The Transverse Stability and Rolling of a Vessel Loaded by Elastically Movable Cargo

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ABSTRACT

Stability and rolling of a vessel loaded with elastically movable cargo (EMC) are studied. Springing and being deformed such cargo shifts down and aside causing variations in mass moment of inertia of an oscillatory system, position of ship's gravity center and appearance of an additional heeling moment. Under assumption of small angles of inclinations the differential roll equation is analyzed for a vessel loaded with EMC which mass moment of inertia is varying harmonically with incoming waves frequency. The parametric resonance conditions are formulated.

The nonlinear differential roll equation for finite oscillations of a vessel loaded with EMC is presented and the loss of stability conditions are studied. Theoretical points are illustrated by calculation results for motor vessel "Rechitsa".

Key words: *elastically movable cargo (EMC), stability of ship in waves, capsizing conditions.*

1. INTRODUCTION

Motor vessel "Rechitsa" of the Soviet Danube shipping company, making trip from port Reni (Danube river) to port Alexandria with a cargo of a rolled wire in coves, loaded by bulk, tipped and sunk on November, 26th, 1976 at an exit to Mediterranean sea.

According to the testimony of the survived crew members during the moment of accident there were force 7 sea waves, thus the vessel was rolling with the amplitude of $10^0 – 12^0$ on both sides. Having wind heel nearby 2^0 and making a sharp turn aside for a divergence with a meeter, motor vessel "Rechitsa" at the next inclination has tilted on a port side up to 20 - 25^0. The vessel has not returned to the vertical position and at several subsequent fluctuations the angle of inclination on a port side continued to increase. After 8 - 10 minutes the vessel has laid down on a port side, then was turned over bottom up and has sweepingly sunk. The increase of heal angle was escorted by a hum, a gnash, perceptible hull vibration.

Onboard a vessel there was a cargo of a steel rolled wire in coves, loaded by bulk in all eight cargo compartments. The cove of a rolled wire has the form of spring ring - a torus of 1,2 m in diameter and weight of 0.8 т.

The analysis of ship documentation has shown that at the departure the vessel completely met the stability requirements of the Register of Shipping of the USSR. The rolled wire in coves had never been mentioned in any official normative document as a cargo which is dangerous for ship stability conditions. Thus no resolutions existed regarding special measures to be taken during the loading of a vessel with EMC.

The official investigation, including both calculational analysis and special natural experiments, was performed after loss of a vessel. This investigation allowed to establish the following facts:

- the cargo of rolled wire steeped by bulk presents a continuous springing mass in which under the act of gravity and vibration appreciable elastic and residual deformations of compression and shifting are obtained,
- the deformability of this cargo was studied experimentally by keeling of a tank filled with it. It was shown that modification of a specific loading volume of that cargo varied from 1.8 - 1.9 m^3/т up to 1.3 m^3/т during the inclination,
- experiments with the tank have also shown that if the heel angle exceeded 18^0 the cargo filling tank space not completely started to shift aside, keeping its free surface close to horizontal level.

The presented special properties of a cargo could render significant effect on actual stability of a vessel (Bondar V.M., 1999; Sizov V.G., 1999). This effect has been strengthened owing to incomplete filling of cargo compartments at loading and the subsequent self-packing of cargo. So at the moment of accident more than 25% of the volume of total space in the upper part of cargo compartments was empty. Formation of voids promoted shifting of cargo after the vessel reached angle exceeding 18^0.

The mentioned special properties of a cargo, manifested during oscillating of a vessel in waves, their probable role in outcome of accident with “Rechitsa”, lack of precise rules on conveyance special cargoes with similar properties (rubber, wool and wooden chips in bales) if they are steeped by bulk and not shared with rigid separation, cause an imperative need of comprehensive study of dynamics of the vessel, loaded with EMC under heavy wave conditions.

Among prime follows:

- to a evaluate necessity of taking into consideration fluctuations of mass moment of inertia of cargo and vessel,
- to find differential rolling equation for a vessel taking into account particular properties of EMC,
- to conduct the analysis of mentioned rolling equation for a vessel with EMC and to outline the zones with stable and unstable solutions,
- to illustrate the obtained results by predicted data for motor vessel “Rechitsa”. The main characteristics of motor vessel “Rechitsa” are given in table 1.

