Stochastic Assessment Using Moment Equation Method for Parametric Rolling of Ships in Random Seaways

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ABSTRACT

Parametric rolling is one of the dangerous dynamic phenomena, and it is important to estimate the exceedance probability of certain dynamical behavior of the ship with respect to a certain threshold level. In this study, the moment equation, which is one of the stochastic methods, is used. To obtain the PDF of roll angle, the method proposed by Maruyama et al. (2022) is used. In this study, the calculation for two subject ships and several sea conditions is conducted, and the result is examined. As a result, it is observed that our proposed method is useful to obtain the PDF of roll angle which is non-Gaussian in some cases.

Keywords: *Parametric Rolling, Moment Equation, Stochastic Differential Equation, Cumulant Neglect, Linear Filtering.*

1. INTRODUCTION

Recently container loss accidents are often reported. For preventing such accidents, the second-generation intact stability criteria developed by the International Maritime Organization (IMO) for the several failure modes could be utilized (IMO2020). The failure modes relevant to the container loss accidents are parametric rolling.

To discuss a vessel's safety for parametric rolling, a stochastic method can be used. In general, by using this method, the probability density function concerning the ship motion can be derived. One method is to use the moment equation (Bover 1978, Wu 1987). Some researchers use a moment equation and a linear filter, which has been applied in the field of naval architecture and ocean engineering. For example, Francescutto et al. (2003) and Su et al. (2011) considered the roll motion in beam seas using a 4th-order linear filter and a moment equation. Chai et al. (2016) analyzed the response of parametric rolling in irregular waves by using Monte Carlo simulation (MCS) and a linear filter. Dostal et al. (2011) used the Local Statistical Linearization in combination with moment equations. Furthermore, Maruyama et al. (2022) showed the procedure to derive the moment equation from the sixth-order ARMA (Autoregressive Moving Average) filter and stochastic differential equation (SDE) of roll motion in longitudinal waves. Here, solving the moment equation numerically was suggested.

In this study, the method proposed by Maruyama et al. (2022) is applied to two subject ships. In addition, several sea conditions are set to calculate the moment equation.

2. LINEAR FILTER

To derive moment equations, the system of the ship motion needs to be represented by an Itô stochastic differential equation (SDE). In this case, it is appropriate to represent the parametric excitation process approximately by the SDE. In this study, a combination of the linear filter and non-memory transformation is used to obtain the parametric excitation process.

Firstly, the method that the effective wave spectrum is approximated by the sixth-order ARMA process spectrum is explained. It is necessary to obtain the coefficients of the ARMA spectrum such that they fit well the effective wave spectrum. It should be noted that the system can become unstable even if these spectra have a good agreement and problems in the modelling of time history may occur. Therefore, the stability criterion of the corresponding system is added as one of the conditions to determine the coefficients of the linear filter. This was proposed by Maruyama et al.(2022).

As a result, the appropriate coefficients of the ARMA spectrum can be derived. An example of this calculation result is shown in Figure 1. It can be seen in this figure, the red dashed line agrees well with the black solid line. Thereby, the time history of the effective wave can be obtained by solving the SDE corresponding to the ARMA spectrum. By using FFT for this time history, the grey solid line in Figure 1 can be plotted.

Secondly, to consider the GM variation in waves, the relationship between the amount of GM variation ΔGM and wave amplitude at amidship is needed. This relationship is called non-memory transformation. The restoring arm for the case when the ship is heeling by two degrees in a regular wave is calculated from hydrodynamic theory (Umeda, 1992) using a wavelength that is the same as the ship length. Then the wave crest or trough is set to be located at amidship, and GM is calculated for each wave amplitude.

The linear filter can generate the Gaussian process only. However, to combinate with the non-memory transformation, the non-Gaussian process can be modeled. The comparison of the calculation result of C11 between solving the SDE numerically and using the superposition principle is shown by Maruyama et al.(2022). In this paper, as an example, the calculation result with ITTCship A1 in a certain sea condition is shown. The body plan, principal particulars, and GZ curve of the subject ships were utilized by Maruyama et al.(2021). Next, the non-memory transformation of each subject ship is shown in Figure 2. From this figure, it is clear that ITTCship A1 has stronger nonlinearity than C11. Thereby, when the actual result of A1 is approximated by a 12th order polynomial in all ranges, there is a discrepancy. By dividing the range and making a polynomial approximation, the polynomial approximation, which agrees with the actual result in all ranges, may be obtained. However, when the moment equation is applied, only one polynomial approximation can be used in all ranges. Therefore, if the nonlinearity is strong, a higher-order polynomial approximation should be used.

