Wave Force Modelling in Time-Domain Ship Motion

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ABSTRACT

The ship motion in waves was traditionally classified into sea-keeping problem, in which the ship motion is restricted in constant heading and speed, so that the encounter frequency and heading of wave are fixed and the analysis can be done in frequency domain. The need for time-domain ship motion has been growing in order to analyze the ship motion in maneuvering operation and in the phenomenon of stern quartering or following sea, and in larger waves. In this paper, authors introduced wave force modelling for time domain that was used in their institute, and a new formulation. It used cross flow concept and long wave approximation, and the modelling was for arbitrary heading and speed, and including maneuvering effect. And the comparisons between calculations and captive model test results were introduced with respect to wave force in time domain.

Keywords: *Wave Force, Wave Diffraction, Time Domain*

1. INTRODUCTION

Traditionally, there are two main streams in the equations of motion of a ship, one is for the maneuvering motion in which the ship's heading and velocity have no restriction and the equation is described in time domain, without waves. The other is for sea keeping motion in the waves in which the ship's velocity and heading are fixed and the equation is described in frequency domain. Efforts to describe the ship motion without any restriction has been made for a long time, and the need for unified equations of motion is growing more to analyze the ship motion in maneuvering operation and in the phenomenon of stern quartering or following sea, and in larger waves. There are two basic modelling of hydrodynamic forces in frequency domain. These are radiation and diffraction problems. The radiation force can be transformed to time domain force by using impulse-response function concept and the resulting modelling becomes in the form of time convolution integral. However the diffraction force is difficult to convert to time domain because of the restriction in the frequency domain problem, that is the velocity and heading should be fixed to waves.

In this paper, the hydrodynamic force modelling is treated, especially focused in the wave diffraction force modelling. The wave force modelling that is introduced in this paper was used for about ten years in our institute, as the one of the time domain wave force modelling. In the wave force model, the instantaneous pressure and velocity of the wave are used, so it is useful in time domain problems. And this model includes maneuvering effects, so it can be used in the unified equations of motion of a ship in waves. And the comparison between calculation and the captive model test was done.

2. HYDRODYNAMIC FORCES ACTING ON A SHIP HULL

The hydrodynamic forces acting on a ship arise mainly due to the relative velocity of a ship and surrounding fluid. These forces can be categorised into three parts. The first is due to the surrounding pressure field; the force can be calculated by integration the pressure over the hull surface. The second is due to the relative acceleration; the pressure field is changed because of the relative acceleration of the fluid so that the force proportional to the relative acceleration is come out. And the third is due to the relative velocity; this one is a complicated one, this includes friction drag, pressure drag, lift etc. The force is assumed to be represented as following.

$$F = F_p + F_a + F_v + F_{vv} \qquad (1)$$

F_p is the force due to the surrounding pressure field, the force due to the static pressure is the buoyancy, and the force due to wave pressure is so called Froude-Krylov force, in non-linear equations of motion it is convenient to calculate these two forces together. F_a is the force due to relative acceleration, this one has a linear property, so that one due to ship's acceleration and the one due to the acceleration of surround fluid can be calculated separately. This force usually is represented by the added mass. F_v is the force proportional to the relative velocity, this includes a part of radiation force and wave force. F_{vv} is the force proportional to the square of relative velocity, this include mainly drag force and forward velocity effects.

Among the force components, the forces except F_{vv} are linear, so that we can separate the force into one from ship's motion and the other from surrounding wave field. However F_{vv} is non-linear so that it cannot be separated, and must be calculate the forces in the relative velocity concept.

The radiation problem is well defined in the frequency domain, and the transformation it to the time domain also is well defined in linear range. The method to calculate the buoyancy and Froude-Krylov force is simple in general. We omit the explanations about these forces, and focus in the wave diffraction and the forward velocity effect i.e. maneuvering forces.