Table 1. – The characteristics of motor vessel “Rechitsa”

Length between perpendiculars	$L_{\perp\perp}$	109.0
Breadth	B	16.6
Depth	D	8.36 m
Draft	d	6.53 m
Displacement	Δ	8420 t
Load weight	P	4032 t
Volume of cargo spaces	V	6885 m3
Mass moment of inertia of vessel (including water added mass moment) round longitudinal axis	J_x	30325 t·sec2/m
Height of gravity center of vessel (at the end of loading)	z_g^0	6.34 m
Height of gravity center of vessel before accident	z_g	6.06 m

Let's notice that lowering of ship's gravity centre before accident in comparison with the moment at the end of her loading is caused by compression of EMC.

2. EFFECT OF EMC MASS MOMENT OF INERTIA FLUCTUATIONS ON ROLLING OSCILATIONS OF VESSEL

Let's consider oscillations of vessel with periodically varying mass moment of inertia of EMC loaded on board (Sizov V.G., 2002).

The mass moment of inertia of vessel is represented in the form

$$J_x = I_1 + I_c\,, \tag{1}$$

where I_1 is a constant component of mass moment of inertia of a vessel (moment of inertia of added mass of water included); I_c mass moment of inertia of strained and displaced part of cargo.

Let's present I_c in the form

$$I_c = i_0 + mr^2\,, \tag{2}$$

where m is a mass of straining and displacing cargo; $r = r_0 \sin\sigma t$ is radius of inertia of mass m; σ is frequency of encounter; i_0 - inertia moment of mass m at it's the most compact disposition relatively axis of fluctuations.

Assuming $I + i_0 = I_0$, we receive

$$J_x = I_0 + mr_0^2 \sin^2\sigma t = I_0\left(1 + \varepsilon \sin^2\sigma t\right), \tag{3}$$

where $\varepsilon = \dfrac{mr_0^2}{I_0}$, and it is accepted $\varepsilon \ll 1$.

In a problem considered it is impossible to use the traditional equation of rolling which is fair for a constant mass moment of inertia of vessel. Using a principle of conservation of angular momentum for oscillating system, we receive in case of small inclinations

$$\frac{d}{dt}\left(J_x \ddot{\theta}\right) + N\dot{\theta} + K\theta = K\alpha_0 \sin\sigma t\,, \tag{4}$$

where N is drag coefficient of rolling; $K = \Delta h$ - coefficient of transverse stability; h - metacentric height; α_0 - an effective wave slop angle of incoming waves with angular frequency σ.

From (4) it is found

$$J_x \ddot{\theta} + \frac{\partial J_x}{\partial t}\dot{\theta} + N\dot{\theta} + K\theta = K\alpha_0 \sin\sigma t\,, \tag{5}$$

and

$$\frac{dJ_x}{dt} = \varepsilon I_0 \sigma \sin 2\sigma t\,, \tag{6}$$

$$\frac{1}{J_x} = \frac{1}{I_0\left(1 + \varepsilon \sin^2\sigma t\right)} = \frac{1}{I_0}\left(1 - \varepsilon \sin^2\sigma t\right) + O\left(\varepsilon^2\right) \tag{7}$$

Let us take a new variable

$$\chi = \theta \exp\left(\frac{1}{2}\int B(t)dt\right), \tag{8}$$

where $B(t) = N + \dfrac{\dot{J}_x}{J_x}$.

From the equation (5) it comes

$$\begin{aligned} &\ddot{\chi} + \left(n^2 - \frac{1}{4}B^2 - \frac{1}{2}\dot{B}\right)\chi = \\ &= n^2\alpha_0 \sin\sigma t\left(1 - \varepsilon \sin^2\sigma t\right) + O\left(\varepsilon^2\right), \\ &B(t) = \nu - \varepsilon\left(\frac{\nu}{2} + \sigma \sin 2\sigma t + \frac{\nu}{2}\cos 2\sigma t\right); \nu = \frac{N}{I_0}, \\ &\dot{B}(t) = \frac{dB(t)}{dt} = 2\varepsilon\sigma^2 \cos 2\sigma t - \\ &- \varepsilon\nu\sigma \sin 2\sigma t + O\left(\varepsilon^2\right), \end{aligned} \tag{9}$$

where $n = \sqrt{K/I_0}$ - frequency of free oscillations of vessel.

