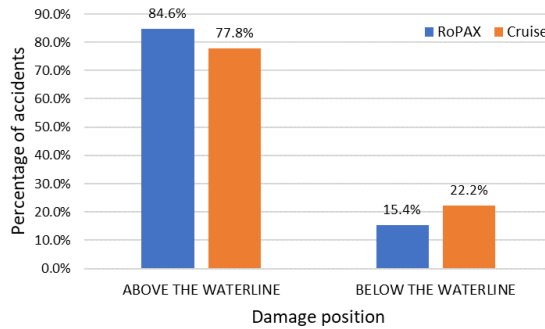


Figure 17: Vertical distribution of damage breaches

The statistical characteristics of damage parameters (length and width) are analysed based on the best-fit probability distribution function (PDF). The goodness-of-fit (GOF) method, using the Kolmogorov–Smirnov (K-S) tests combined with probability plots for a 95% confidence interval, is used to verify the selected PDF compatibility. The most well-known PDFs were chosen based on their popularity and relevance. The selected PDF is further confirmed using the lowest test statistics, which is the difference between the data sample and the fitted empirical CDF. Based on the results of the statistical analysis, the PDF and CDF of the damage characteristics were established for collision, bottom grounding, and side grounding. Figure 18 shows the

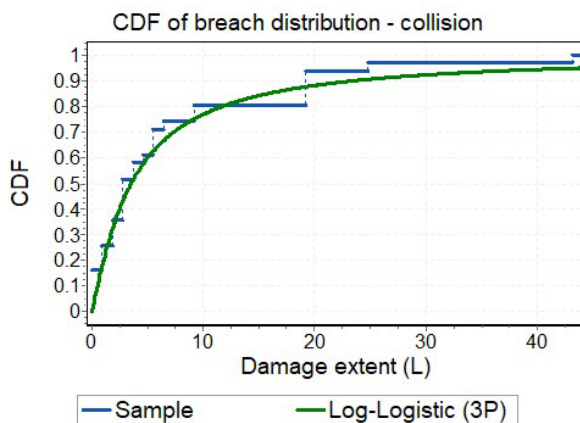
breach probability distributions for damage length and breadth for the 3 hazards (collision, side and bottom groundings). These distributions need to be normalised by accounting for the fleet at risk for each one of the hazards and, of course, as indicated earlier, 6 distributions are needed for each hazard to completely describe the breach distributions. The results presented here are early work in the Project FLARE to be completed in due course. The purpose of presenting it here is to demonstrate the methodology that needs to be followed in the statistical approach for damage breach definitions.

Table 3 summarizes the details of the types of distributions selected and their parameters, along with the corresponding p-factors.

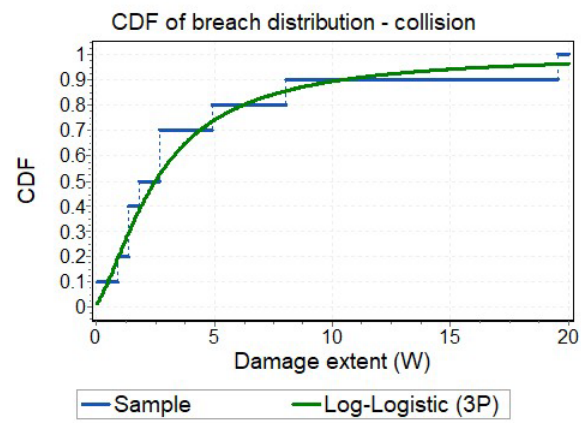
Table 3: The probability distribution of breach extents for collision, bottom, and side grounding.

Accident type	Damage characteristics	PDF	Parameter
Collision	Damage length (L)	3-P Log-logistic	$\alpha = 1.2086$ $\beta = 3.64$ $\gamma = 0.0042$
	Damage width (W)	3-P Log-logistic	$\alpha = 1.5891$ $\beta = 2.6846$ $\gamma = 0.1695$
Bottom grounding	Damage length (L)	2-P Weibull	$\alpha = 0.5055$ $\beta = 13.22$
	Damage width (W)	3-P Weibull	$\alpha = 0.4146$ $\beta = 4.939$ $\gamma = 0.008$
Side grounding	Damage length (L)	3-P Log-logistic	$\alpha = 0.5635$ $\beta = 1.219$ $\gamma = 0.07$
	Damage width (W)	General extreme value	$\alpha = 0.9275$ $\beta = 0.4160$ $\gamma = 0.3089$

Note: 3-P and 2-P denotes three-parameter and two-parameter, respectively.