In this study, the parametric excitation process is modelled by combining the linear filter with non-memory transformation. The probability density function (PDF) of this process can be obtained from this time history. It is observed in Figure 3 that this PDF is a non-Gaussian distribution and the PDF of our proposed method agrees with the PDF by the superposition principle. Therefore, our proposed method can generate the non-Gaussian process.

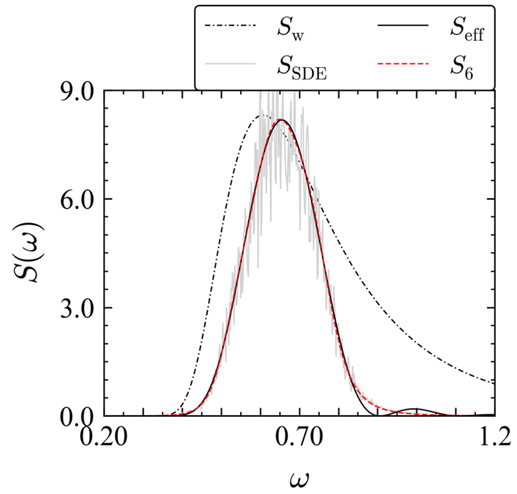

Figure 1: Comparison among ITTC spectrum: S_w, the effective wave spectrum: S_{eff}, 6th-order ARMA spectrum: S_6, and spectrum analysis result of time history obtained by solving SDE: S_{SDE}, sea state with T_{01}=7.987[s] and $H_{1/3}$=7.5[m].

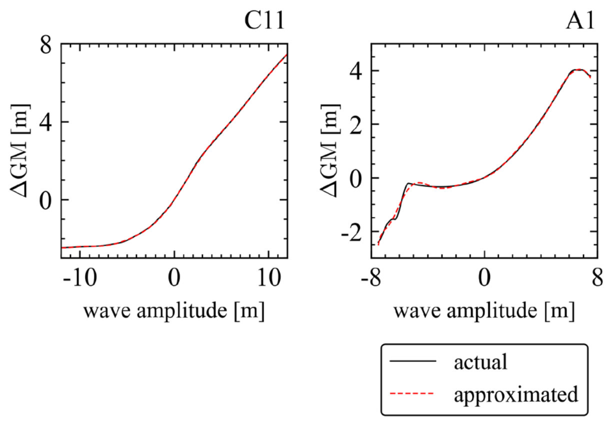

Figure 2: Relationship between ΔGM and wave amplitude at amidship, subject ship C11 and ITTCship A1.

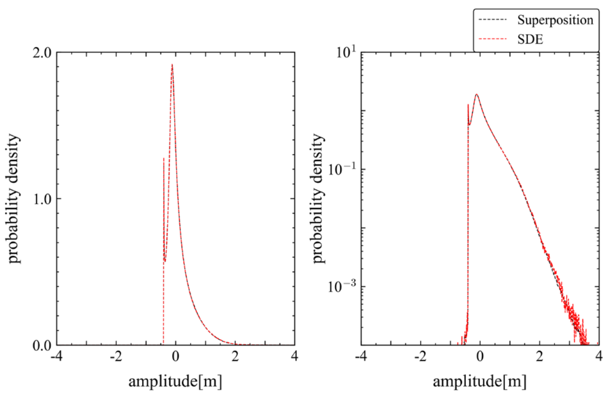

Figure 3: Comparison of GM variation's PDF between the superposition principle and result obtained by solving SDE, sea state with T_{01}=7.987[s] and $H_{1/3}$=7.5[m], subject ship ITTCship A1.

