Measures for avoiding self-repetition effect in the direct stability assessment

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ABSTRACT

In the direct counting method used in the direct stability assessment, it is essential whether a self-repetition effect of roll motion is present or not. In this study, three methods of modelling irregular waves are examined for self-repetition. The first method discretises a wave spectrum with uniformly distributed frequencies; the second method divides the spectrum into equal areas by changing the sampled frequency intervals, and the third method linearly filters the white noise. In the latter two methods, we observe that time history does not have a self-repetition effect from the result of the autocorrelation function, and we discuss the accurate modelling of irregular waves by using the probability density function and the spectrum of the time history. Furthermore, using these methods, the self-repetition effect in the time history of roll motion is investigated. As a result, we confirm that the non-uniform frequencies method and the linear filtering of white noise can obtain a time history that does not have any self-repetition effects for up to 9 hours.

Keywords: *Self-repetition, Irregular waves, Inverse Fourier transformation, Linear filtering, White noise, Roll motion*

1. INTRODUCTION

The second-generation intact stability criteria developed by the International Maritime Organization (IMO) opened the door to using the stochastic time-domain numerical simulation as the direct stability assessment (IMO, 2020). In the direct stability assessment, if self-repetition of roll motion appears, it could result in under evaluation of the occurrence probability of the stability failures, such as capsizing. The calculation of ship motion requires the modelling of irregular waves. The time history of wave elevation is modelled by the inverse Fourier transformation or linearly filtering white noise. The former method includes two techniques to sample wave frequencies: one way discretises a wave spectrum with uniformly distributed frequencies; the other way divides the spectrum into equal areas by changing the sampled frequency intervals (Hirayama et al., 2009). In other words, the difference is that the sampled frequency interval is uniform or non-uniform. For the inverse Fourier transformation, Belenky (2011) concluded that the nature of self-repetition of irregular waves is a numerical error caused by the highly oscillatory character of an integrand in cosine Fourier transformation for a large value of time such as 1 hour. Concerning the linear filtering of white noise, Spanos (1983, 1986), Flower (1983, 1985), and Thampi (1992) applied a linear filter for the generation of stochastic time series based on wave spectra using the Autoregressive Moving Averaging (ARMA) process, which was a good approximation of the process obtained from sea wave spectra. Furthermore, Degtyarev and Reed (2011) and Degtyarev and Gankevich (2012) discussed that the autoregressive model was used to describe the wave surface and incident random waves.

In this study, these different methods for modelling irregular waves and roll motion are examined not only for the incident waves but also for the roll motions. Here, we generate the time histories for up to 9 hours, evaluate the autocorrelation functions and discuss the self-repetition. As a result, a guide for the sampled frequency number requiring a longer duration is provided for the inverse Fourier transformation. A guide is provided for the required time step for the linearly filtered white noise.

2. MODELLING OF WAVES

In this study, each method for modelling irregular waves is explained mathematically, and the calculation results are considered. In these cases, Fast Fourier Transform (FFT) is applied to the generated time history of irregular waves. As a result, the spectra and autocorrelation functions are obtained. The smoothing is not used in this analysis. Throughout the paper, the calculation condition is set that the wave mean period is 9.99s and the significant wave height is 5.0m.

Inverse Fourier transformation with uniform frequency sampling (Method 1)

As expressed in Eq.(1), the time history of wave elevation $\zeta_w(t)$ can be computed by a sum of trigonometric functions(Hino, 1977). In this case, this process is assumed to be a Gaussian process.

$$\begin{aligned}\zeta_{\mathrm{w}}(t) &= \int_0^\infty \sqrt{2S_{\mathrm{w}}(\omega)d\omega}\cos(\omega t+\delta) \\ &= \sum_{n=1}^{N}\sqrt{2S_{\mathrm{w}}(\omega_n)\Delta\omega_n}\cos(\omega_n t+\delta_n)\end{aligned} \tag{1}$$