3. DIFFRACTION FORCE MODELLING

Wave diffraction forces are due to the effect that a ship blocks the flow of waves. In frequency domain, there are two calculation methods, one is from solving diffraction potential problem, and the other is a long wave approximation. But these methods are difficult to apply for the case of arbitrary speed and heading. In ocean engineering field, well known formula has been used for calculating the force acting on a body in the region without free surface effect, this is Morrison's formula. Long wave approximation assumes the wavelength is long relative to ship's dimension, so the flow of wave can be treated to be uniform on the ship surface. Thus the resulting force can be calculated from added mass and wave damping by assuming that the force is identical for the case that the ship moves in apposite direction. Morrison's formula treated the flow consists of acceleration and velocity, and the force is sum of the forces due to added mass and drag force. The drag force is proportional to the square of the velocity. This formula is independent of frequency.

Summing up the above, diffraction force has added mass term and drag force term if one neglects the frequency effects. For the added mass, that of infinity frequency will be used for the vertical mode because it does not differ quite from that of finite frequency, and the added mass by using strip method has singular behavior in the zero frequency. In lateral mode, the added mass of zero frequency will be used, it has finite value at zero frequency. The force dependent on the frequency is wave-making damping, and is linear. Basically it must be dependent on frequency because the radiated wave energy is related to the frequency. However it is difficult to use the frequency independent term because the wave making damping vanishes at zero frequency and infinite frequency.

3.1 Vertical Force Modelling

Consider the case that the ship is in the vertical flow. If the flow has acceleration, the force exist proportional to the acceleration of flow. And the drag force exists, and the wave making forces if there is the free surface. The force can be represented as follows.

$$dF_2^D = dF_{2a}^D + dF_{2vv}^D + dF_{2vw}^D \tag{2}$$

The suffix 2 means the vertical direction, 'a' means the force due to acceleration, 'vv' means that proportional to the square of velocity, 'vw' means that proportional to velocity and the force is mainly from wave making effect. The force due to acceleration can be represented by

$$dF_{2a}^D = a_{22}[\dot{w}]\frac{d(t)}{d_0} \tag{3}$$

$$\text{where} \quad [\dot{w}] = \frac{\int \dot{w} dS}{\int dS}$$

where, a_{22} is 2-D heave added mass at infinite frequency. And $d(t)/d_0$ is the ratio of current draft and original draft, this term is included for the consideration of the non-linear effect. If the section is located totally out of fluid, the above representation gives no force.

The drag force proportional to the square of velocity can be derived from momentum change. Consider the force acting on the surface as in figure 1.

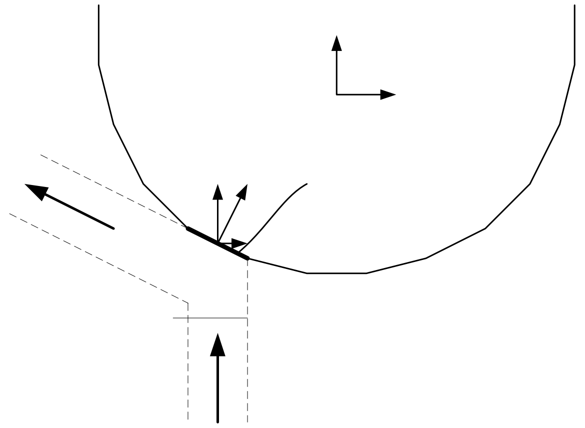

Figure 1. The flow direction near the surface.

The direction of vertical flow changes its direction near the surface, so its components of out flow can be written as follows.

$$V_{oz} = V\left|n_y\right| \quad , \quad V_{oy} = -Vn_z \tag{4}$$

The vertical force acting on the surface is derived from the momentum change in the vertical direction, it is proportional to the square of the velocity.

$$\begin{aligned} dF_{2vv}^D &= \frac{1}{2}\rho\int w^2(1-n_y)dA \\ &= \frac{1}{2}\rho\int w^2(1-n_y)\left|n_z\right|dS \end{aligned} \tag{5}$$

