Parametric Rolling of Ships – Then and Now

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ABSTRACT

Modern research on parametric rolling of ships was first conducted in Germany in the late 1930s. This work was initiated in an effort to explain the capsizing of some small ships such as coasters and fishing vessels in severe following seas. The work included experiments with models in open water as well as numerical and theoretical computations. The phenomenon was thought to be of concern principally in following seas and for small, low freeboard ships. In the 1990s, however, there were reports of containerships and even some cruise ships experiencing heavy rolling in head seas. These were ships having a hull form characterized by great flare forward and wide flaring stern sections, features that are known to lead to significant stability variations in waves. The APL CHINA casualty in October 1998 focused attention on parametric rolling in head seas. The investigation of this casualty included theoretical computations of GZ variations, model experiments and numerical simulations as well as meteorological studies of the wind and sea conditions prevailing at the time. Results of the investigation received wide dissemination in the technical press. Since the APL China casualty and other similar incidents, much theoretical and experimental work has been focused on head seas parametric roll. IMO and many of the Class Societies now have recommendations to designers and masters for avoiding head seas parametric rolling situations.

1. INTRODUCTION

The phenomenon of parametric roll has been recognized by naval architects for more than fifty years. It may be described as a spontaneous rolling motion of the ship moving in head or following seas that is not directly excited by the waves but comes about as a result of a dynamic instability of the motion. In pure head or following seas the transverse symmetry of the ship would imply that no wave-induced roll exciting moment should be present. Nevertheless, for certain frequencies of wave encounter, it is found that a small initial disturbance in roll can trigger an oscillatory rolling that can grow to appreciable amplitude after only a few cycles. This is explained in terms of a dynamic instability or bifurcation in the motion characteristics. In 1863, William Froude identified a ship roll-heave coupling phenomenon akin to parametric roll consisting of heave oscillations at the heave natural frequency excited by the small roll induced variations in the buoyant force which occur at twice the roll frequency.

The occurrence of parametric roll is directly related to variations in the transverse stability of the ship as it moves through head or following waves. As the ship encounters successive waves, the geometry of the immersed portion of the hull varies as the waves move along the ship length. The underwater geometry changes with time due to the pitch-heave motion combined with the wave profile. This results in a time varying transverse stability as measured by both GM for small angles of heel and the GZ curve at large angles. Figure 1 contains such righting arm curves for a general cargo ship of 1960s vintage and a modern post Panamax containership, both in waves of length equal to ship length and height equal to L/20.

The general character of the time varying curves is illustrated here consisting of diminished stability when the wave crest is amidships and enhanced stability when a trough is amidships. For intermediate longitudinal positions of the wave the stability values vary accordingly.

