Influence of Current on the Probability Distribution of Wave Asymmetry and Steepness

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ABSTRACT

The empirical distributions of wave asymmetry and steepness were analyzed in order to determine how the presence of a current influences the distributions. This investigation is based on the analysis of data generated on a wave basin. The data set was a large number of sea states that were available without current and with following or opposing current. Influence of current on wave asymmetry was also described as well as ratios of increase and decrease of wave steepness and nonlinearity of sea states analyzed. The empirical steepness and asymmetry distributions were fitted by theoretical distributions.

Keywords: *steepness, influence of current, deep water waves, wave statistics*

1. INTRODUCTION

Wave asymmetry and steepness are not as much studied as other wave parameters such as height, period and crest heights, which have been the ones that have attracted the attention of most researchers in the field.

However a recent study has shown that in general the larger density of bad weather accidents coincide with the ocean areas where the steepest sea states can be found (Guedes Soares et al, 2001). Steep waves can cause damages in ships and offshore structures (Kjeldsen, 1997, Guedes Soares et al, 2004) and they can also induce the capsizing of smaller vessels (Dahle et. al., 1988). In fact, Dahle et. al., (1988) also brought the attention to the fact that currents can increase the wave steepness and make them even more dangerous for small vessels.

The interaction of waves and current has been much studied related to numerical approaches that describe the velocity fields in the fluid but less work has been done on how the wave elevation is modified by the presence of current, as discussed in the recent paper of Guedes Soares and Pablo (2006).

Myrhaug and Kjeldsen (1984) developed parametric model of joint probability distribution of vertical asymmetry factor and wave height, as well as model of joint probability distribution of wave steepness and height. These models however are valid for extreme waves on the Norwegian continental shelf. Another model presented in Myrhaug and Kjeldsen (1987) describes joint distribution of crest front steepness and wave height. The probability of occurrence of waves with different steepness is estimated with each parametric model for a family of JONSWAP spectra. These models do not describe influence of current on steepness and asymmetry of waves.

The main objective of this work is to study the probability distribution of the asymmetry and steepness of individual waves, based on the analysis of wave data generated in an offshore wave basin without current and with following and opposing current.

The changes of the distributions of sea state parameters without current were analyzed for the sea states with following and opposing currents. In general a following current decreases wave steepness and an opposing one increases it.

In this work a short description of the changes of asymmetry of individual waves due to following and opposing current is also presented.

2. DATA DESCRIPTION

The data used in this study were recorded in January 2000 in a wave tank belonging to The Danish Hydraulic Institute in Hørsholm (DHI). There are all together 110 records of deep water waves sampled in intervals of $\Delta t = 0.217$ seconds. In the majority of files there are $N = 24001$ ordinates per record corresponding to a duration of 87 minutes. An example of one of the records is shown in figure 1.

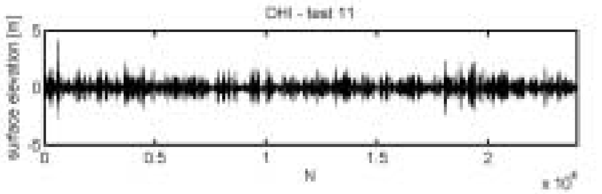


Figure 1. Example of time series recorded in the DHI basin.

The records contain sea states with 2D waves with and without current in different combinations. All sea states were generated from a JONSWAP spectrum with a peak enhancement factor of 3.0. A description of the characteristics of the 30 sea states studied here is given in table 1. In the table, in addition to significant wave heights and spectral peak period of the wave systems, there are also their directions and the direction and velocity of the current.

Table1. Description of sea states used in this study.