The equation (9) takes a form

$$\begin{aligned} &\ddot{\chi} + \left[n^2 - \frac{\nu^2}{4}(1 - \varepsilon) - \varepsilon\left(\sigma^2 - \frac{\nu^2}{4}\right)\cos 2\sigma t\right]\chi = \\ &= n^2\alpha_0 \sin\sigma\tau\left(1 - \varepsilon \sin^2\sigma\tau\right). \end{aligned} \tag{10}$$

Let's enter dimensionless time $\tau = \sigma t$, so $\ddot{\chi} = \dfrac{d^2\chi}{dt^2} = \sigma^2\dfrac{d^2\chi}{d\tau^2}$ and the equation of oscillations (10) rewrite in the form

$$\frac{d^2\chi}{d\tau^2} + (a - 2q\cos 2\tau)\chi = \frac{n^2}{\sigma^2}\alpha_0 \sin\tau\left(1 - \varepsilon \sin^2\tau\right) \tag{11}$$

$$a = \frac{n^2}{\sigma^2} - \frac{\nu^2}{4\sigma^2}(1 - \varepsilon), \quad q = \frac{\varepsilon}{2}\left(1 - \frac{\nu^2}{4\sigma^2}\right).$$

Besides forced oscillations under exciting moment action described by (11) parametric oscillations can arise defined by the uniform equation

$$\frac{d^2\chi}{d\tau^2}+(a-2q\cos 2\tau)\chi=0. \qquad (12)$$

The equation (12) is an initial form of Mathieu equation. Solutions of this equation have oscillatory character. Depending on values of parameters a and q these oscillations have limited or unlimited amplitude increasing by the exponential law. Borders of parameters a and q, zones corresponding to stable and unstable condition of the oscillatory system described by the equation (12) can be find on Eince - Strett diagram (Smirnov, 1968). In our case the value of q is small, and the least ε value of relative change in mass moment of inertia of a displaced cargo at which amplitude of oscillations increases infinitely can be find from inequality

$$\varepsilon>\frac{n^2-\frac{\nu^2}{4}}{\sigma^2-\frac{\nu^2}{2}}. \qquad (13)$$

On returning from χ to roll angle θ an exponential multiplier appears

$$exp\left[-\frac{1}{2}\int B(t)dt\right]= =exp\left[-\frac{\nu}{2}\left(1-\frac{\varepsilon}{2}\right)+\frac{\varepsilon}{4}\sqrt{1+\frac{\nu^2}{4\sigma^2}}\cos(2\sigma t-\delta)\right] \qquad (14)$$

This multiplier ensures damping action on oscillations. The solution of equation (12) can be find in the form

$$\chi=e^{\mu\tau}f_1(\tau)+e^{-\mu\tau}f_2(\tau), \qquad (15)$$

where $f_1(\tau)$ and $f_2(\tau)$ - are periodic functions of τ.

It allows to write down additional condition for excitation of parametric oscillations with increasing amplitude

$$\mu>\frac{\nu}{2}\left(1-\frac{\varepsilon}{2}\right), \qquad (16)$$

as soon as the average value of second component (14) for a period of oscillations is equal to zero.

It is obvious that damping action narrows area where parametric resonance exists.

The described method is applicable for an estimation of a principal opportunity of existence of a parametric resonance in rolling for vessel “Rechitsa” in her last trip. Fluctuation of cargo mass moment of inertia which has arisen in connection with elastic and residual strains of EMC has been considered only. Change of mass moment of inertia of cargo due to its transverse displacement at heel is not considered here. It is made in following section of work.

The analysis of documents concerning the loading of the vessel, results of experiment and calculations allowed to find that inertia moments in (2) have values $I_c=11959t\cdot sec^2/m$, $i_0=10025t\cdot sec^2/m$, $I_0=28391t\cdot sec^2/m$, so that $\varepsilon=0.068$.

The value of $\nu=N/I_0$ as function of a wave length (frequency of oscillations) was calculated according to (Remez J.V.,1983).

On the basis of (13) maximum values of metacentric height h^M were defined as function of wave length λ. If for a given loading conditions $h<h^M(\lambda)$, parametric resonance under such wave excitation is possible, but if $h>h^M(\lambda)$ the parametric resonance does not occur. Values of h^M for wave lengths λ are given in table 2. Here the minimal values of μ_{min} from (16) are also presented.