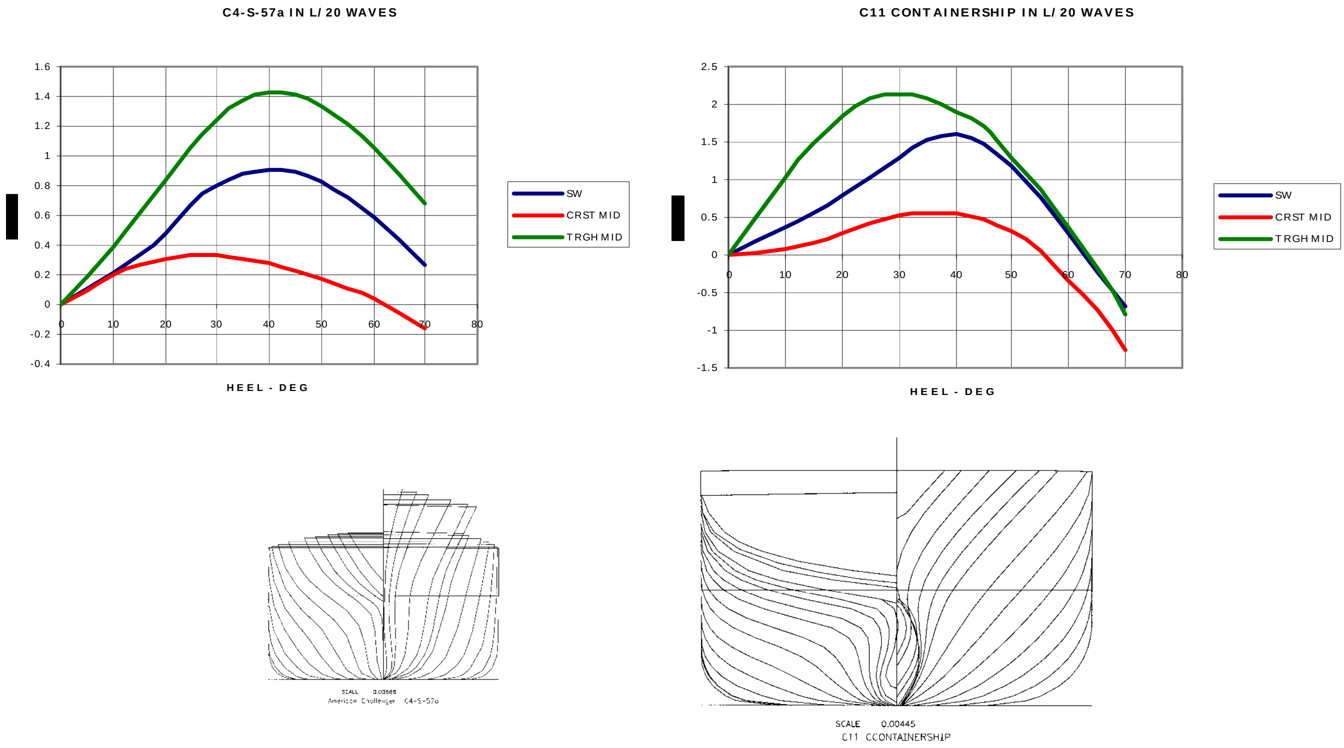

Figure 1. Righting arm curves for two ships in calm water, head and following seas: C4 - American Challenger Cargo Ship of 1960s vintage and C11 Post Panamax Containership (Body plans not to scale)

2. A SIMPLE DYNAMIC MODEL OF PARAMETRIC ROLL

The dynamic rolling motion a ship undergoing these stability variations in head or following seas is analogous to that of a spring-mass system in which the spring constant varies sinusoidally in time. For the ship the frequency of these variations equals the frequency of wave encounter. Let us now examine the small amplitude roll dynamics of a ship with periodic stability variations using a simplified model as follows. As our ship moves through head or following seas the time-varying GM may be expressed as Equation (1).

$$GM(t) = GM_o(1 + C\cos\omega t) \qquad (1)$$

Here, GM_o = still water GM,
C = fractional variation of GM due to waves, heave and/or pitch,
ω = encounter frequency

The equation of motion for uncoupled roll motion without excitation is now given by Equation (2).

$$I_x \frac{d^2\phi}{dt^2} + \Delta\phi(GM_o + CGM_o\cos\omega t) = 0 \qquad (2)$$

Here, ϕ = angle of roll,
Δ = ship displacement,
I_x = mass moment of inertia in roll, including added mass effect

Now divide both sides of (2) by I_x and make the change of variable,

$$\tau = \omega t \qquad \text{for which} \qquad \frac{d^2}{dt^2} = \omega^2 \frac{d^2}{d\tau^2}\,.$$

We note that $\omega_n^2 = \dfrac{\Delta GM_o}{I_x}$,

where $\omega_n =$ natural frequency of roll,

we define

$$\delta = \frac{\Delta GM_o}{\omega^2 I_x} = \frac{\omega_n^2}{\omega^2}\ ,$$

$$\varepsilon = \frac{C\Delta GM_o}{I_x\omega^2} = C\frac{\omega_n^2}{\omega^2}\,.$$

The equation of roll motion now becomes,

$$\frac{d^2\phi}{d\tau^2} + (\delta + \varepsilon\cos\tau)\phi = 0 \qquad (3)$$

This ordinary differential equation with sinusoidally varying spring constant is known as the *Mathieu* equation and the behavior of its solutions have been determined. Figure 2, known as the Ince-Strutt diagram illustrates the stability of solutions for the Mathieu equation. In this diagram the shaded regions represent stable solutions to the equation and unshaded regions correspond to unstable solutions. Thus, if the two parameters (δ,ε) for the system lie in a stable region, an arbitrarily small initial disturbance will die out with increasing time, while if they lie in an unstable region, the disturbance will grow with time.

We see that δ is equal to the square of the ratio of the natural frequency of roll to the frequency of variation of GM and ε is proportional to the fractional change in GM.