Here, ω_n, $\Delta\omega_n$ and δ_n describe the wave frequency, the frequency interval and the phase of the wave. N denotes the number of elements. In this study, the ITTC spectrum is used to approximate the ocean wave spectrum $S_{\mathrm{w}}(\omega)$, which is given by

$$S_{\mathrm{w}}(\omega) = A\omega^{-5}\exp(-B\omega^{-4})$$

where (2)

$$A = 173{H_{1/3}}^2{T_{01}}^{-4}, \qquad B = 691{T_{01}}^{-4}$$

Here, $H_{1/3}\ (m)$ and $T_{01}\ (s)$ are the significant wave height and the mean period, respectively.

The method, which divides a wave spectrum into uniform frequency intervals, is examined. In this paper, this method is called “Method 1”. It is widely known that the time history of irregular waves by Method 1 has a self-repetition depending on the frequency interval (Hirayama et al., 2009). Especially, this time history depends on the minimum wave frequency. The autocorrelation function of the time histories generated by this method is discussed. The component wave of the minimum wave frequency can be expressed as follows:

$$x_{\min}(t) = a_{\min} cos(\omega_{\min}t+\delta_{\min})\ . \tag{3}$$

Here, $x_{\min}$, $a_{\min}$, $\omega_{\min}$, and $\delta_{\min}$ denote the wave elevation of the component wave, the amplitude of the component wave, the minimum wave frequency, and the phase of the component wave. Here, the index “min” denotes the element of the component wave of the minimum wave frequency. The autocorrelation function can be derived as

$$\begin{aligned}R_{min}(\tau) &= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x_{\min}(t)x_{\min}(t+\tau)dt \\ &= \frac{{a_{\min}}^2}{2}\cos\omega_{\min}\tau\end{aligned} \tag{4}$$

Based on Eq.(4), it follows that the spike in the autocorrelation function occurs at a period $\pi/\omega_{\min}$.

The calculations were executed with the two different uniform frequencies $\Delta\omega$ of Method 1: one is 0.02[rad/s], and the other is 0.06[rad/s]. The generated time histories of irregular waves are shown in Figure 1, and the autocorrelation functions are in Figure 2. The minimum wave frequency means the median of $\omega = 0$ and $\omega = \Delta\omega$. Therefore, the minimum wave frequencies for the two cases are 0.01[rad/s] and 0.03[rad/s]. Thereby, the self-repetition periods of the time histories are mathematically obtained as 628[s] and 209[s]. As shown in Figure 1, wave groups repeat at these periods. In addition, the spike period of the autocorrelation function can be calculated as 314[s] and 105[s] for these cases. As shown in Figure 2, it is observed that these mathematically calculated results are validated.

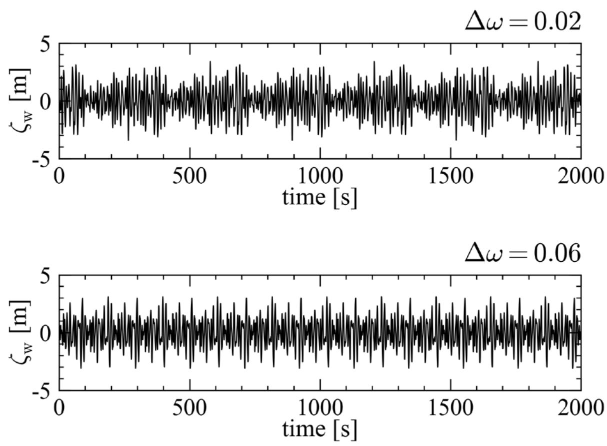

Figure 1: Time series of wave elevations by Method 1.