(a)



(b)

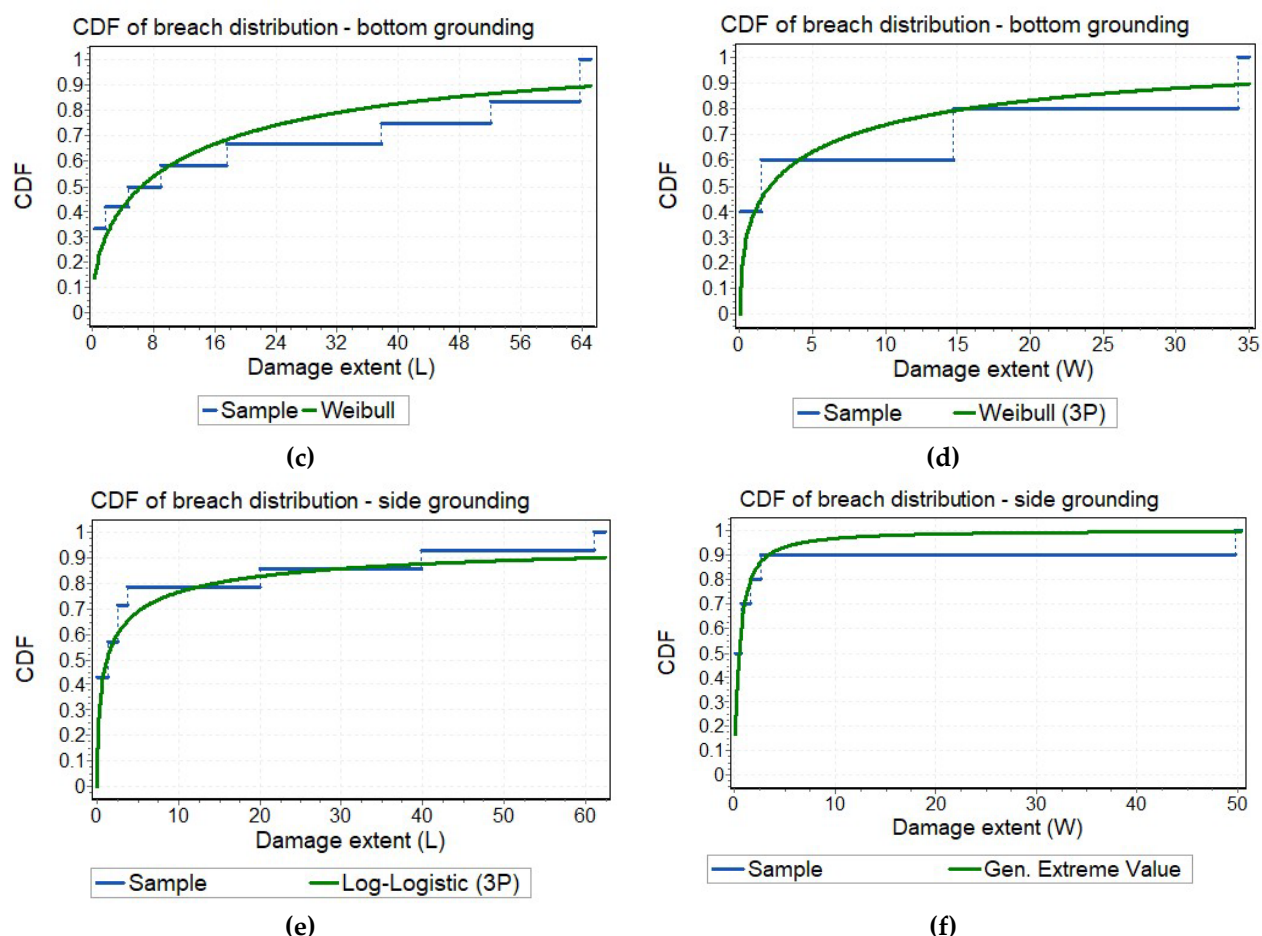


Figure 18: (a) CDF of damage length (L) for collision; (b) CDF of damage width (W) for collision; (c) CDF of damage length (L) for bottom grounding; (d) CDF of damage width (W) for bottom grounding; (e) CDF of damage length (L) for side grounding; (f) CDF of damage width (W) for side grounding.