Table 2 – Values of metacentric height h^M

λ, m	45	67	95	140	160
h^M ,m	0.310	0.210	0.148	0.100	0.090
μ_{min}	0.020	0.024	0.025	0.028	0.030

It is visible that the accepted scheme does not allow to find the possibility of parametric resonance appearance as soon as actual metacentric height values for motorship "Rechitsa" both on departure ($h = 0.36$ m) and before capsizing ($h = 0.64$ m due EMC compression) exceeded values given in table 2.

Nevertheless, results of the analysis have proved the necessity of taking into account change of mass moment of inertia for EMC in calculations of rolling for vessel with specific cargoes of EMC type.

3. THE DIFFERENTIAL ROLL EQUATION OF A VESSEL WITH EMC AND ITS ANALYSIS

Let's find a differential roll equation of the vessel transporting EMC, taking into account its special properties related with the advent of variations in mass moment of inertia, position of ship gravity center, appearance of additional heeling moment.

Considering the form of equation (4), the differential roll equation of a vessel in regular beam sea can be presented in the form

$$J_x \ddot{\theta} + \frac{\partial J_x}{dt}\dot{\theta} + N\dot{\theta} + \kappa_\theta\, m(\theta) + M(\theta) = \\ = \kappa_\theta m(\alpha_0 \sin\sigma t). \tag{17}$$

Here in addition to (4) following symbols are introduced

$m(\theta)$ - restoring moment,

$M(\theta)$ - heeling moment called by shifting EMC aside, $M(\theta)$ and $m(\theta)$ are acting opposite,

κ_θ - reduction coefficient for exciting and restoring moments (Remez, 1983)

Let's note, that

$$J_x = I_0 + I(\theta), \tag{18}$$

where $I(\theta)$ is a variable part of mass moment of inertia of oscillating system due to compression and shifting aside of EMC.

Considering (18), we find

$$\frac{dJ_x}{dt} = \frac{dI(\theta)}{d\theta}\frac{d\theta}{dt} = \frac{dI(\theta)}{d\theta}\dot{\theta}, \tag{19}$$

Let's introduce the dimensionless time $\tau = \sigma t$ so that $\frac{\partial}{\partial t} = \sigma\frac{\partial}{\partial\tau}$, $\dot{\theta} = \sigma\frac{d\theta}{d\tau}$, $\ddot{\theta} = \sigma^2\frac{d^2\theta}{d\tau^2}$ and rewrite the equation (17)

$$\frac{d^2\theta}{d\tau^2}[1 - Q(\theta)] - \frac{dQ(\theta)}{d\theta}\left(\frac{d\theta}{d\tau}\right)^2 + \overline{N}\frac{d\theta}{d\tau} + \\ + \frac{\kappa_\theta \overline{m}(\theta)}{\sigma^2} + \frac{\overline{M}(\theta)}{\sigma^2} = \frac{\kappa_\theta}{\sigma^2}\overline{m}(\alpha_0 \sin\tau). \tag{20}$$

In (20) magnitudes of $Q(\theta) = \frac{I(\theta)}{J_x}$, $\overline{m}(\theta) = \frac{m(\theta)}{J_x}$, $\overline{M}(\theta) = \frac{M(\theta)}{J_x}$ are introduced.

The damping coefficient $\overline{N}$ is determined under the supposition that the viscous damping moment is proportional to square of oscillation velocity and linearised using energy reasons (Remez, 1983), so that $N = \frac{\sigma\overline{N}}{J_x}$, and $\overline{N}$ depends on displacement Δ, mass moment of inertia J_x, metacentric height h, the ration B/d and hull coefficients δ and α. Thus all items in (20) are dimensionless.

The stability diagram $m(\theta)$, accepted in the subsequent calculations, corresponds to a maximum compression of EMC directly before an accident (h=0,64 m, z_g=6,06 m). The mass moment of inertia $I(\theta)$ $\theta > 18°$ is taken under supposition, that the side shifting of cargo arises when roll angles θ exceed the angle φ=18°. It is accepted that from a state of maximum vertical compression the free surface of cargo, remaining flat, is turned to tilted side on angle $\theta-\varphi$ when the vessel's heel angle equals θ. The cargo fills free volume of compartment and amount of cargo remains constant.