No Test	Hs [m]	Tp [s]	Wave Dir [deg]	Curr. Dir [deg]	Curr. Vel [m/s]
1	3,6	7	0	-	-
2	3,6	10	0	-	-
3	3,6	14	0	-	-
4	3,6	20	0	-	-
5	4,6	7	0	-	-
6	4,6	14	0	-	-
7	4,6	20	0	-	-
8	2,3	7	0	-	-
9	2,3	14	0	-	-
10	2,3	20	0	-	-
15	3,6	7	0	0	1,5
16	3,6	10	0	0	1,5
17	3,6	14	0	0	1,5
18	3,6	20	0	0	1,5
19	4,6	7	0	0	1,5
20	4,6	14	0	0	1,5
21	4,6	20	0	0	1,5
22	2,3	7	0	0	1,5
23	2,3	14	0	0	1,5
24	2,3	20	0	0	1,5
25	3,6	7	0	180	1,5
26	3,6	10	0	180	1,5
27	3,6	14	0	180	1,5
28	3,6	20	0	180	1,5
29	4,6	7	0	180	1,5
30	4,6	14	0	180	1,5
31	4,6	20	0	180	1,5
32	2,3	7	0	180	1,5
33	2,3	14	0	180	1,5
34	2,3	20	0	180	1,5

The first ten sea states have no current but they are different combinations of three significant heights with four spectral peak periods. The next group of 20 sea states are of repeated values of Hs and Tp however opposite or following current was added to them.

This variety of sea states with different characteristics allows a comprehensive investigation on the effect of current on the asymmetry and steepness distributions. Such distributions calculated and plotted for full field data are usually inconclusive because not all sea state parameters are measured, as usually there is no simultaneous data on direction and velocity of current.

3. INFLUENCE OF CURRENT ON ASYMMETRY OF INDIVIDUAL WAVES

There are different definitions of horizontal and vertical asymmetry of an ocean wave. The magnitudes used in this study were defined as follows (Guedes Soares et al., 2004):

$$a_v = \frac{H_D}{cr} \tag{1}$$

$$a_h = \frac{T_b}{T_f} \tag{2}$$

where H_D is wave height from down-crossing definition, cr is wave crest height, T_b is wave back period and T_f is period of wave front. Wave back period is the time between maximum of the wave crest and the minimum of the wave trough following the crest. Wave front period is the time between maximum of the wave crest and the minimum of the wave trough before that crest.

On the basis of table 2 changes of horizontal and vertical asymmetry were investigated. The table shows mean values of vertical and horizontal asymmetry calculated from respective asymmetries of individual waves.

The presence of a current, regardless of its direction, increases the vertical asymmetry of waves in case of high sea states and decreases it in case of low sea states. Medium sea states can go either way when current is added. Horizontal asymmetry, on the other hand, increases more with following current.

It is difficult to be sure about the results of the vertical asymmetry, because it seems already very high with absence of the current. It is possible that with opposite current waves started to break and in this situation it would be impossible to say precisely, how values of steepness changed.

Table 2. Mean asymmetry for sea states with and without current; Hs=3.6m: 1-4, Hs=4.6m: 5-7, Hs=2.3m: 8-10.

No Sea State	No Current		Following Current		Opposite Current	
	vertical asymmetry	horizontal asymmetry	vertical asymmetry	horizontal asymmetry	vertical asymmetry	horizontal asymmetry
1	2,15	1,10	2,39	1,10	2,48	1,07
2	2,65	1,09	2,29	1,10	2,30	1,11
3	3,16	1,18	4,47	1,10	2,70	1,11
4	3,54	1,14	3,37	1,26	3,12	1,14
5	2,16	1,06	3,28	1,08	2,37	1,06
6	2,87	1,16	2,88	1,14	3,05	1,14
7	2,65	1,14	3,93	1,33	3,81	1,20
8	2,32	1,07	2,29	1,07	2,22	1,08
9	3,11	1,16	2,63	1,16	4,55	1,16
10	5,31	1,22	2,95	1,29	4,83	1,21

4. INFLUENCE OF CURRENT ON INDIVIDUAL WAVE STEEPNESS

Several definitions of wave steepness have been proposed. Myrhaug and Kjeldsen (1986), Stansberg (1998), Stansell et al. (2003) and also Guedes Soares et al. (2004) presented different sets of definitions to investigate steepness of an irregular waves. In this study the classical steepness s given by equation (3) will be investigated, as it is most often used parameter.

The classical wave steepness s is calculated according to equation:

$$s = \frac{H}{L} \tag{3}$$

The wave length L for this formula can be calculated for each wave separately from the dispersion relation as:

$$L = \frac{gT^2}{2\pi} \tag{4}$$

where g is a gravity acceleration and T is the wave period [s]. After replacing L in equation of wave steepness (eq. 3) by equation 4 the wave steepness formula looks as follows:

$$s = \frac{2\pi H}{gT^2} \tag{5}$$

Wave steepness is directly proportional to the wave height and inversely proportional to the square of its period. Thus wave steepness is not statistically independent of period and height.