4. DIRECT APPROACH TO MODELLING BREACH DISTRIBUTIONS

The first step of the methodology is to run a very large number of scenarios, for each hazard in question, namely collision, side grounding and bottom grounding, considering a reference ship. The aim is to simulate a large range of representative breaches, adopting for example, a design of experiments strategy and using suitable crash analysis software. In the example presented here, the Super-Element software SHARP is utilised and the hazard considered is collision. In the particular example considered, a collision scenario is defined by the following parameters: (a) striking ship type, (b) striking ship initial surge velocity, (c) struck ship initial surge velocity, (d) impact longitudinal position, (e) collision angle, (f) striking ship draft and (g) struck ship draft. For each of these parameters, a range of values has been defined in order to build a load case matrix capable of inducing

a large range of pertinent breaches. 1,980 collision scenarios have been defined by considering the combination of parameters presented in Table 4. As indicated earlier, current SOLAS damage distributions for collision are developed based on all ship types. In this respect, considering that the number of accidents leading to flooding of large passenger ships are rare, as demonstrated by the data presented in Section 3, this poses a statistical challenge to obtain desired and accurate distributions. This, in turn, makes a direct approach much more attractive, especially considering that the right tools are available for this purpose.

Table 4: Parameters used in collision crash analysis

Parameter	Unit Values
Striking ship type	11 ships (see Table 5)
Striking ship initial surge velocity (m/s)	2, 4, 6, 8, 10
Impact longitudinal position (m)	95.2, 103.6, 112
Collision angle (degrees)	30, 45, 60, 90

Table 5: Striking ships general characteristics

ID	Vessel Type	Length overall (m)	Moulded breadth (m)	Draft (m)	Depth (m)	Displacement (Tonnes)
1	Cargo	92.2	14.0	4.9	10.0	3,500
2	CSV	80.0	17.6	6.8	13.8	3,500
3	Chemical carrier	110.0	19.5	7.6	10.6	11,064
4	Gas carrier	155.0	22.70	6.92	17.95	16,006
5	Cargo	145.0	15.87	8.00	11.15	15,415
6	RoRo	180.0	30.50	6.80	15.80	22,062
7	Passenger	251.0	28.80	6.60	19.35	29,558
8	RoPax	221.0	30.00	6.90	15.32	30,114
9	Bulk carrier	180.0	30.00	10.00	15.00	50,000
10	Container	300.0	48.20	12.50	24.60	119,130
11	Tanker	274.0	42.00	14.90	21.00	140,000

With respect to the definition of the collision scenarios, it is to be noted that:

- Since SHARP considers the structural description of one half of the ship (collisions are modelled at port side), the structure of the ship has been considered symmetrical and hence a unique model is used.
- In all simulations, the struck ship is supposed to be at rest (no initial surge velocity). This is in accordance with Lützen (2002), who observed from the collision accident statistics that the most likely surge velocity of the struck ship would be zero. Furthermore, the ship considered for the case study having very limited draft variability, the struck ship was assumed to be at design draft.
- According to the probabilistic damage analysis model, the longitudinal position is independent of all other damage variables. On this basis, only impacts at the midship section are modelled. However, the actual longitudinal position varied so that transverse bulkheads can also be directly hit.

In simulating collision scenarios, a large range of striking ships is considered, as it drives the damage size obtained but also the relationship between the damage longitudinal, transverse and vertical extents. For the analysis presented here, 11 striking ships of various types and dimensions were modelled. The general characteristics of the striking ships considered represent the world fleet and are shown in Table 5.

For this case study, all the calculations have been carried out considering a reference ship the FLOODSTAND SHIP B Cruise ship, Luhmann (2009), the main particulars of which are given in Table 6. The super-element structural description has been modelled for a section that is 100 m long

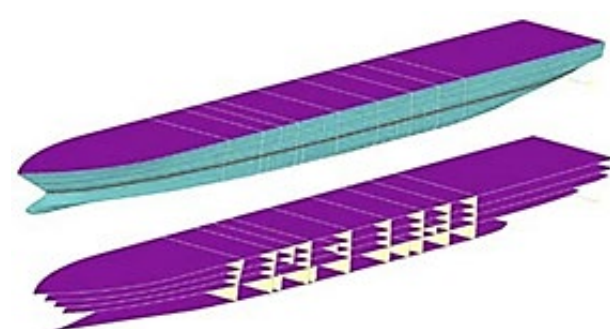
along the ship parallel body and centred on the mid-ship section. All materials have been modelled as rigid-perfectly plastic with S235 mild steel properties (see Table 7). The failure strain - which in SHARP is compared to the averaged tension stress within the super-elements - has been considered equal to 10%. Similar values have been observed by other authors to provide a good fit between super-element predictions and experimental results, Zhang (1999), Lützen, (2002), Buldgen et al. (2012). The SHARP super-element model of the struck ship is shown in Figure 19. Its hydrodynamic properties as required by MCOL have been obtained using the BV Hydrostar software, (BV, 2019).