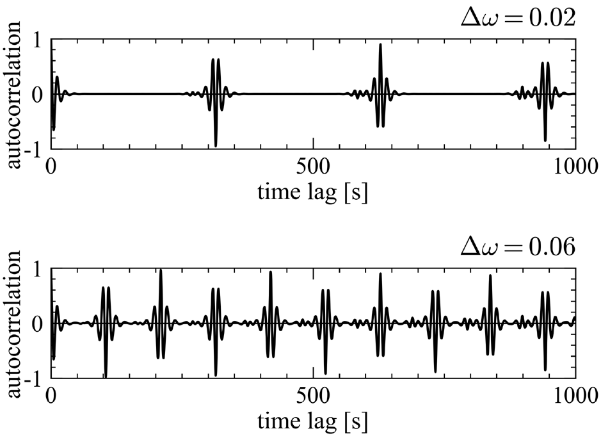

Figure 2: Autocorrelation function for irregular waves of Figure 1, calculated by FFT.

Inverse Fourier transformation with non-uniform frequency sampling (Method 2)

The method, which divides the spectrum so that the energies of component waves are equal to each other, is discussed. In this paper, this method is called "Method 2". In this method, the wave amplitude of each component wave is constant. Based on Shuku et al. (1979), the amplitude and the wave frequency of the arbitrary component wave in the case of the ITTC spectrum can be derived using the following equations.

$$a_n = \sqrt{\frac{A}{2BN}} \cong 0.3538 \times \frac{H_{1/3}}{\sqrt{N}} \tag{5}$$

$$\omega_n = \frac{1}{B}\left[\ln\frac{2N}{2n-1}\right]^{-\frac{1}{4}} \cong \frac{5.127}{T_{01}}\left[\ln\frac{2N}{2n-1}\right]^{-\frac{1}{4}} \tag{6}$$

From Eq.(6), the ratio of the wave frequency of two arbitrary component waves is defined as Eq.(7).

$$\rho = \frac{\omega_i}{\omega_j} = \frac{\left[\ln\frac{2N}{2i-1}\right]^{-\frac{1}{4}}}{\left[\ln\frac{2N}{2j-1}\right]^{-\frac{1}{4}}} \tag{7}$$

where ρ is a positive value. In this method, if more than one of the ratios between different two component wave's frequencies are rational number, the time history has a self-repetition period. Thereby, the ratio ρ is used for discussing the occurrence of a self-repetition. Firstly, for the convenience of explanation, k_i is defined as $k_i = \ln(2N/2i-1)$. Here, we discuss whether k_i is a rational or irrational number. For this purpose, k_i is assumed to be a positive rational number. Thereby, $k_i = n/m$ is expressed by a natural number n and m. Based on the above facts, Eq.(8) can be obtained.

$$(2N)^m = e^n(2i-1)^m \tag{8}$$

where e denotes Euler's number. Due to the power of a natural number, the left-hand side of Eq.(8) and $(2i-1)^m$ are natural numbers. Due to the power of an irrational number, e^n is an irrational number. Thus, the right-hand side of Eq.(8) follows as an irrational number from the multiplication of an irrational and a natural number. Therefore, there is a contradiction between the left-hand side of Eq.(8) and the right-hand side of Eq.(8). k_i is an irrational number, which is proved by *reductio ad absurdum*. Furthermore, ${k_i}^{0.25}$ is an irrational number because the power root of an irrational number is an irrational number. Thereby, ${k_i}^{-0.25}$ is an irrational number because the inverse of an irrational number is an irrational number. Therefore, it is clear that the wave frequency ω_n of each component wave is an irrational number. From the above proof, the ratio ρ is the ratio of an irrational number to an irrational number. When $i \neq j$, it cannot be determined whether ρ is an irrational number or not.