Functions $Q(\theta), \frac{dQ(\theta)}{d\theta}, \overline{m}(\theta)$ and $\overline{M}(\theta)$ are calculated using arrangement plans in the form of polynomials of heel angle θ. It has been considered, that

$Q(-\theta)=Q(\theta)$, $\frac{dQ(\theta)}{d\theta}=-\frac{dQ(-\theta)}{d\theta}$,
$\overline{m}(\theta)=-\overline{m}(-\theta)$, $\overline{M}(\theta)=$
$=-\overline{M}(-\theta), \theta>0$.

Damping coefficient $\overline{N}$ and reduction coefficient κ_θ are presented in the form of polynomials of incoming wave length λ.

The primal problem for examination of dynamics of the system governed by a differential equation (20) consists in studing the structure of phase plane $\left(\theta, \dot{\theta}\right)$ stuffing with phase trajectories. (Butenin N.V., Naimark Y.I, Fufaev N.A., 1987).

For this purpose it is enough to examine the behavior of special trajectories for the homogeneous equation corresponding to equation (20). The interior forces are counterbalanced in fixed points where $\dot{\theta}=\ddot{\theta}=0$. Limit cycles are closed curves on a phase plane to which aspire eventually $(\tau \to +\infty)$ certain set of phase trajectories. Separatrices separate those parts of phase plane where phase curves describe qualitatively equivalent behavior of oscillatory system.

Let's find the fixed points of equation (20) on a phase plane where $\dot{\theta}=\ddot{\theta}=0$.

Introducing a new variable

$$K=\dot{\theta}[1-Q(\theta)]+\overline{N}\theta, \qquad (21)$$

instead of equation (20) we find system of two first order differential equations for fixed points $\dot{\theta}=0$ and $\dot{K}=0$

$$\begin{cases} \dot{\theta}=\dfrac{K-\overline{N}\theta}{1-Q(\theta)}; \\ \dot{K}=-\dfrac{\overline{M}(\theta)+\kappa_\theta \overline{m}(\theta)}{\sigma^2} \end{cases} \qquad (22)$$

From the first equation of system (22) it follows that $\dot{\theta}=0$ along straight line $K=\overline{N}\theta$ on the plane (θ, K). From the second equation of (22) it comes that $\dot{K}=0$ in points θ_i which are zeroes of the function $\overline{M}(\theta)+\kappa_\theta \overline{m}(\theta)$. Getting from (21)
$\dot{K}=\ddot{\theta}[1-Q(\theta)]-\frac{\partial Q(\theta)}{\partial\theta}\left(\dot{\theta}\right)^2+\overline{N}\dot{\theta}$, we discover that in the fixed points $\ddot{\theta}$=0, so function $Q(\theta)$ is continuous. Taking into consideration oddness on θ of function $\overline{m}(\theta)$ as well as the fact that $\overline{M}(\theta)\equiv 0$ for $|\theta| \le 18°$ and for $|\theta| > 18°$ this function also is odd on θ, we find that system (22) and naturally the equation (20) have three fixed points: one in zero of phase plane ($\theta=\dot{\theta}=0$) and two others with abscissas corresponding to zeroes of polynomial function $\overline{M}(\theta)+\kappa_\theta \overline{m}(\theta)$. This function is governed by properties of vessel as well as reduction coefficient κ_θ that is additionally defined by incoming wave length. The calculations fulfilled for "Rechitsa" have shown that in a range of $0<\lambda\le 160$ m the fixed points have abscissas θ_1= 0,38, θ_2= -0,38, θ_3= 0 which practically do not depend on wave length λ.

Topological type and stability of fixed points we can define, exploring the linearized system, gained from (22), in a neighborhood of these points. Let u_ε be a small vicinity of fixed points $(\theta_i, 0)_i = 1,2$. For any point of

$(\theta_i + \varepsilon_\theta, \varepsilon_K)$ from the vicinity of u_ε it comes with an accuracy $O(\varepsilon_\theta^2 + \varepsilon_K^2)$

$$\begin{cases} \dot{\varepsilon}_\theta = \dfrac{\partial F_1}{\partial K}\varepsilon_K + \dfrac{\partial F_1}{\partial \theta}\varepsilon_\theta; \\ \dot{\varepsilon}_K = \dfrac{\partial F_2}{\partial K}\varepsilon_K + \dfrac{\partial F_2}{\partial \theta}\varepsilon_\theta, \end{cases} \tag{23}$$

where $F_1 = \dfrac{K - \overline{N}\theta}{1 - Q(\theta)}$, $F_2 = -\dfrac{\overline{M}(\theta) + \kappa_\theta \overline{m}(\theta)}{\sigma^2}$.