Table 6: Reference ship main particulars

Parameter	Value
LPP [m]	216.8
Breath moulded B [m]	32.2
Depth D [m]	16
Draft T [m]	7.2
Displacement [tonnes]	33,923

Table 7: Material parameters considered

Parameter	Value
Yield strength [MPa]	235
Tensile strength [MPa]	400
Flow stress [MPa]	317.5
Failure strain [-]	10%

**Figure 19: Struck ship SHARP model**

As far as the striking ships are concerned, the bow shape has been modelled in SHARP and the ships have been assumed to be rigid. For the studied ship, this assumption is supported by the finite element analysis (FEA) computations carried out during the benchmark of SHARP using striking ship 8, which showed a good agreement between the FEA and SHARP results.

After simulation of all collision scenarios and filtering damages not compatible with SOLAS description (i.e., mainly damages with lower vertical limit above the waterline), it was examined to which extent potential SOLAS damages can be practically simulated. This is demonstrated in Figures 20-24, where the main damage parameters (L_x, L_y, z_{UL}, z_{LL}) are presented by pair plots. Overall, it is deemed that the SOLAS domains are well populated by the simulation results. Some unpopulated areas are discussed below:

- Figure 20 shows that no damages of length higher than 50 m are obtained. A potential explanation is that the calculation matrix lacks very severe scenarios. Another explanation would be that for the reference ship considered, the SOLAS damage limit of 60 m cannot be physically reached when considering realistic scenarios.
- Figure 20 also shows that longitudinal damages higher than 20 m ($L_x > 20\text{ m}$) with low penetrations ($L_y < 2.5\text{ m}$) cannot be simulated. This may be due to the fact that no initial surge velocity was considered for the struck ship. It could also come from the underlying SOLAS model, which considers that for such type of damages, the longitudinal and transverse extents are independent.
- Figure 21 shows that the domain is well populated due to the large striking ships database. No damages have been simulated with the damage upper limit slightly above the waterline and the damage lower limit slightly below. The simulation of such damages would typically require that the damage is due to the bulb of the striking ship only and that the combination of striking ship draft and bulb height is adequate.

- From Figure 22, it can be noted that no longitudinal damage can be simulated with vertical position just above the waterline. However, this was expected since long damages mainly correspond to the more massive striking ships with high freeboard.
- Figure 23 shows that simulated damages with large penetration have lower vertical limit close to the ship bottom. This was expected given the bow shapes of the striking ships.

In Figure 24, the results from Figure 20 are shown after clustering the data into either striking ship initial velocity or collision angle. It is observed, as expected, that the striking ship initial velocity has a significant influence on the damage extent and that the collision angle has a strong impact on the damage length.