3. MOMENT EQUATION

The parametric rolling focused on in this study results from ship motion induced by irregular excitation. This phenomenon is a case that is non-Gaussian (Belenky, 2011). The purpose of this paper is to determine the probability density function of roll angle, based on the moment values obtained by determining the moment equations. To obtain the moment equations, the system of the ship motion needs to be represented by a stochastic differential equation (Sobczyk, 1991). In this case, the SDE should be represented by a polynomial expression. In the previous chapter, we discussed that the irregular excitation of a non-Gaussian process is derived by the SDE mathematically. In this study, the resulting system of equations is represented by the following 8th-order Itô stochastic differential equation, which consists of a second-order SDE for the ship motion and a 6th-order SDE for the effective wave. The moment equations are derived from this SDE. This derivation process is summarized in Maruyama et al.(2022). The n-th order moment equations can be mathematically obtained from the 8th-order SDE. In general, a nonlinear system generates an infinite hierarchy of moment equations. To form a closed set of moment equations, higher-order moments need to be truncated. Therefore, the cumulant neglect closure method (Sun 1987 and 1989, Wojtkiewicz 1996) is used. In this study, firstly, the second-order cumulant neglect closure method, which ignores cumulants higher than the third-order, is used. The third and higher-order moments include in the first and second-order moment equations. Comparing the coefficients of series expansions of a moment generating function and a cumulant generating function, the relations between moments and cumulants can be obtained. Thereby, the third and higher-order moments can be represented using first and second-order moments. Therefore, a closed set of moment equations can be obtained. Furthermore, this closure method is the same as the Gaussian closure method, because the third-order and higher-order cumulants of a Gaussian distribution are zero. Therefore, to reflect non-Gaussian, the third-order cumulant neglect closure method is used additionally. This method ignores the fourth and higher-order cumulants. Thereby, the fourth and higher-order moments can be approximated by the first, second, and third-order moments.

4. RESULT

In this study, for two subject ships, the moment equations are solved and the moment values are obtained. These subject ships are C11 and ITTCship A1. As we already mentioned, the body plan, principal particulars, and GZ curve of the subject ships were utilized by Maruyama et al.(2021). Furthermore, it can be seen from Table1 that several sea conditions are set.

Table 1: Calculation condition. T_{01} and $H_{1/3}$ denote the wave mean period and the significant wave height, respectively.

name	T_{01}	$H_{1/3}$
C11 - 1	8.00	5.0
C11 - 2	9.99	5.0
C11 - 3	12.0	5.0
C11 - 4	9.99	3.0
A1 - 1	6.00	7.5
A1 - 2	7.98	7.5
A1 - 3	10.0	7.5
A1 - 4	7.98	5.5

In this study, it should be noted that the moment equations are calculated in an unsteady state. The calculation of moment equations in steady-state needs to solve simultaneous nonlinear equations. Then the solutions can be obtained by using the Newton-Raphson method and Jacobian matrix. The convergence and the corresponding matrix calculation are complex. On the other hand, the region of the steady-state can be determined easily from the computation of the corresponding ordinary differential equation. Furthermore, we consider that it is an appropriate method from the perspective of unaffected the number of moment equations.

These moment equations are computed by using the 4th order Runge–Kutta method, and the time step is 0.01[s]. For the initial condition, each moment in the second-order cumulant closure method is set as 0.001 or 0.01. On the other hand, in the third-order cumulant closure method, the initial values are set to zero, and only the second-order moment of roll angle and roll velocity are set to 0.01. As a result, the time histories of moments are obtained. The average of the moment at a steady state in the obtained time history denotes the moment value derived from the moment equation.

In this study, as an example, the second-order moment values are shown. In Figures 4 - 6, the calculation results of moment equations can be observed. In several sea conditions for both subject ships, compared with Monte Carlo simulation (MCS) results, which were indicated by horizontal solid line, the order's magnitude of moment values derived from the moment equation is correct. Furthermore, we can confirm that the result of the third-order cumulant closure method is closer to the SDE result. However, like A1-2, a case is identified in which the result of the second-order cumulant closure method is closer to the SDE result. Therefore, we will conduct research on calculating by the higher-order cumulant closure method, and the calculation result should be examined and discussed more.