The subject of influence of uniform steady current on waves has been investigated for some time. The interaction theory of current on waves in a constant depth was first derived by Longuet-Higgins and Steward (1961). When waves propagate through a region with variable current, some of their characteristic parameters, such as their length, height, steepness, velocity and direction will suffer modifications. The presence of a current alters the velocity of the waves and affects the relation between the observed wave length and period. The current also produces changes in other properties of the waves, as happens with the velocity (and acceleration) of water particles.

Peregrine (1976) identified the "stopping current" velocity for finite amplitude waves, which is the velocity for which breaking of waves occurs.

Huang et al (1972) were probably the first who dealt with the effect of currents in an irregular sea state, showing how wave spectra were changed by the presence of currents, a theory that was the basis of several other developments which were compared with experimental data in Guedes Soares and Pablo (2006).

This study concerns the statistical aspects of individual wave steepness, as influenced by a simultaneous current.

Table 3. Individual wave mean steepnesses and their ratios.

			Mean Wave Steepness for Test with		
No Sea State	Hs	Tp	No Current	Following Current	Opposing Current
1	2,3	7	0,019	0,014	0,032
2	2,3	14	0,007	0,006	0,015
3	2,3	20	0,004	0,004	0,009
4	3,6	7	0,030	0,022	0,040
5	3,6	10	0,019	0,016	0,032
6	3,6	14	0,011	0,009	0,020
7	3,6	20	0,006	0,005	0,010
8	4,6	7	0,037	0,029	0,042
9	4,6	14	0,014	0,012	0,025
10	4,6	20	0,008	0,007	0,012

The presence of a current in a wave field always influences steepness of waves: an opposing current increases amplitudes of waves and decreases their wavelengths (increases steepness) and a following one decreases wave amplitudes and increases wavelengths (decreases steepness).

Table 4. Steepness changes due to current.

			Ratio of Mean Steepnesses for Sea States	
No Sea State	Hs	Tp	Foll. Current / No Current [%]	Opp. Current / No Current [%]
1	2,3	7	74,0	168,5
2	2,3	14	85,3	204,6
3	2,3	20	91,6	209,1
4	3,6	7	73,8	134,1
5	3,6	10	79,7	162,2
6	3,6	14	79,5	175,7
7	3,6	20	88,5	171,9
8	4,6	7	79,0	113,2
9	4,6	14	86,3	174,3
10	4,6	20	89,5	161,9

Tables 3 and 4 show the ratios of increased and decreased steepness caused by opposing and following currents for sea states 1-10, where opposite and following currents have the same velocity equal to 1.5 m/s.

Looking at tables 3 and 4 it is visible that all ratios of increase and decrease of steepness grow with the growth of the spectral peak period value for given significant wave height.

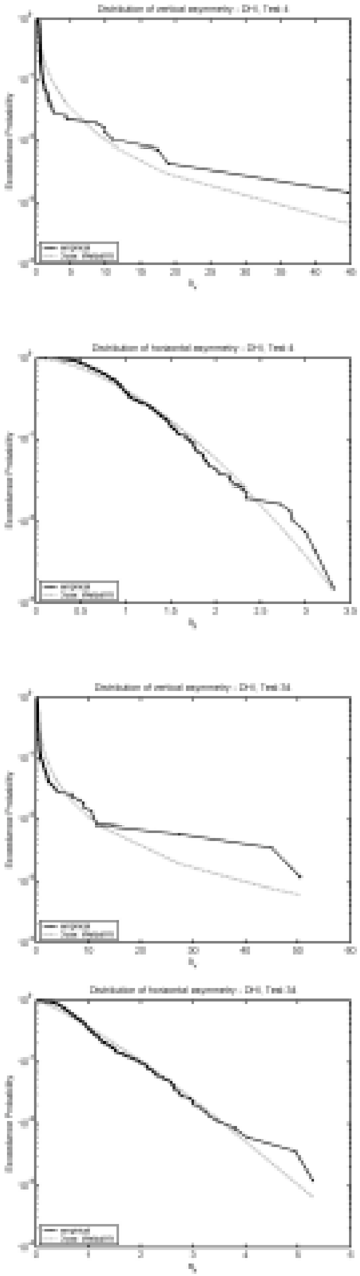


Figure 2. Distribution of vertical asymmetry (first column), distribution of horizontal asymmetry (second column). Asymmetry values were normalized.