Roots $p_{1,2}$ of characteristic polynomial of linear system

$$\sigma^2[1 - Q(\theta_i)]p^2 + \sigma^2 \overline{N} p + \\ + \frac{\partial}{\partial \theta}\left[\overline{M}(\theta_i) + \kappa_\theta \overline{m}(\theta_i)\right] = 0, i = 1,2 \tag{24}$$

are determined in the form

$$p_j = \frac{-\sigma^2 \overline{N} \pm \sqrt{D}}{2\sigma^2[1 - Q(\theta_i)]} \quad (j = 1, 2). \tag{25}$$

Here D is discriminant of a characteristic polynomial (24).The topological type of the fixed points and behavior of dynamic system in their neighborhood are defined by roots (25). If roots p_j $(j = 1,2)$ are complex conjugate the corresponding fixed point is a focal point. The focal point is stable, if the real part of roots is negative and it is unstable if the real part is positive. When roots p_j $(j = 1,2)$ are real and have one sign the corresponding fixed point is a knot. If both roots are negative this is a stable knot, otherwise a knot is unstable. If roots p_j $(j = 1,2)$ are real with different signs this is saddle.

From (25) it follows, that the type of the roots p_j $(j = 1,2)$ for dynamic system is completely determined by discriminant D.

For $|\theta_i| \le 18^0$ the discriminant is

$$D = \sigma^4 \overline{N}^2 - 4\sigma^2 \kappa_\theta [1 - Q(\theta_i)] \frac{\partial \overline{m}(\theta_i)}{\partial \theta}. \tag{26}$$

In this range of θ the only fixed point $\theta_i = 0$ exists.

It is a stable focus if

$$h > \frac{J_x \sigma^2 \overline{N}^2}{4\Delta\kappa_\theta [1 - Q(0)]}. \tag{27}$$

When $|\theta_i| > 18^0$ the discriminant is

$$D = \sigma^4 \overline{N}^2 - \\ -4\sigma^2 \kappa_\theta [1 - Q(\theta_i)] \frac{\partial}{\partial \theta}\left[\overline{M}(\theta_i) + \kappa_\theta \overline{m}(\theta_i)\right] \tag{28}$$

.

For fixed points θ_1 = 0.38, θ_2 = – 0.38 $\dfrac{\partial}{\partial \theta}\left[\overline{M}(\theta_i) + \kappa_\theta \overline{m}(\theta_i)\right] < 0$. From (25) and (28) it comes that for fixed points θ_i $(i = 1,2)$ among two roots (25) one is positive and another one is negative, so both fixed points θ_i $(i = 1,2)$ are saddles.

Trajectories on phase plane (θ, K) are determined from obvious equality $\dfrac{dK}{d\theta} = \dfrac{\dot{K}}{\dot{\theta}}$. Substituting into this equality corresponding expressions from system (22), we find the differential equation of trajectories in the form

$$(K - \overline{N}\theta)\frac{dK}{d\theta} + \frac{1}{\sigma^2}\left[\kappa_\theta \overline{m}(\theta) + \overline{M}(\theta)\right][1 - Q(\theta)] = \tag{29}$$

If there is no damping $\overline{N} = 0$, this equation is easy to integrate and considering (21) to write down the equation of a set of phase trajectories

$$\left(\dot{\theta}\right)^2 + \frac{2}{\sigma^2[1 - Q(\theta)]}\int \frac{\kappa_\theta \overline{m}(\theta) + \overline{M}(\theta)}{1 - Q(\theta)} d\theta = C \tag{30}$$

.