Since the non-existence of self-repetition is not mathematically provided, the numerical calculation of Method 2 is conducted for two different numbers of sampled frequencies for elements: one is 100 and the other is 10000. FFT analyses the generated time history of irregular waves for 9 hours. As shown in Figure 3, we can observe the comparison of the spectra between the FFT result and the spectrum specified by Eq.(2). As the number of elements increases, marked spikes of the spectra decrease. As shown in Figure 4, we can observe the autocorrelation function of the two cases. In the upper panel of Figure 4, the spreading error can be observed in the autocorrelation function. Belenky (2011) observed a comparable result of the autocorrelation function. As the number of elements increases, the autocorrelation functions converge to zero drastically and overall are less noisy. In contrast to Figure 2, it can be stated that the irregular waves generated by Method 2 here do not have a self-repetition. Especially, the greater number of elements there are, a self-repetition hardly occurs.

Furthermore, the spectra obtained from the 1 hour-time history for different element numbers and the specified spectrum are compared in the Mean Squared Error (MSE). The result can be shown in Figure 5. Concerning the spectrum, if the number of elements is 1000 or more, it can be seen that Method 2 can adequately generate the time history of irregular waves.

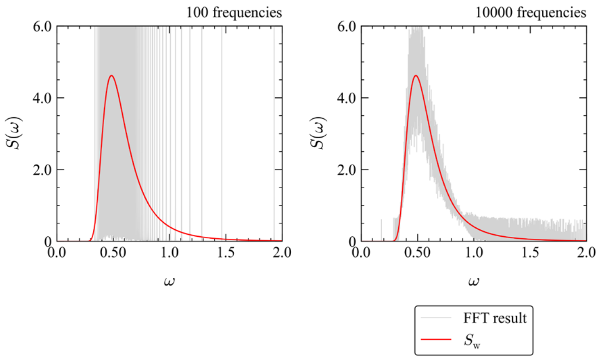

Figure 3: Comparison between ITTC spectrum and the spectrum obtained by FFT.

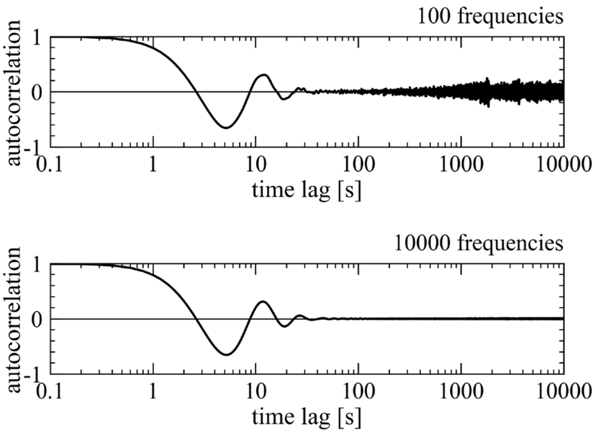

Figure 4: Autocorrelation function for irregular waves of Method 2, calculated by FFT.

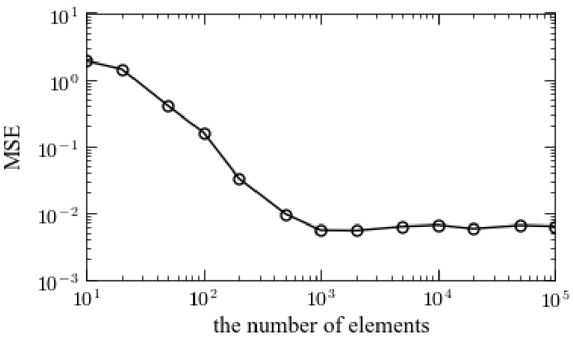

Figure 5: Relationship between MSE and the number of elements with equal wave energies.

Linearly filtered white noise (Method 3)

The real noise, such as sea waves, is not white but coloured. This process could be generated from filtered white noise via the stochastic differential equation for the stochastic method. In this study, to realise real noise from white noise, the time history of irregular waves is modelled using an Autoregressive Moving Average (ARMA) process. We presume a more accurate approximation can be obtained using a higher-order linear filter. Therefore, the following 6th-order ARMA filter is used as Eq.(9). Here, x_1 denotes the wave amplitude, and dW/dt is the white noise. The spectrum S_6 corresponding to Eq.(9) is expressed as Eq.(10).