The ship surface blocks the vertical flow, and the flow is altered, so the wave generated by the ship hull. This wave making diffraction force can be calculated in frequency domain. With respect to fluid, the flux is arises due to the ship hull, this flux can be calculated as follows.

$$q = \int wn_z dS \tag{6}$$

The mean depth and velocity of this flux is as follows.

$$z_i = \frac{1}{q}\int wn_z z dS \quad , \quad \overline{w} = q / \int n_z dS \tag{7}$$

In the far field, the flux can be represented as the source located at mean depth z_i, and its strength is q. In the far field, the wave amplitude generated by the above flux is as follows.

$$A = q\frac{\omega}{g}e^{kz_i} \tag{8}$$

The above wave radiated outward in both directions. The force due to wave radiation can be written as follows.

$$\begin{aligned} F &= 4\frac{d\overline{E}}{dt} / \overline{w} \\ &= \rho\overline{w}\,\omega e^{2kz_i}\left(\int n_z dS\right)^2 = \rho\overline{w}\,\omega e^{2kz_i}B^2 \end{aligned} \tag{9}$$

where, B is the instantaneous breadth of the section. In the above derivation of wave making diffraction force, the concept of Haskind relation was used.(Newman, 1977) For the practical application in the time domain formulation, the condition that the maximum wave energy radiated outward can be drawn as follows.

$$\begin{aligned} & f = \omega e^{2kz_i} = \sqrt{gk}\,e^{2kz_i} \\ & \frac{df}{dk} = \frac{1}{2}\sqrt{\frac{g}{k}}e^{2kz_i} + 2z_i\sqrt{gk}\,e^{2kz_i} \\ & \therefore k = -\frac{1}{4z_i} \quad , \quad \therefore \omega = \sqrt{gk} = \sqrt{\frac{g}{-4z_i}} \end{aligned}$$

Assume the wave energy radiated as half of maximum energy flux for the practical application in the time domain formulation.

$$\begin{aligned} dF_{2vw}^{D} &= \frac{1}{2}\rho\overline{w}\,\omega e^{2kz_i}B^2 \\ &= \frac{1}{2}\rho\overline{w}\sqrt{\frac{g}{4|z_i|}}e^{-1/2}B^2 \end{aligned} \tag{10}$$

The above expression has no frequency dependent term explicitly, and can be used in time domain formulation.

3.2 Lateral Force Modelling

The lateral force can be derived in the same concept as in the vertical force formulation. In the vertical mode the restoring force blocks the drift motion, however, in the lateral mode the drift motion can be arises, and this makes the hydrodynamic phenomena more difficult due to eddy separation and boundary layer effects, specially in the case of forward speed. For the forward speed effect, we have maneuvering coefficients, and these coefficients give information on the hydrodynamic forces of the forward speed effect. This will be explained in the next section. In the same concept of previous section, the lateral forces can be represented as follows.

$$dF_1^{D} = dF_{1a}^{D} + dF_{1vv}^{D} + dF_{1vw}^{D} \tag{11}$$

The force due to flow acceleration is as follows.

$$dF_{1a}^{D} = a_{11}[\dot{v}]\frac{d(t)}{d_0} \tag{12}$$

$$\text{where} \quad [\dot{v}] = \frac{\int \dot{v}dS}{\int dS}$$

where, a_{11} is sway added mass at zero frequency.

The drag force proportional to the square of velocity is as follows.

$$\begin{aligned} dF_{1vv}^{D} &= \frac{1}{2}\rho\int v^2\left(1-|n_z|\right)dA \\ &= \frac{1}{2}\rho\int v^2\left(1-|n_z|\right)|n_y|dS \end{aligned} \tag{13}$$

The wave making phenomena is somewhat different from that of vertical mode. In the case of vertical mode, the flux can be treated as source in the far field, but in this case the flux is to be treated as dipole. The dipole strength, mean velocity, and mean depth are as follows.