The first unstable region is centered on a value δ=1/4 or a ratio of natural frequency to frequency of GM variation of ½. If the frequency of GM variation does not exactly satisfy this value, unstable motion can still occur for a sufficiently large value of C, i.e., if the amplitude of the variation in GM is sufficiently large.

The effect of linear damping is to merely raise the threshold value of C at a given frequency of variation, ω. The unstable motion will still take place if C is sufficiently large and, in general, will grow without bound. In order for a limit on the ultimate amplitude to exist, there must be nonlinear damping in the system of which quadratic roll damping is an example.

From the foregoing simplified analysis we can expect that, if the ship encounters regular head or following seas at a frequency near one-half the natural frequency of roll, a small disturbance in roll will grow to appreciable amplitude depending on the amplitude of the stability variation and the roll damping. In real situations, the initial disturbance is almost always present and supplied by some external effect such as wind or oblique wave components.

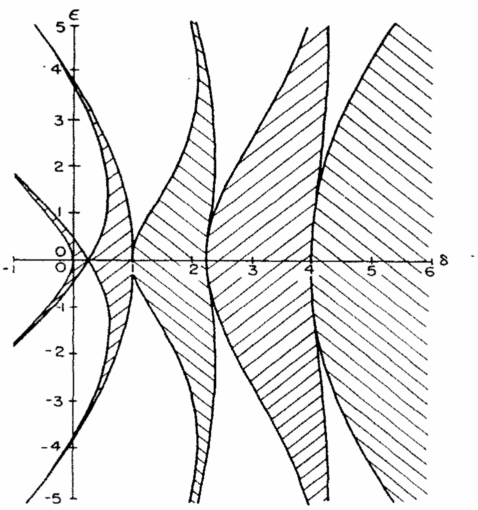

Figure 2. Ince-Strutt diagram illustrating stability of solutions of the Mathieu Equation

3. EARLY PARAMETRIC ROLL RESEARCH

Much of the earliest work on parametric roll was conducted in Germany starting in the late 1930s and focused on smaller working ships such as fishing boats, seagoing tugs and small coastal cargo craft which had experienced a number of casualties. Some of the research involved experiments with free running models which demonstrated the phenomenon quite convincingly.

The reasons for concentrating on smaller ships, aside from their casualty records, were two in number. First, smaller ships are more likely than large ships to encounter waves that are high and steep in relation to the ship dimensions. Second, they generally operate at Froude numbers that are more likely to result in a frequency of encounter within the critical range for the ship of marginal stability. Such ships are, of course, the ones in greatest danger of capsize.

Further, it was felt that the transient behavior of the roll motion was rather gradual, i.e., a large number of cycles would be required in order for the rolling motion to grow to appreciable amplitude. This was due in part to observations during the aforementioned model tests and in part to some early numerical simulations of the phenomenon which showed a slow growth of amplitude of the roll motion.

This slow transient growth was interpreted to mean that in order for the rolling motion to grow to appreciable amplitude, a rather large number of nearly regular waves would have to be encountered.

Waves in the real ocean, however, are random in character although in a system of random waves there will, from time to time, be found groups of several waves that appear to be almost regular. Furthermore, the waves as observed from a moving point appear to be somewhat different from the waves at a fixed point. To a moving ship, therefore, following waves appear to have distinctly different characteristics compared to head seas.

This is a consequence of the dispersion relation, i.e., the dependence of wave velocity on wave length. When observed from a point moving in the same direction as the waves as in following seas, the wave spectrum is compressed into a narrower band of frequencies than that for a fixed point, while head seas result in a spreading of the spectrum over a wider band of frequencies. The resulting appearance of the wave time history is shown in Figure 3. Here the upper graph contains the wave profile observed from a point moving in the same direction as the waves, the center graph is the profile observed from a fixed point and the lower graph corresponds to a point moving opposite to the wave motion.