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 - \alpha_1 x_1 \\ \dfrac{dx_2}{dt} = x_3 - \alpha_2 x_1 + \sqrt{\pi}\Gamma \dfrac{dW}{dt} \\ \dfrac{dx_3}{dt} = x_4 - \alpha_3 x_1 \\ \dfrac{dx_4}{dt} = x_5 - \alpha_4 x_1 \\ \dfrac{dx_5}{dt} = x_6 - \alpha_5 x_1 \\ \dfrac{dx_6}{dt} = -\alpha_6 x_1 \end{cases} \tag{9}$$

$$S_6\ (\omega) = \frac{\Gamma^2 \omega^6}{\left(-\omega^6 + \alpha_2\, \omega^4 - \alpha_4\, \omega^2 + \alpha_6\right)^2 + \left(\alpha_1\, \omega^5 - \alpha_3\, \omega^3 + \alpha_5\, \omega\right)^2} \tag{10}$$

It is necessary to determine the coefficients α_i and Γ included in Eq. (10) to fit the ITTC spectrum well. In this case, the stability criterion of the system proposed in Maruyama et al. (2022) is applied. This criterion means the numerical stability and uses the denominator of the transfer function derived from Eq.(9). As a result, the solid red line in Figure 6 can be obtained. This spectrum agrees with the specified ITTC spectrum plotted by the solid black line well. The time history of irregular waves can be modelled by calculating the stochastic differential equation (SDE) corresponding to Eq.(9) numerically. In this paper, this method is called "Method 3". The time history is computed using the Euler-Maruyama method (Maruyama, 1955). The time step is set as 0.001[s]. The solid grey line of Figure 6 shows the spectrum, while Figure 7 shows the autocorrelation function. These results are obtained using FFT for the time history generated by Method 3 for 9 hours. This spectrum agrees with S_6 and S_W well, and it is clear that the spectrum characteristics reflect on the time history modelled by computing the SDE. For this autocorrelation function, there is no significant spike. Compared with Figure 2, it is considered that irregular waves for modelling by Method 3 do not have a self-repetition. Furthermore, in Figure 8, the difference between the spectrum obtained from the time history of Method 3 for 1 hour at each time step and the ITTC spectrum is evaluated by MSE. In Figure 9, the difference between the PDF obtained from the time history of Method 2 for 1 hour and the PDF obtained from the time history of Method 3 is evaluated by MSE. As a result, from the viewpoint of the spectrum and the PDF, if the time step in the

Euler-Maruyama method is 0.02[s] or less, it can be seen that Method 3 can adequately generate the time history of irregular waves.

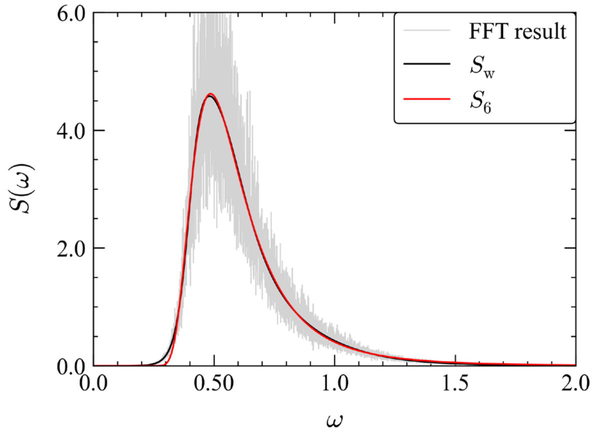

Figure 6: Comparison of the ITTC spectrum, the ARMA spectrum, and the spectrum obtained by FFT.