$$m = \int vyn_y dS \quad (14)$$

$$z_i = \frac{1}{m}\int vyn_y zdS \quad , \bar{v} = \frac{\int v|n_y|dS}{\int |n_y|dS}$$

The wave amplitude radiated outward at far field is as follows.

$$A = m\frac{\omega}{g}ke^{kz_i} \quad (15)$$

The force due to wave energy radiation is as follows.

$$\begin{aligned} F &= 4\frac{d\bar{E}}{dt}/\bar{v} = \rho g A^2 \frac{\omega}{k}/\bar{v} \\ &= \rho\bar{v}\omega k e^{2kz_i}\left(\int yn_y dS\right)^2 \end{aligned} \quad (16)$$

As the same manner in the previous section, the force can be represented as the half of the force in the condition of maximum energy.

$$dF_{1vw}^{D} = \frac{1}{2}\rho\bar{v}\sqrt{g\left(\frac{3}{4|z_i|}\right)^3} e^{-3/2}\left(\int yn_y dS\right)^2 \quad (17)$$

3.3 Forward Speed Effect

In the previous section, the velocity used is assumed to be wave particle velocity, but there is no limitation. Therefore the equations in the previous sections can be applied to relative velocity as explained in chapter 2. If the ship has forward speed, there remain some hydrodynamic effects to consider. A certain section altered the surrounding fluid, and this flow property flows to backward, so the aft sections are affected by the flow pattern produced in the forward sections. This phenomenon takes place whether the wave exists or not. The maneuvering coefficients include this phenomenon implicitly and abstractly. One can obtain some information on the forward speed effect by inspecting the linear maneuvering coefficients Y_v, Y_r, N_v, N_r. And if we formulate the forward speed effect in the form of sectional representation, one can formulate the wave force and maneuvering forces together in one type of equation.

METHOD 1. The concept of forces due to maneuvering motion introduced in PNA (Vol. III, pp.236-240, Lewis 1989). Y_v consists of lift and drag forces, N_v consists of the moment due to Y_v and the Munk moment. Y_r consists of the center of pressure times Y_v and longitudinal added mass. In their notation, the center of pressure was defined as the center point of real fluid effect. N_r consists of moments due to rotational added mass and drag. All the formula, Y_v term that is lift and drag force included. So the sectional force can be assumed as follows.

$$dF = \frac{1}{2}\rho d V^2 C_C(x)\frac{v}{V} \quad (18)$$

where, V is the total speed of the ship, and v is the lateral component of the velocity. C_C is the cross flow coefficient, this force comes from mainly the lifting phenomenon. The force and moment can be represented using the above expression.

$$\begin{aligned} Y &= \frac{1}{2}\rho VD\int C_C(x)\frac{d(x)}{d_0(x)}v_r(x)dS \\ N &= \frac{1}{2}\rho VD\int\left(\frac{2}{\rho D}k_N a_{11} + C_C x\right)\frac{d}{d_0}v_r(x)dS \end{aligned} \quad (19)$$

In the above expression, the cross flow coefficient C_C and factor k_n are unknown. Assume C_C and k_n are constant in forward half and aft half region, and their values are different at forward and aft region. If the relative velocity is interchanged by v and r, the expressions for Y_v, Y_r, N_v, N_r are obtained. We can get the value of C_C from Y_v, Y_r, and the value of k_n from N_v, N_r. The values of $d(x), v_r$ are the mean values at the section.

METHOD 2. Various efforts were made on calculation of the maneuvering coefficients theoretically. Among these, there is one method in which the slender body theory and vortex shedding phenomenon are used to obtain the maneuvering coefficients.(Kim et al 2000, Fuwa 1973, Kijima et. al 1995, Kose et. al 1996, Xiong et. al 1996) This method solves the flow of 2-D section and vortex shedding and its effects, and calculates the maneuvering forces. In this method, the force due to lateral velocity can be expressed as the combination of the change of lateral flow and the effects of the forward speed, and the vortex shed at the position in which the flow changes abruptly. The lateral force was written as in the following expression.

$$\frac{dY}{dx} = -\rho UV \frac{dI_2(x)}{dx} - \rho U \frac{dI_3(x)}{dx}$$

$$I_2(x) = \int \phi_2 N_y dl$$

$$I_3(x) = \int_C \phi_3 N_y dl + \sum_{i=1}^{N} \gamma_i \delta_i$$

Figure 2 shows the results of sectional force distribution when a ship has drift motion, left figure shows the results of above method, and right figure shows the CFD results. The agreement is fairly good.