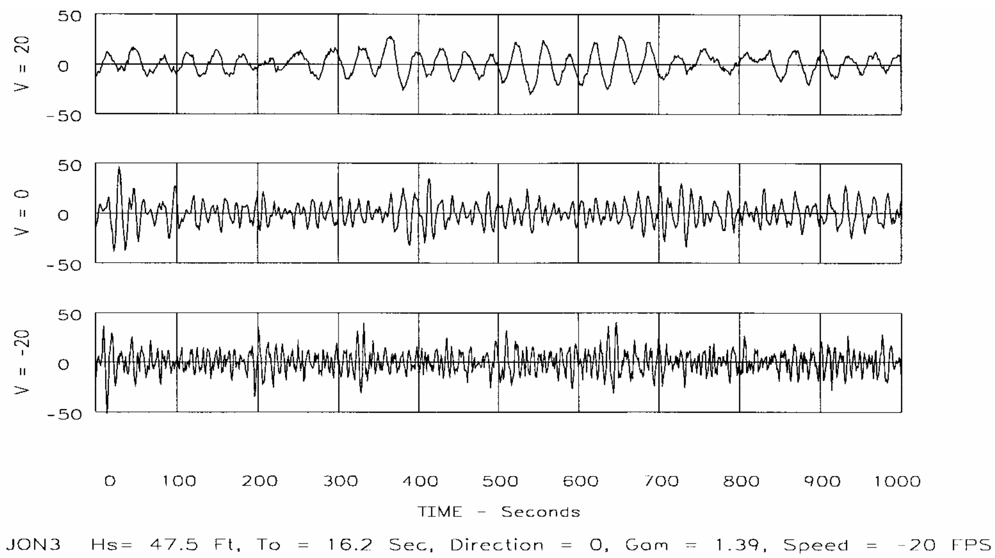

Figure 3. Unidirectional random waves observed from moving and stationary points

4. THE APL CHINA CASUALTY

In the early 1990s several containerships reported the occurrence of heavy rolling in head seas and the description of the incidents indicated the possibility of parametric rolling. Some of the reports indicated that the rolling motion built very quickly, during the encounter of three or four waves not several tens of waves as would have been expected.

These containerships had hull forms characterized by great flare forward and wide flaring stern sections. The reason for these form features was a desire to obtain more container space on deck while keeping a fine underwater form to minimize resistance at the high operating speeds of the ships.

These form features, however, lead to large stability changes as the ship moves through waves from the crest amidships to trough amidships positions. Furthermore, the typical metacentric heights at which these ships operate put their natural roll frequencies into the range of one-half the head sea encounter frequencies for wave lengths approximately equal to ship length. These are typically the waves causing the most pronounced stability variations..

The effect on stability of the flared form can be seen in Figure 1. At the smaller angles of heel, the righting arm curves, trough versus crest amidships, diverge much more strongly for the C11 containership than for the older, more wall-sided dry cargo ship of traditional form.

This implies a larger value for C, the coefficient of variation of GM in Equation (1) and would lead to increased susceptibility to occurrence of parametric roll, all other things being equal.

The APL CHINA casualty in October 1998 focused worldwide attention on head seas parametric roll. While eastbound in the North Pacific Ocean, the ship found itself in a rapidly moving weather system that was formed by the convergence of two low pressure systems and, despite weather routing advice, was unable to avoid the rapidly changing situation. The resulting combined system was described as an “explosively intensifying low” or meteorological ‘bomb” and referred to by some as a Pacific version of the “perfect storm”. At the height of the storm, the ship’s deck log recorded estimated wind of Beaufort force 11 and sea state 9. A weather and seastate hindcast performed after the accident gave peak sustained winds of 30.8 m/s or 60 knots and a significant wave height of 14.9 m

During the height of the storm, the master attempted to maintain a heading into the seas at a speed just sufficient to retain control. The ship reported rolls of over 40 degrees and violent pitching in the very heavy and confused seas. As a result of the violent motions combined with boarding seas, heavy damage to- and loss of a large number of containers was sustained and this is illustrated in Figure 4

Figure 4. APL CHINA container damage

In order to determine the causes of this casualty, extensive studies including weather analyses, model experiments, numerical simulations, and theoretical analyses were undertaken. At the conclusion, it was felt that head seas parametric roll was clearly established as the major cause. One significant outcome of this and related work is the development by classification societies as well as IMO of rules and guidelines for the avoidance of parametric roll situations both in the design and operation of ships.