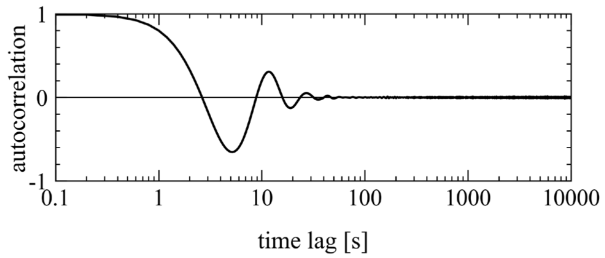

Figure 7: Autocorrelation function for irregular waves of Method 3, calculated by FFT.

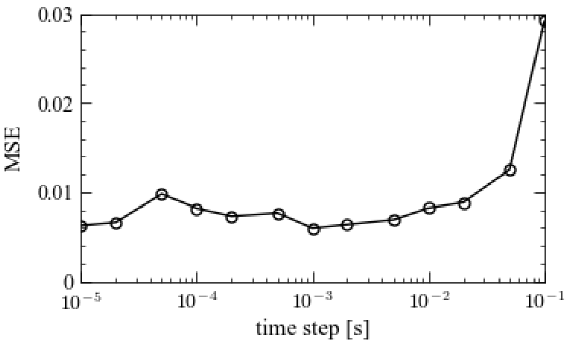

Figure 8: Relationship between MSE of spectra and time step.

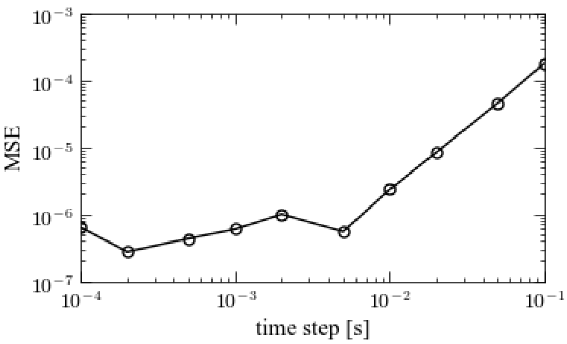

Figure 9: Relationship between MSE of the wave amplitude's PDFs and time step.

3. ROLL MOTION

Although we confirmed no self-repetition of the incident waves generated by Method 2 or 3, it is still not certain whether the roll motion due to the incident waves has a self-repetition. The roll motion itself is a kind of band-pass filter so that the element preventing the self-repetition of the incident waves could be excluded. Therefore, using these three methods, self-repetitions of the roll motions are investigated. In this study, to derive the time history of roll angle, the self-repetition effect is discussed using the equation for nonlinear parametric roll motion in irregular longitudinal waves modelled with Eq.(11).

$$\ddot{\phi} + \beta_1\dot{\phi} + \beta_3\dot{\phi}^3 + \sum_{i=1}^{5}\gamma_{2i-1}\phi^{2i-1} + \sum_{j=1}^{12}\lambda_j A_w\phi^j = 0 \tag{11}$$

Here, the roll angle, the roll velocity, and the roll angular acceleration are denoted by ϕ, $\dot{\phi}$, and $\ddot{\phi}$, respectively. The parameter β_1 is the linear and β_3 is the cubic damping coefficient, γ_{2i-1} $(i = 1,2,3,4,5)$ is the coefficient of the i-th component of the polynomial fitted GZ curve, $\lambda_j (j = 1,2,\cdots,12)$ is the coefficient of the j-th component of the polynomial that fits the relationship between ΔGM and wave elevation at amidships, shown in Maruyama et al. (2022). Moreover, A_w is the effective wave amplitude. The role of this relationship is to translate a Gaussian process such as the effective wave into a non-Gaussian process such as the parametric excitation. In this study, the subject ship is a post-Panamax containership of the C11 class, which is utilised in our previous study (Maruyama et al., 2022).