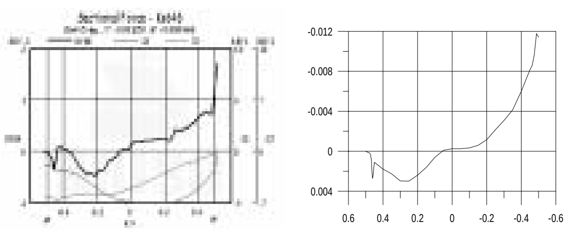

Figure 2 Sectional Force Distribution with Drift Motion(Kim et. al 2000)

Inspecting the above expression, it is known that I_2 is the sway added mass at zero frequency, and I_3 is the accumulation of the shed vortex, so it behaves like the integral values of the lateral velocity. If the lateral velocity is v' , the I_2, I_3 behave like the following expression.

$$I_2 \propto a_{11}(x) v'(x)$$

$$I_3 \propto \int_x^{L/2} v'(x) dx$$

Sectional force has the expression in which the differentiation of the above is used.

$$dF \propto \left(-\frac{da_{11}}{dx} v' - a_{11} \frac{dv'}{dx} + v' \right)$$

The above expression can be rewritten as follows.

$$dF = \frac{1}{2} \rho D V^2 C_c \frac{v}{V} \tag{20}$$

$$\text{where,} C_c v = -k_2 \left(\frac{da'_{11}}{dx} v' + a'_{11} \frac{dv'}{dx} \right) + k_3 v'$$

$$v' = \begin{cases} v(x) & \text{for } x > 0 \\ v(x)\left(1 + k_1 \dfrac{x}{L/2}\right) & \text{for } x < 0 \end{cases}$$

$$a'_{11} = 2 \frac{a_{11}(x)}{\rho D}$$

According to Kim et. al(2000), the appropriate value of k_1 is 0.4. The factors that will be seek are k_2, k_3 , and these values are different in the forward and aft region. The number of unknown factors is 4, and the 4 values Y_v, Y_r, N_v, N_r are given, so the 4 factors can be determined.

The above expression changes abruptly in the region of bow and stern. The smoothing is made only for the differentiation values. In forward region, the assumed values of a'_{11} is the quadratic lines in which the value at midship is the same of a'_{11}, and it vanishes at the foremost section. The differentiation is made for the value of mean values of a'_{11} and assumed value. In the region $-0.5L < x < -0.4L$, the section' shape changes abruptly, and the flow separates in large amount. The most part of vortex shedding is related to the resistance, and about 20% of vortex shedding is known to be related to lateral force. So if the value of differentiation is larger than mean differentiation of aft part by 50%, the value of differentiation is limited to 150% of mean differentiation of aft part of ship. And because the force must vanish at the aft most section, the linear factor is multiplied to the coefficient so that the force vanishes linearly aft –0.4L.

In this formulation, the sectional lateral force can be rewritten as follows.

$$dF_{1v}^{D} = \frac{1}{2}\rho DV\left(C_C + C_R \frac{d}{dx}\right)v \qquad (21)$$

where, C_C is the cross flow coefficient, and C_R is the rotational component of the cross flow force. Integration of the above formula over the ship length, one can obtain the lateral force and yaw moment.

$$\begin{aligned} Y_{1v}^{D} &= \frac{1}{2}\rho DV\int \frac{d(x)}{d_0(x)}\left(C_C + C_R \frac{d}{dx}\right)v dx \\ N_{1v}^{D} &= \frac{1}{2}\rho DV\int \frac{d(x)}{d_0(x)}x\left(C_C + C_R \frac{d}{dx}\right)v dx \end{aligned} \qquad (22)$$

Typical sectional force distribution from the above formulation is given in figure 3. It is known from this figure that the center of pressure is located forward of the midship, and this means that the force comes from mainly lift forces.