Much more fundamental research on parametric roll has been conducted since the APL CHINA studies. Many model tests have been conducted, simulation codes have been developed and exercised, and theoretical work has been conducted on the probabilistic nature of nonlinear phenomena. A considerable body of literature on the subject exists, much of it the product of the STAB symposia. Rather than attempting to summarize it we will instead take a look at two specific side issues, one concerning the nature of the transient roll response in regular waves and wave groups and the second concerning the difference in behavior of the stability variations in head seas in contrast to following seas

5. A SOMEWHAT MORE SOPHISTICATED MODEL

Let us consider a slightly modified form of Equation (2) as follows,

$$I_x \frac{d^2\phi}{dt^2} + B\left(\frac{d\phi}{dt}\right) + \Delta GZ(\phi,t) = 0 \qquad (4)$$

Here, $B(\frac{d\phi}{dt})$ is a general roll damping which may include linear and nonlinear (quadratic) terms.

$\Delta GZ(\phi,t)$ is the time varying restoring moment function.

The most suitable method of solving equation (4) is stepwise numerical integration and a number of suitable methods are available.

The forth order Runge-Kutta method was used in the present examples. The function $GZ(\phi,t)$ may be visualized as a tabulated family of curves similar to those in Figure 1 computed for successive closely spaced positions of the wave along the ship length.

The damping term may be estimated from data found in the literature and many ship hydrostatics codes are available for constructing the family of GZ curves.

A simple interpolation scheme is used for finding the instantaneous value of GZ for the angle of heel and wave position at each time step of the integration.

Having coded this numerical integrator, is easy to play "what if" games that may lead to understanding of some of the basic features of parametric roll response. Two examples are presented here, the first consisting of the response time history for the ship in regular head seas where the computation is started with different initial conditions in each case. The second looks at the response in head seas consisting of a series of repeated wave groups.

The groups are formed by modulating the fundamental sinusoidal wave train by a sinusoidal envelope function of lower frequency chosen to produce groups somewhat resembling those encountered in a random seaway.

The C11 at its designed draft having a GM of 0.05B is used as the example in both of these computations. The family of GZ curves was constructed for 20 equally spaced positions of the wave crest along the ship length and the nonlinear roll damping was estimated using the method embedded in the US Navy's SMP code.

The natural undamped roll period of the ship was estimated to be about 23 seconds assuming a mass radius of gyration in roll of 0.4B. In head seas of length equal to the ship length, the critical frequency ratio of ½ occurs at a speed of about 5.5 knots.

In constructing the wave groups for the second set of computations, we first examine Figure 3 for some impression of the behavior of random seas. The middle graph of Figure 3 corresponds to observation from a fixed point in space and shows wave groups at about 20, 400 and 700 seconds. Each group consists of four to six waves.

At the higher speed of the lower graph, there are groups at about 10, 320, 500 and 650 seconds. The latter groups appear to have fewer component waves and persist for shorter time periods. For the present example, we have constructed a wave system in which each group contains five principal waves formed by multiplying the fundamental ship length sine wave whose period is about 12.9 seconds by a modulating sine wave with period of 129.5 seconds.

This is found to produce individual wave groups having an appearance similar to that which might be found in a real wave system.

6. INITIAL CONDITION SENSITIVITY

Figure 5 presents results in regular waves for an initial roll of 0.1, 1.0 and 5.0 degrees. As one would expect, this demonstrates that the transient build up of the motion to steady state is strongly dependent on the initial disturbance. Reported cases of unexpected head seas parametric roll have indicated that the motion built up rapidly over the occurrence of five or six extreme rolls and then subsided. In heavy head seas some component waves from directions other than dead ahead will always exist and these could very likely set up roll motions of five degrees or so that would then trigger the rapid build up of parametric response.

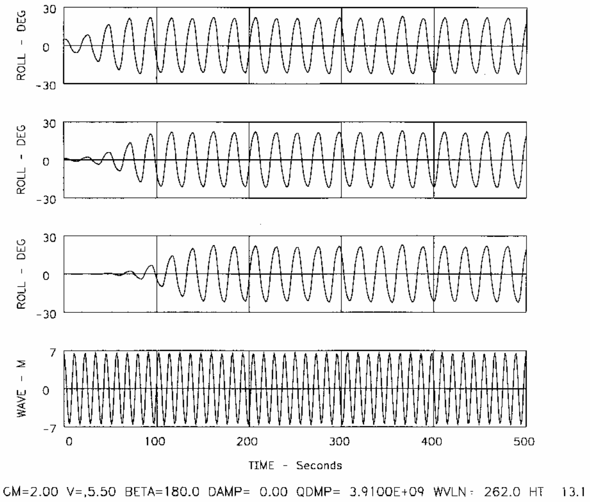

Figure 5 Sensitivity of transient response to initial disturbance for initial roll angle = 0.1, 1.0, 5.0 deg.