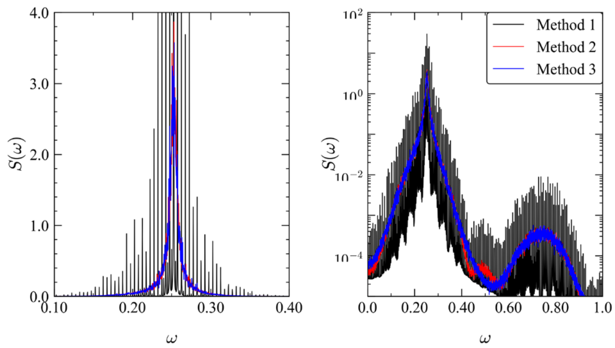

Figure 10: Comparison among the roll motion spectra obtained using three methods.

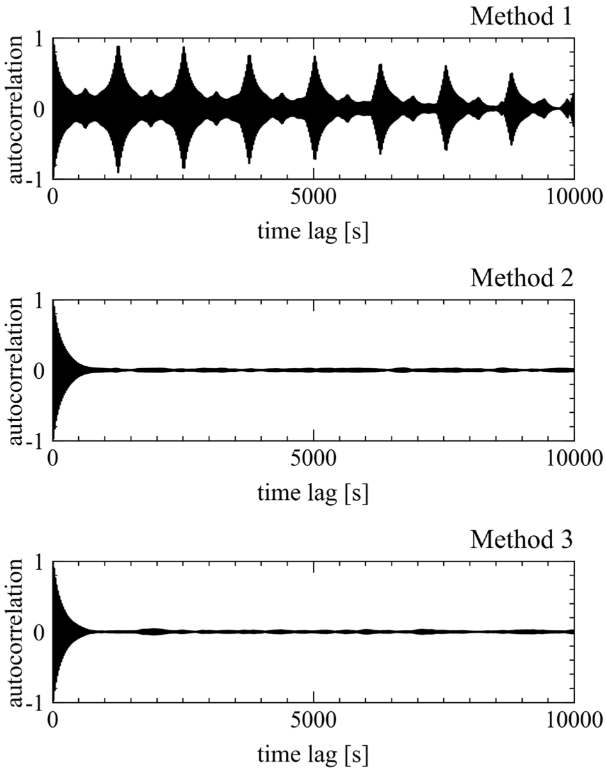

Figure 11: Autocorrelation functions of roll motion obtained by using three methods.

The time history of roll angle is calculated using the waves generated by three methods, and each simulation duration is 9 hours. The uniform step width $\Delta\omega$ of Method 1 is set at 0.02[rad/s], the number of elements of Method 2 is set at 1000, and the time step in the numerical calculation of Method 3 is set at 0.001[s]. Firstly, the spectra of Figure 10 are obtained by FFT for the time history of roll angle. Secondly, the autocorrelation functions of Figure 11 are obtained by applying FFT to the spectra with no smoothing. In Method 1, the repetition period of irregular waves is 628[s]. The spectrum of Method 1, plotted by the solid black line in Figure 10, has spikes at uniform intervals. Even as the time lag increases, spikes can be observed occurring in the autocorrelation function of Method 1 in Figure 11 at the same time step. On the other hand, the spectra of Method 2 and 3 in Figure 10 are noisy but have the same results. Furthermore, as the time lag increases, the autocorrelation functions of Method2 and 3 in Figure 11 generally converge to zero.

CONCLUDING REMARKS

In this study, by setting a sufficient number of elements, i.e. 10000 elements, we could observe that self-repetition does not occur in the nine hour-time history of the method, which divides the spectrum into equal areas by changing the sampled frequency intervals. Furthermore, we confirmed that the required time step to generate from filtered white noise via the stochastic differential equation is 0.001s. The marked spike does not occur for the autocorrelation function of the time history by the linearly filtered white noise for 9 hours. Moreover, each method modelling irregular waves is applied to the equation of roll motion due to the incident waves for investigating the self-repetition of roll angle. As a result, no self-repetition of the roll motion due to the incident waves was confirmed when the incident waves do not have self-repetition.

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