The highest ratios of increase of wave steepness in the presence of an opposing current appeared for the lowest sea states. The ratios grow with the growth of spectral peak period values. The highest decrease of wave steepness in the presence of a following current indicate ratios for sea states with lowest values of the spectral peak value.

5. PROBABILITY DISTRIBUTION OF ASYMMETRY OF INDIVIDUAL WAVES

The vertical and horizontal asymmetry have a very different range of values and statistical behaviour, in particular in the tail of the distributions. Because of a very long tail that most of vertical asymmetry distributions have, it is quite difficult to model values close to the median of the distribution.

Nevertheless, the tail of the vertical asymmetry distributions was fitted quite successfully with 3-parameter Weibull distribution. However the probability of the highest values of the vertical asymmetry is underestimated by the Weibull distribution – see figure 2.

The horizontal asymmetry does not reach so high values, thus the tail is not so long, and it was possible to model successfully the horizontal asymmetry with a 3-parameter Weibull distribution. The best fits were obtained for series without current and with opposing current.

A comparison of asymmetry distributions with current and without it is shown in figure 3. It is visible that current does not seem to have much influence on the extreme values of vertical asymmetry, as it is difficult to notice any pattern on plots from the first column of figure 3. The highest values of horizontal asymmetry were most often provoked by a following current.

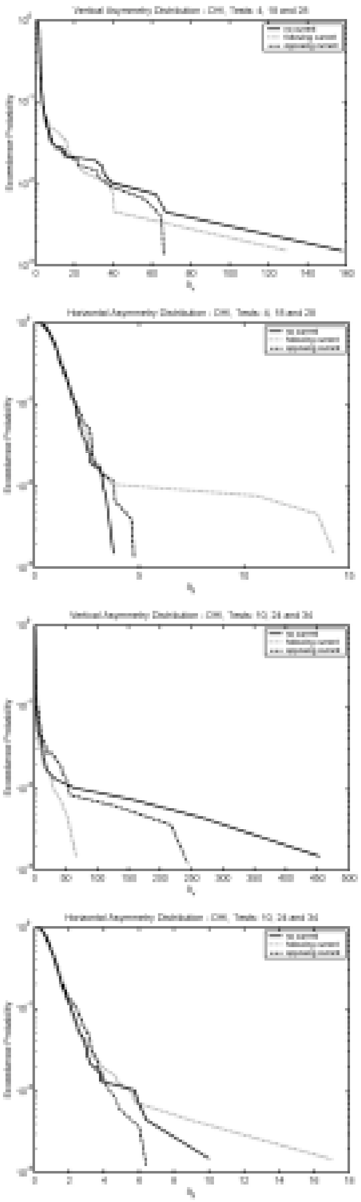


Figure 3. Comparison of distributions of asymmetry of sea states with following, opposing and no current. First column – vertical asymmetry, second column – horizontal asymmetry.

6. PROBABILITY DISTRIBUTION OF STEEPNESS OF INDIVIDUAL WAVES

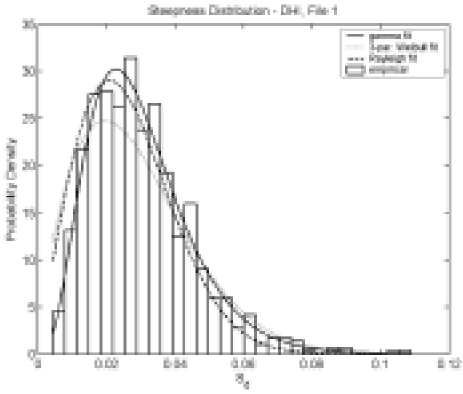
Marginal distributions of non-normalized wave steepness were also analyzed. Some histograms are presented in figure 4. They are relatively smooth due to the large duration of the time series, which were equal to 87 minutes. This regular structure allowed attempts of fitting theoretical distribution to the empirical one. Thus Rayleigh, Gamma and 3-parameter Weibull formulae were tried.