7. WAVE GROUP RESPONSE

The transient response in group waves appears somewhat more sluggish than in regular waves with initial conditions of both one and five degrees as shown in Figure 6. There is some sensitivity to the time of application of the initial roll disturbance with respect to the peaks and nodes of the envelope, but computations in which the time of application was varied showed it to be of secondary importance to the magnitude of the initial roll angle.

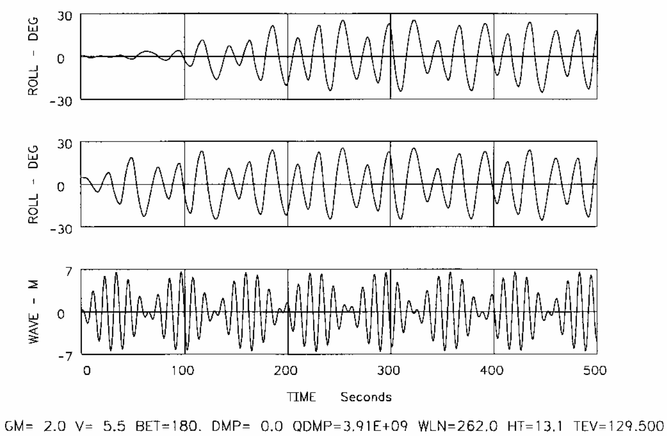

Figure 6. Response of C11 to wave groups, head seas, initial roll angle = 1.0 and 5.0 degrees.

8. SENSITIVITY OF GZ VALUES TO FORWARD SPEED

Several of the hydrodynamic stability computer codes in common use by naval architects today are capable of computing the GZ curves for the ship assumed poised in static equilibrium on the wave. It has been shown in some experiments by Paulling (1961) that this assumption of static equilibrium is usually satisfactory for moderate speeds in following seas. In these experiments, a model was towed at a fixed angle of heel using an apparatus that allowed the model freedom in pitch and heave. As the model was towed on a course at right angles to the wave crests the righting moment was continuously measured and recorded.

Results from experiments conducted at various model speeds in following seas revealed very little variation with model speeds in the range of the normal speeds for seagoing ships. These experiments were carried out on a family of models representing the DTMB Series 60 constructed with variations in beam and freeboard. The experiments were conducted in following seas only and did not explore the possible effects that might be associated with the greater pitch-heave motions to be expected when the ship operates in head seas. Examples of results obtained in following sea experiments compared with hand computations are shown in Figure 7.

The computations assumed static equilibrium of the ship on wave and the agreement is seen to be good for the following seas case.

In head seas, the assumption of static equilibrium of pitch and heave is probably not as valid an approximation as it is in following seas. This is especially true at higher speeds for which the encounter frequencies may approach resonance in the pitch-heave modes.

In order to test the sensitivity of the GZ curve to forward speed, an approximate computation procedure was adopted which would simulate the model experiments. It was assumed that the pitch-heave motion in head seas of the heeled ship would differ only slightly from the corresponding motion of the upright ship.

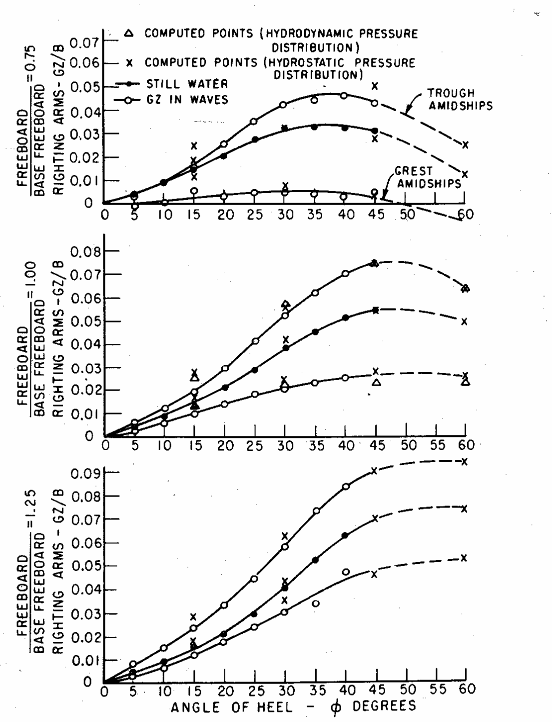

Figure 7 Model measurements of GZ in following seas. Series 60 C_b=0.60, Varied freeboard.