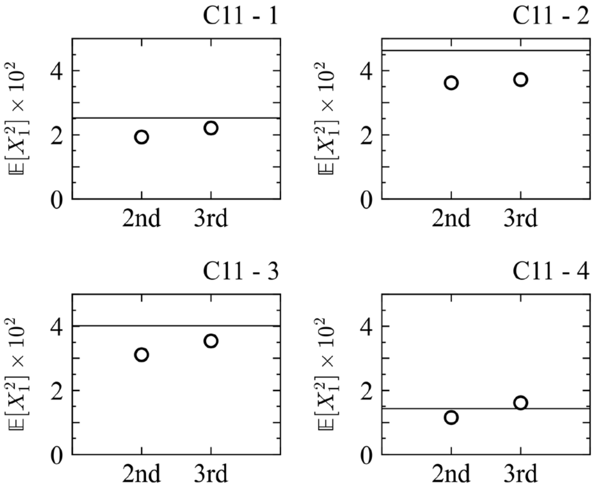

Figure 4: Second-order moment values of roll angle, the subject ship is C11.

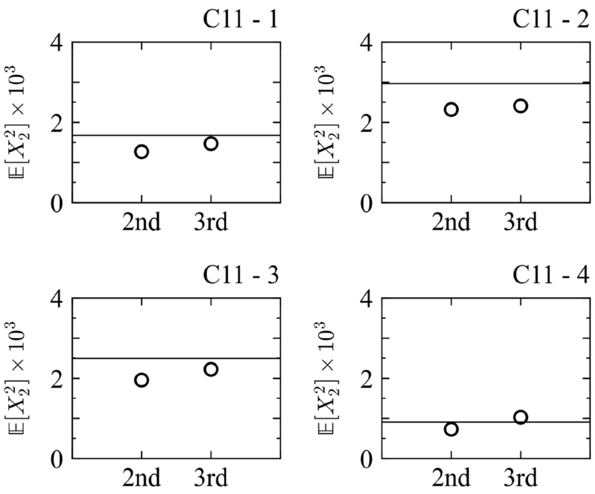

Figure 5: Second-order moment values of roll velocity, the subject ship is C11.

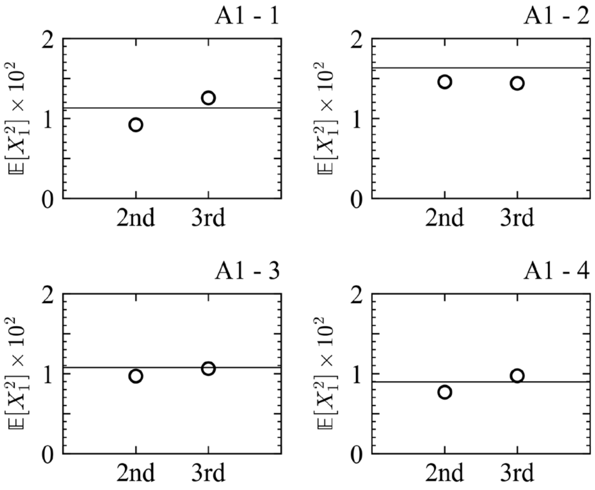

Figure 5: Second-order moment values of roll angle, the subject ship is ITTCship A1.

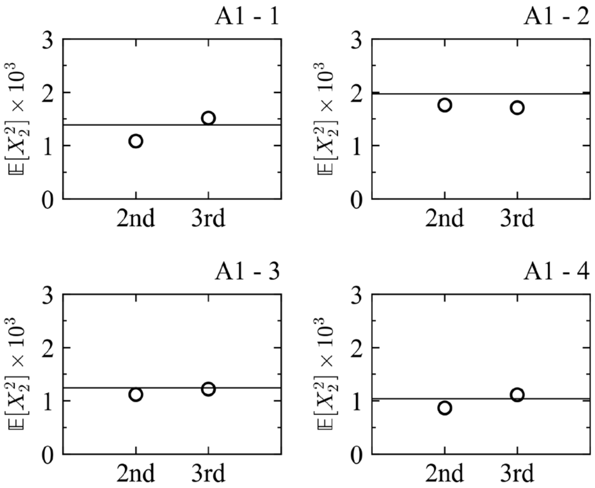

Figure 6: Second-order moment values of roll velocity, the subject ship is ITTCship A1.