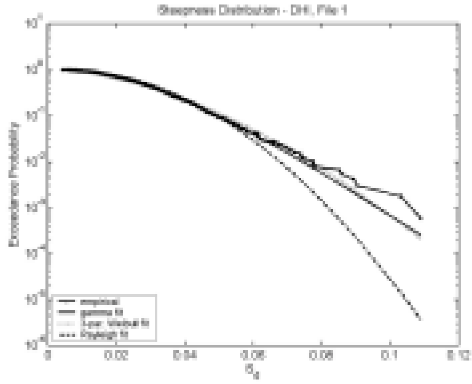
The best fits in the region close to the median of the distribution were obtained with the Gamma distribution as shown in figure 4. On the tail of the distribution the Gamma and Weibull distributions are very similar. They fit well in some cases, but usually they underpredict the probabilities of large wave steepness.

On the basis of all the plots of histograms of wave steepness it was also possible to state that the opposite current provokes a clear shift of the distribution peak towards higher values of steepness, while the shift of the distribution peak towards lower values of steepness with the presence of the following current is not very clear. Currents also influence the range of steepness values, which is increasing when an opposing current is added.

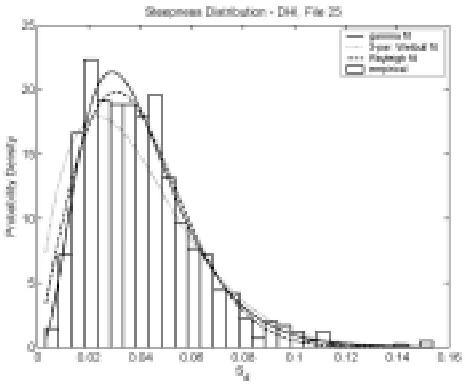
A different comparison of changes of steepness distribution under influence of a current is presented in figure 5. It is visible that values of wave steepness decrease with growth of spectral peak period value and sea states with opposing current have waves of highest values of steepness. More interesting though is that the difference between values of steepness of sea states with opposing current and values of steepness of sea states with following or no current grows with growth of spectral peak period of the sea state. The situation is the same for higher sea states (Hs = 5 m) and sea states with weaker current exhibit similar behaviour.