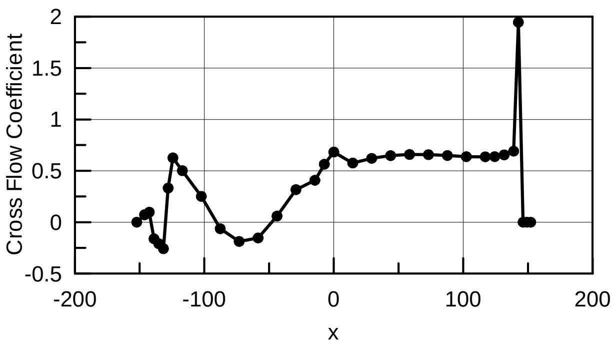

Figure 3 Cross flow coefficient of 9000TEU Container ship.

The roll moment can be obtained by multiplying the lateral force component(from section 3.2 and 3.3) and moment arm, in which the moment arm is defined as the distance from coordinate origin to the mid point of the current draft.

In section 3.1, 3.2, 3.3, the sectional force component formulations are derived. The sectional representation of the force has some advantages; the effect of sectional shape is included, and it can be applied for arbitrary relative velocities, so it can be applied for calculation of wave forces, for calculation of maneuvering forces, and for maneuvering forces on waves. Furthermore, in final formulation there is no frequency dependent term, so this formulation can be applied easily in time domain analysis.

4. COMPARISONS WITH CAPTIVE MODEL TEST

Recently in the authors' institute, the captive model test was performed to analyse the wave force. 9000TEU container ship was chosen, and the scale ratio was 1/72, so the length of the model is 4.0625m. Test conditions are in the Table 1.

Table 1. Test Conditions

Scale Ratio	72
Speed, Fn	0, 0.235
Wave Length, Lw/L	0.5, 1.0
Wave Height, H/Lw	1/30
Wave Direction, deg	0, 45, 90, 135, 180