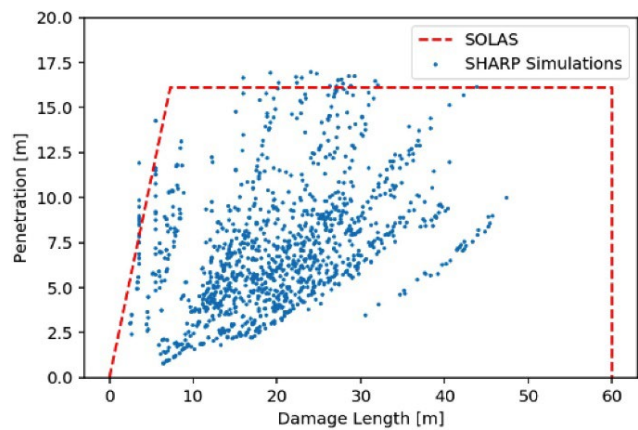


Figure 20: Penetration versus damage length for simulated damages

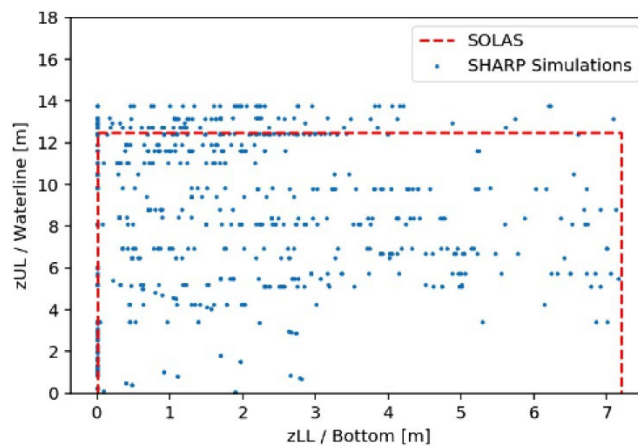


Figure 21: Damage vertical position upper limit versus lower limit for simulated damages

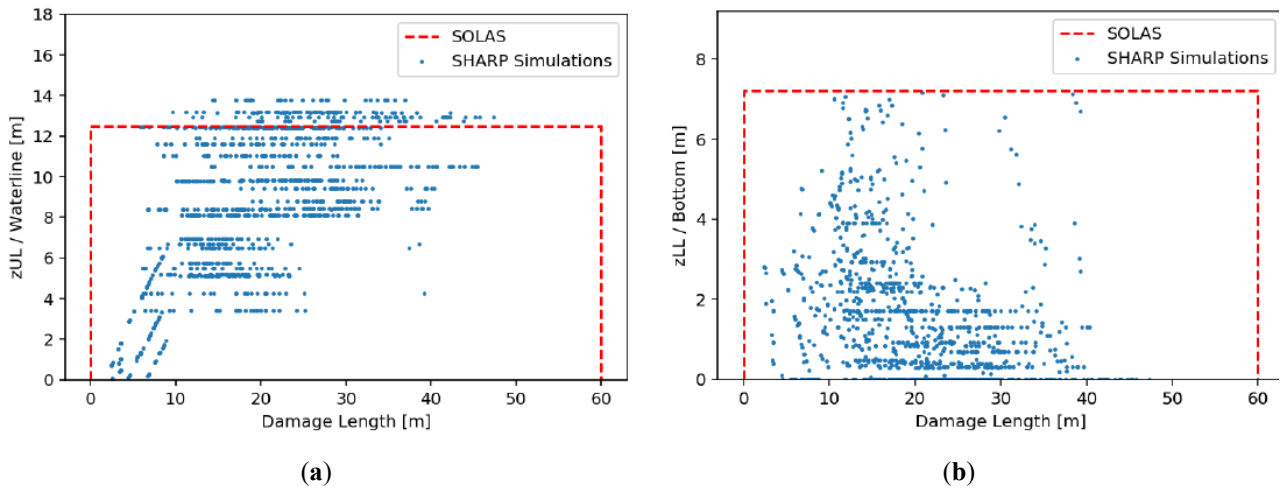


Figure 22: Damage vertical positions versus damage length for simulated damages: (a) upper limit; (b) lower limit.