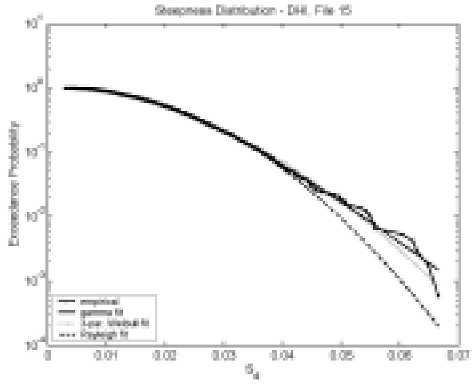
- No current



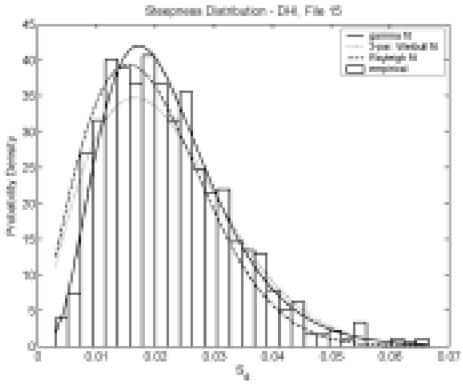


- Opposing Current





- Following Current



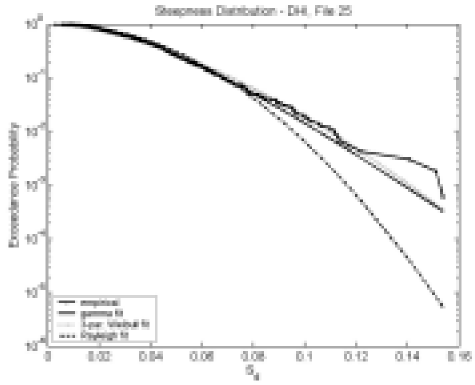
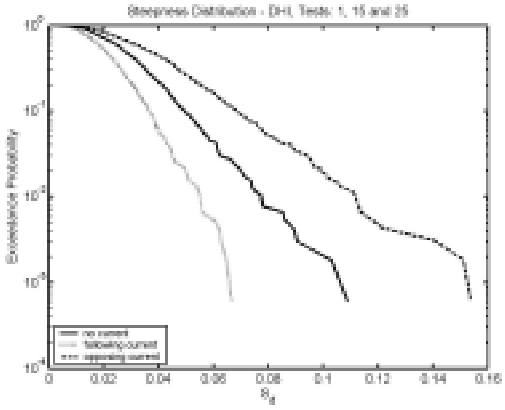
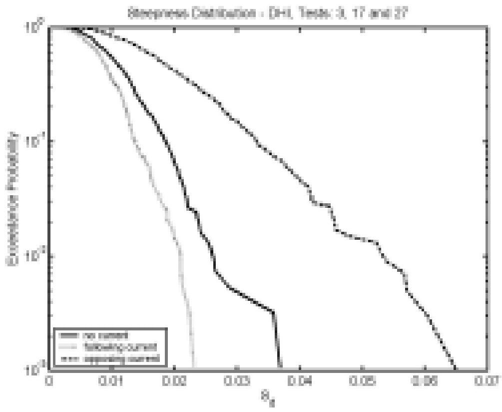


Figure 4. Marginal distribution of steepness for a sea state (Hs = 3.6 m, Tp = 7 s) – comparison of sea state with following, opposing and no current.

a)



b)



c)

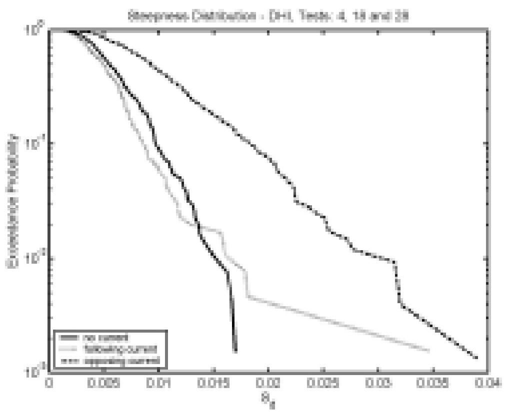


Figure 5. Comparison of steepness distributions without current and with following and opposing currents for sea states: a) Hs = 3.6 m and Tp = 7 s, b) Hs = 3.6 m and Tp = 14 s, c) Hs = 3.6 m and Tp = 20 s.

7. CONCLUSIONS

Vertical and horizontal asymmetry changes caused by following and opposing currents were described. Vertical asymmetry changes appeared to be quite difficult to interpret. Nevertheless the presence of a current, regardless its direction, seems to increase vertical asymmetry of waves in case of high sea states and decrease it in case of low sea states. Horizontal asymmetry appears to increase more with following current.

Modelling vertical asymmetry with a theoretical distribution is difficult because of the presence of a very long tail in majority of distribution. The tail itself could be modelled by 3-parameter Weibull distribution; however the distribution underestimates the probabilities of the highest values of the vertical asymmetry.

The horizontal asymmetry distributions do not have equally long tail and they could be modelled by a theoretical distribution. The tail of this empirical distribution was fitted quite well with 3-parameter Weibull distribution. The highest values of horizontal asymmetry most often provoked a following current.

An analysis of the marginal steepness distribution was performed and quite successful fits of empirical steepness distribution with Gamma theoretical distributions were presented for sea states with or without current. The Weibull distribution also gave satisfying fits.

Comparisons of changes of cumulative distribution of wave steepness for sea states with and without current with growth of significant wave period were presented. It appeared that the difference between values of steepness of sea states with opposing current and values of steepness of sea states with following or no current grows with growth of spectral peak period of the sea state.

The highest ratios of increase of wave steepness in the presence of an opposing current appeared for the lowest sea states. The ratios grow with the growth of spectral peak period values. The highest decrease of wave steepness in the presence of a following current indicate ratios for sea states with lowest values of the spectral peak value.

8. ACKNOWLEDGMENTS

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