The phase trajectories corresponding to stable solutions with various values of C are closed curves. They lay inside of area limited by separatrices for which constant C in equation (30) is easy to find, having substituted in it coordinates of saddle points $\theta_{1,2}$. The position of these points do not depend on magnitude of $\overline{N}$, therefore the trajectories for stable solutions of system with damping $\overline{N} \neq 0$ will not go out the area limited by separatrices (30). On fig. 1 the phase portrait of the homogeneous system without damping, on fig. 2 for this system with damping and on fig. 3 phase portrait of the system with damping and exterior periodic perturbation are shown. On fig. 4 free oscillations of system without damping (1) and with damping (2) both in phase and time spaces are shown.

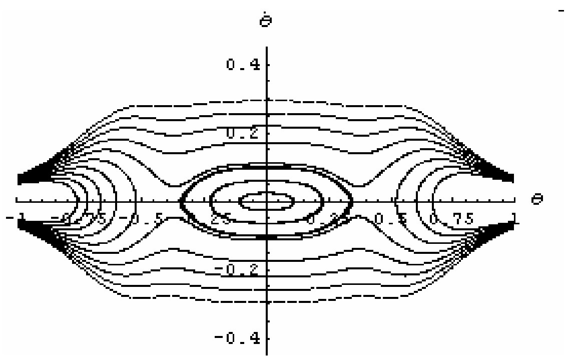

Figure 1 The plane portrait of the homogeneous system without damping ($\overline{N} = 0$)

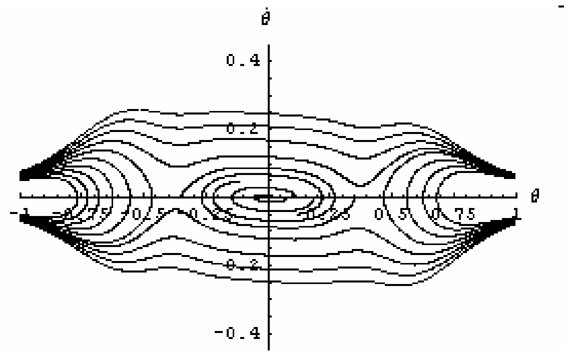

Figure 2 The plane portrait of the homogeneous system with damping ($\overline{N} \neq 0$)

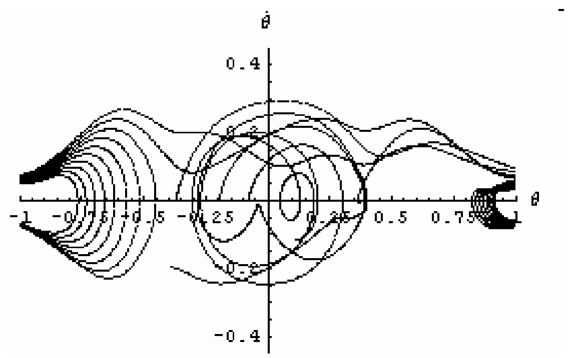

Figure 3 The plane portrait of system with damping and exterior periodic perturbation

For comparison the dynamic system is studied for rolling of the same vessel under supposition that the load is unmovable. In this case $Q(\theta) \equiv 0$ $\frac{dQ(\theta)}{d\theta} \equiv 0$, $\overline{M}(\theta) \equiv 0$ and differential roll equation looks like

$$\ddot{\theta} + \overline{N}\,\dot{\theta} + \frac{\kappa_\theta}{\sigma^2}\,\overline{m}(\theta) = \frac{\kappa_\theta}{\sigma^2}\,\overline{m}(\alpha_0 \sin\tau). \qquad (31)$$

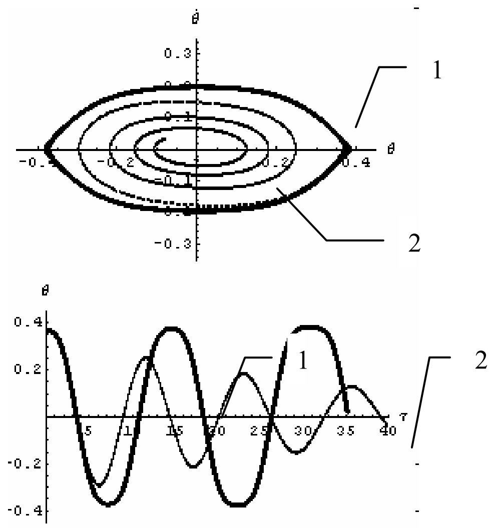

Figure 4 Free oscillations of system with and without damping in phase and time spaces

Comparing phase portraits of the homogeneous differential equations corresponding (20) and (31), it is easy to reveal qualitative differences in behavior of corresponding oscillating systems.