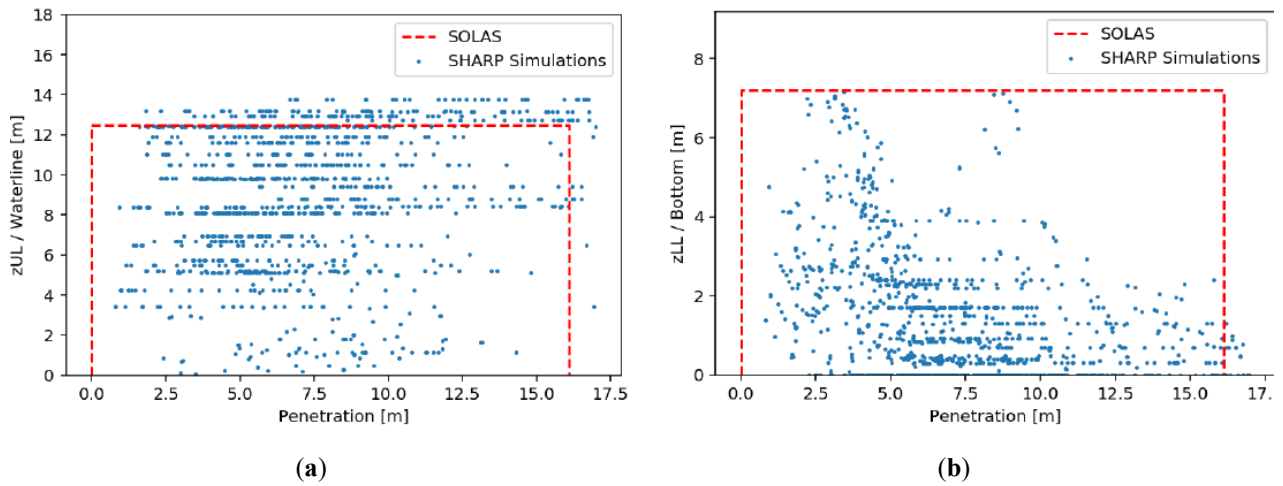


Figure 23: Damage vertical positions versus penetration for simulated damages: (a) upper limit; (b) lower limit.

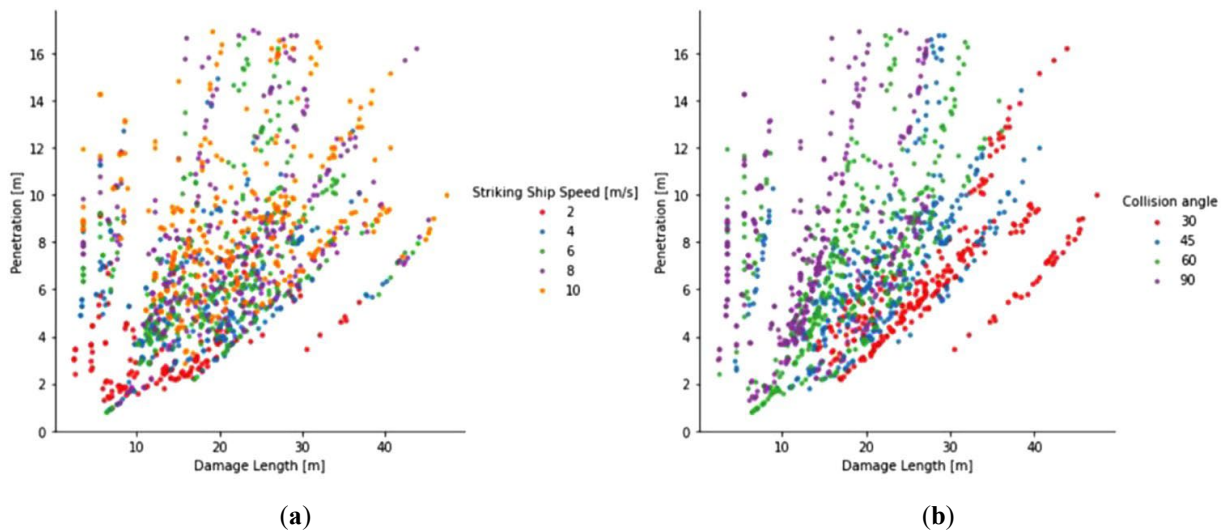


Figure 24: Penetration versus damage length for simulated damages: (a) data clustered by collision angle; (b) data clustered by striking ship initial velocity

5. CONCLUSIONS

Despite a late start and slow early development in the subject of probabilistic damage stability, the past three decades have seen remarkable progress in the evolutionary development of this subject. Such progress covers specific elements in the probabilistic damage stability calculation/simulation process as well as the process itself. Focussing on the requisite data for such calculation/simulation, no input is more important than the damage breaches for each related hazard (collision, side grounding, bottom grounding) and associated probabilistic content, so called p-factors. Pursuing clarification in such determination, the following areas and concerns have been addressed, leading to specific conclusions and recommendations for further work to improve knowledge in this specific subject:

- Clarification on what exactly p-factors are and how they are defined in terms of marginal distributions of six parameters: length, breadth, height, location, side of ship, upper and lower location.
- How to sample such distributions in order to ensure sufficient accuracy in the damage sample.
- Explanation of what constitutes zonal or non-zonal methods in damage breach generation.
- Derivation of the marginal breach distributions based on statistical methods, describing, and using a new accidents database, specific for passenger ships and addressing all pertinent hazards (collision, side grounding, bottom grounding).
- Explanation and demonstration of a direct approach to deriving pertinent p-factors, using a passenger ship operating in the Gulf of Finland.

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