With this assumption, these motions could be computed using a standard linear ship motions code. The pitch-heave motion time history so obtained was then input to the wave stability computations to give a closer approximation to the exact dynamic attitude of the ship in waves than given by the usual static equilibrium assumption. The results could then be regarded as an approximation to the GZ for the dynamically pitching and heaving ship.

The dynamic pressure distribution on the ship due to the motions would be neglected as would the effect of heel on the pitch-heave motion. Since the Froude-Krylov pressure terms are usually dominant in determining the forces on the ship in waves, this procedure can be thought of as providing a first order correction for the exact dynamic attitude of the ship in the waves. An example of the ship's changing attitude with passage of the waves is shown in Figure 8.

Figure 9 contains results of these computations in the form of maximum and minimum GZ curves. These correspond to a wave trough and a crest near amidships, for the ship moving at 18 knots in L/20 head seas. Also shown are GZ curves for an assumed static pitch-heave attitude of the ship which simulates following seas. The curves are labeled "F" for following and "H" for head seas. From these results we conclude that the effect of pitch-heave dynamics is to reduce somewhat the variation of the GZ curves about the still water curve.

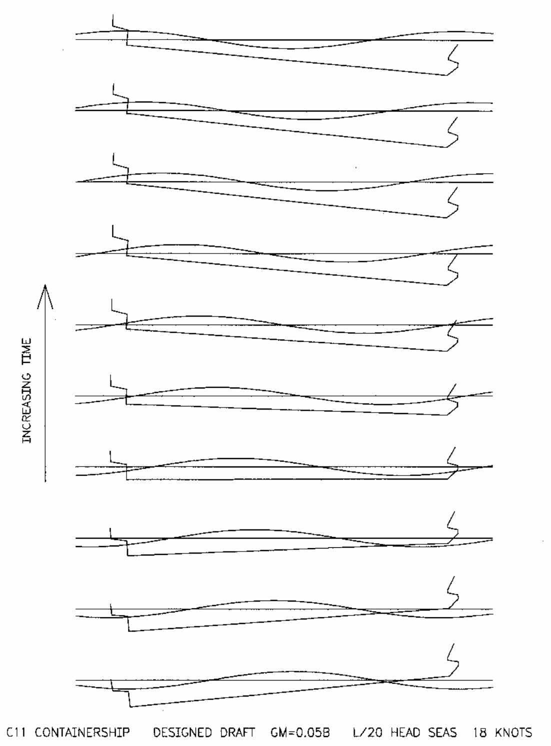

Figure 8. C11 Containership in L/20 head seas at 18 knots.

9. CONCLUSIONS

A very brief background is given of some of the practical implications of parametric roll for ships in following and head seas. Early work on the phenomenon had focused almost entirely on rolling and capsize in following seas but the APL CHINA casualty brought out quite dramatically the possibility and consequences of head seas parametric roll. Much research has come about as a result and we now find that regulatory bodies and shipowners are beginning to take steps to avoid a repetition of the incident both in the ship design process and in the operation of ships.

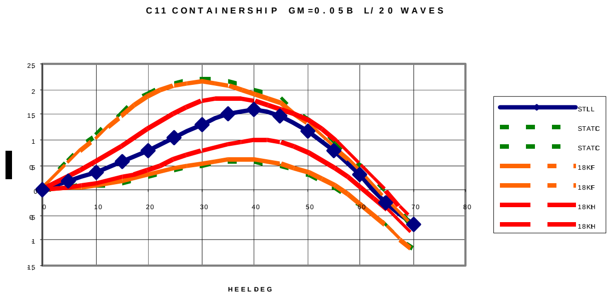

Figure 9. Max – min GZ curves for C11 containership – dynamic pitch-heave attitude at 18 knots

10. ACKNOWLEDGEMENT

The work reported upon here has been supported over a period of many years principally by the American Bureau of Shipping and the Society of Naval Architects and Marine Engineers

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