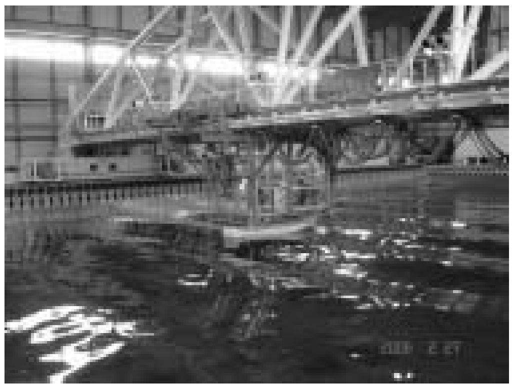

Figure 4 Captive Model Test

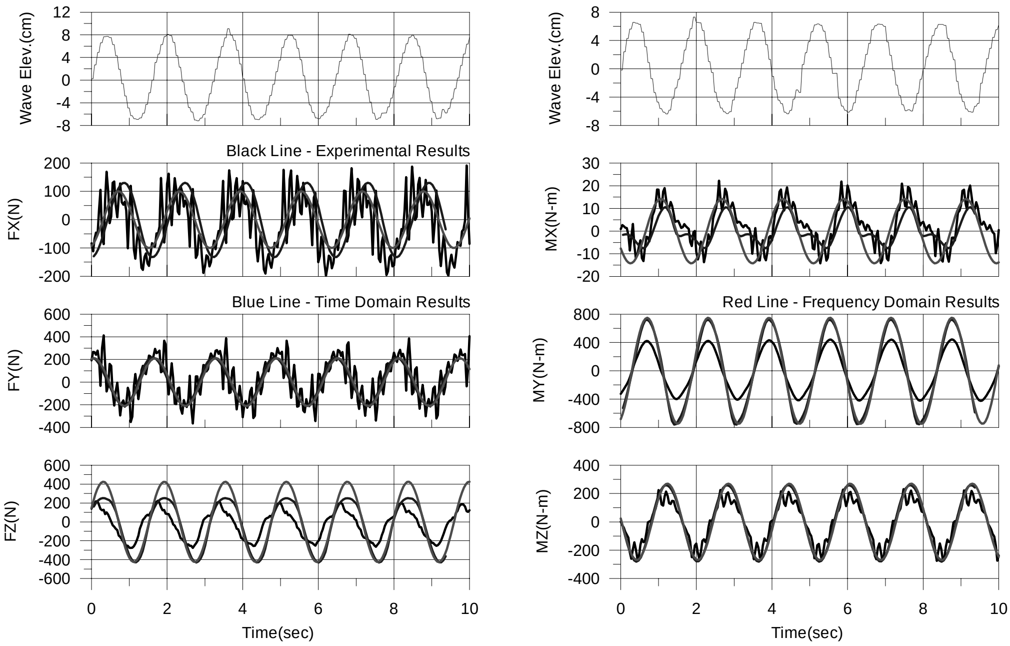

Figure 5 Comparisons of experiments, calculation in time domain, frequency domain. (Fn=0, Lw/L=1, H/Lw=1/30, heading=135º)

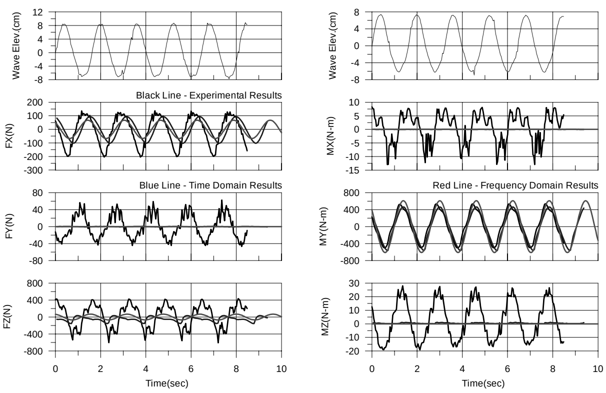

Figure 6 Comparisons of experiments, calculation in time domain, frequency domain. (Fn=0, Lw/L=1, H/Lw=1/30, heading=0º)

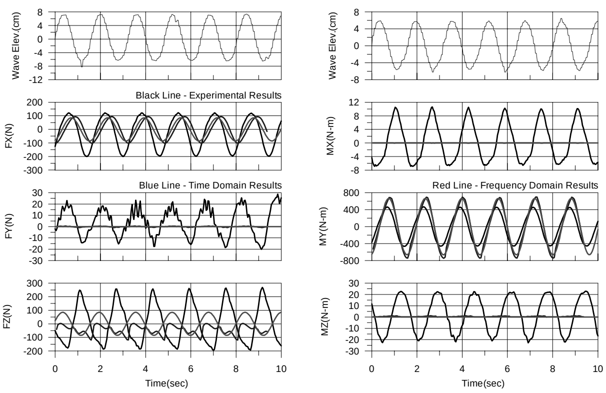

Figure 7 Comparisons of experiments, calculation in time domain, frequency domain. (Fn=0, Lw/L=1, H/Lw=1/30, heading=180º)

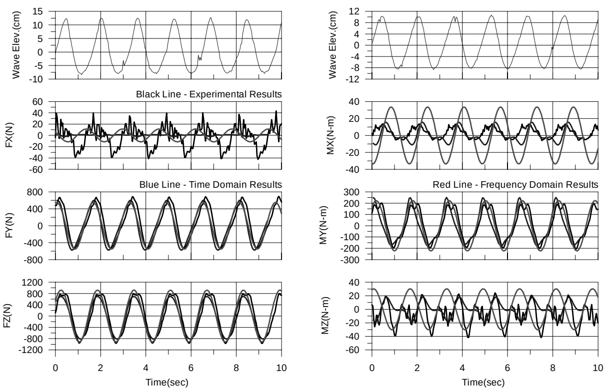

Figure 8 Comparisons of experiments, calculation in time domain, frequency domain. (Fn=0, Lw/L=1, H/Lw=1/30, heading=90º)