The differential equation $\ddot{\theta} + \overline{N}\,\dot{\theta} + \frac{\kappa_\theta}{\sigma^2}\,\overline{m}(\theta) = 0$ has three equilibrium points at phase plane, one of which $\left(\theta = \dot{\theta} = 0\right)$ is a focal point, and two others θ_1 and θ_2 are saddles ($\theta_1 = -\theta_2, \theta_1 > 0$). It is easy to show that θ_1 and θ_2 correspond to zeroes of function $\overline{m}(\theta)$ i. g. the angles $\pm\theta_v$ of vanishing stability on starboard and port sides. Thus, the area with stable oscillations of system appears for differential equation (31) significantly larger than analogous area for differential equation (20). It means that some situations defind for vessel with EMC by means of differential equation (31) as safe, can lead to real capsizing of a vessel if the capsizing comes from solution of equation (20).

Two rolling processes of motor vessel "Richitsa" with EMC under similar wave conditions are given in fig. 5. The process marked by 1 corresponds to supposition that loaded cargo is unmovable and equation (31) is used, the process marked by 2 correspond to equation (20) where all special properties of EMC are taken into consideration. The result is obvious: to predict dangerous situation one has to take in mind specific qualities of EMC and use for calculation differential equation (20).

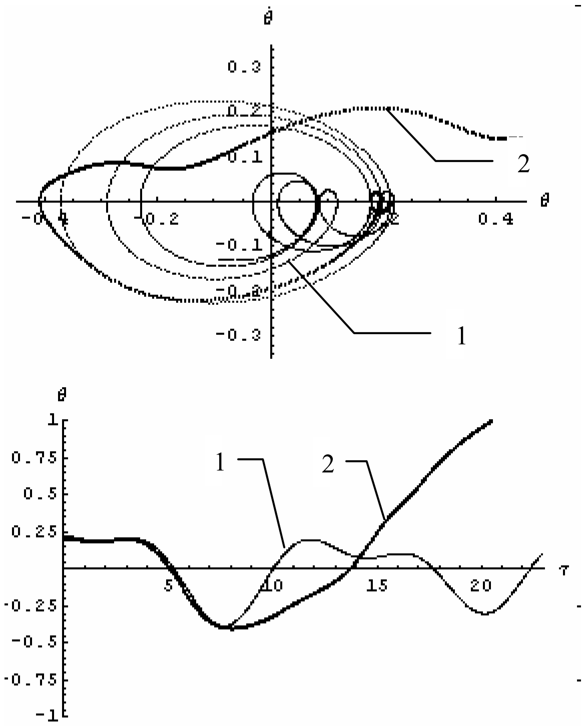

Figure 5 The comparison of rolling processes

4. CONCLUSIONS

The vessel, loaded with EMC, possesses special dynamic properties cousing the unusual response to incoming waves when navigations in heavy seas.

Special properties of a cargo lead to:

- fluctuation in mass moment of inertia of a cargo that provocates a parametric resonance of a vessel rolling;
- appreciable changes on vertical position of ship's gravity centre due to condensation and deformation of a cargo;
- action of additional heeling moment due to lateral cargo motion when ship is oscillating.

These reasons have been considered when deducing differential roll equation for vessel with EMC. The specified differential equation has basic differences from a standard roll equation in connection with the account of fluctuations in mass moment of inertia and occurrence of additional heeling moment of a cargo transferred due to rolling.

Applying practice of introduction reduction coefficient to disturbing moment, authors have considered it natural to enter the same coefficient into restoring moment as the hydrodynamic nature of both moments is identical.

The situation is demonstrated that under same loading and wave conditions the solution of traditional roll equation which doesn't take into consideration specific cargo properties demonstrates a normal rolling process while specialized equation when the decision of the standard differential equation demonstrated in this report and accounting special properties of EMC shows the capsizing of vessel.

Results of this research demonstrate the vital necessity of introducing into a known line of moving cargoes (liquid, hanging, rolling, loosing) an additional one – elastically movable cargo (EMC). It is necessary to supply the navigators transporting this cargo with reliable regulating documents.

5. REFERENCES

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