5. PROBABILITY DENSITY FUNCTION

Based on the moment values by computing the moment equations, the PDF of roll angle is determined. In this study, the following non-Gaussian PDF shape types are set:

$$P(X_1) = Cexp\{-(d_1|X_1| + d_2|X_1|^2 + d_3|X_1|^3 + d_4|X_1|^4)\} \quad (1)$$

Here, X_1 denotes the roll angle, and C denotes a normalization constant. To determine the coefficients of Eq.(1), the following expression is suggested.

$$J_n = \int_{-\infty}^{+\infty} {X_1}^n P(X_1) dX_1 - \mathrm{E}[{X_1}^n] \quad (2)$$

Furthermore, the following objective function $J(d_1, d_2, d_3, d_4)$ is set.

$$J(d_1, d_2, d_3, d_4) = \sum_{i=1}^{6} l_i |J_i| \quad (3)$$

Here,l_i are weights and $l_i = 1$. As shown in Eq.(3), up to the sixth-order moment value is used.

As shown in Figures 7 and 8, the comparison of the PDFs of the roll angle can be observed. The black solid line shows the MCS result obtained by solving the SDE. The black dashed-dotted line shows the Gaussian distribution. In this case, the mean and the variance are obtained from the MCS result obtained by solving the SDE. The blue solid line shows the optimized result that the coefficients of Eq.(1) are decided by using the moment values derived from the third-order cumulant closure method. The red solid line shows the optimized result that the coefficients of Eq.(1) are decided by using the moment values obtained from the MCS result. This result is obtained by solving the SDE. It is observed from these figures that the PDF of the roll angle does not agree with a Gaussian distribution. In Figures 7 and 8, the red solid line agrees better with the black solid line than the blue one. Therefore, if the appropriate moment values and Eqs.(1) – (3) are used, the theoretical PDF of the roll angle agrees with the PDF from the MCS result.

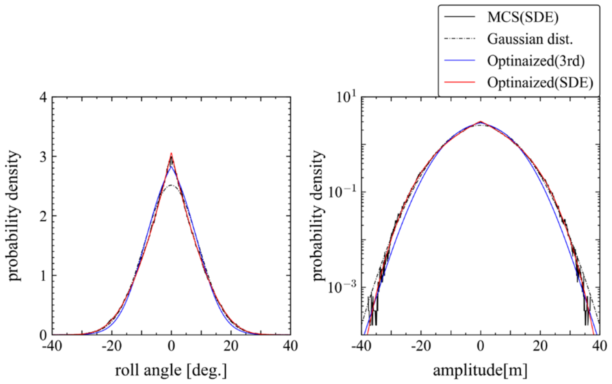

Figure 7: Comparison of PDF of roll angle among four results, the subject ship is C11, sea condition is C11-1 in Table1.

6. CONCLUDING REMARKS

In the two subject ships which are different in the GZ curve and non-memory transformation, the non-Gaussian excitation process could be modelled by using the linear filter and the non-memory transformation.

In several sea conditions and two subject ships, this study showed that the moment values of roll angle and roll velocity could be obtained from the moment equations. Furthermore, considering the non-Gaussian properties, the moment values of the third-order cumulant neglect closure method are better than those of the second-order cumulant neglect closure method.

It is observed that our proposed PDF shape is useful to obtain the PDF of roll angle which is non-Gaussian in some cases.

In future work, the effect of the higher-order cumulant neglect closure method on the moment values must be investigated.

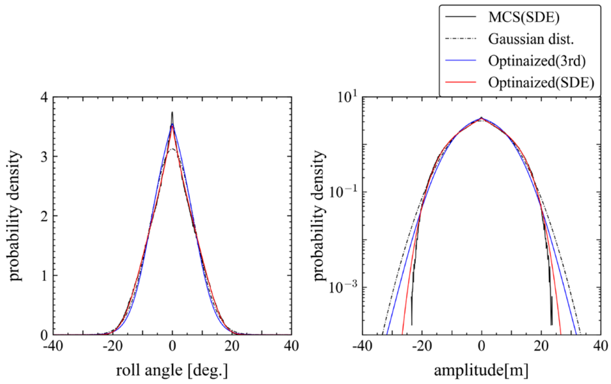

Figure 8: Comparison of PDF of roll angle among four results, the subject ship is A1, sea condition is A1-2 in Table1.

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