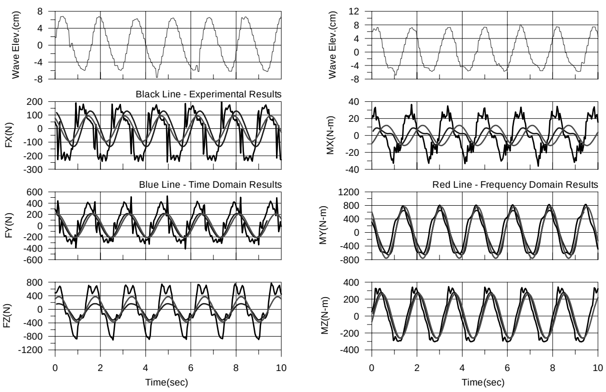

Figure 9 Comparisons of experiments, calculation in time domain, frequency domain. (Fn=0, Lw/L=1, H/Lw=1/30, heading=45º)

When the speed is zero, the comparisons of experiment, calculation in time domain, and in frequency domain are drawn in Figure 5 to Figure 9. Calculation in frequency domain is reproduced in time series from amplitude and phase. Calculation in time domain used Method2 in section 3.3. In figure 6 and 7, the heave forces disagree each other, but the pitching moments agree well. And the other figures show that the agreement is fairly good.

In the case of forward speed, the comparisons are shown in Figure 10, 11, 12. These figures show that the agreement is fairly good too.

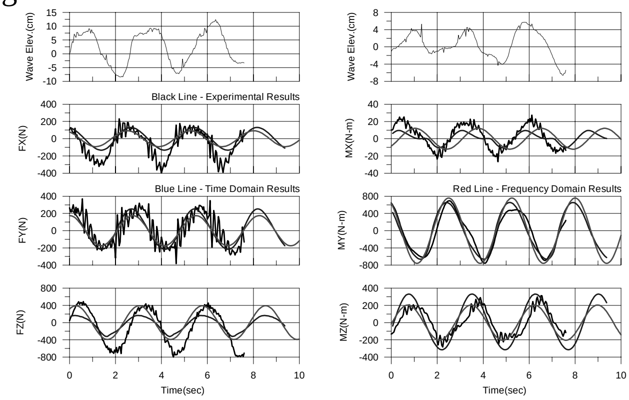

Figure 10 Comparisons of experiments, calculation in time domain, frequency domain. (Fn=0.235, Lw/L=1, H/Lw=1/30, heading=45º)

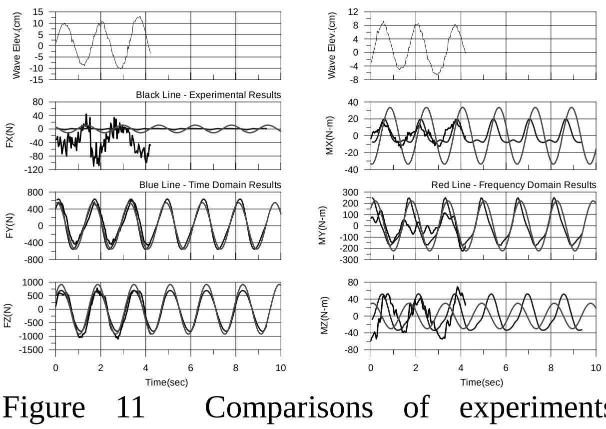

Figure 11 Comparisons of experiments, calculation in time domain, frequency domain. (Fn=0.235, Lw/L=1, H/Lw=1/30, heading=90º)

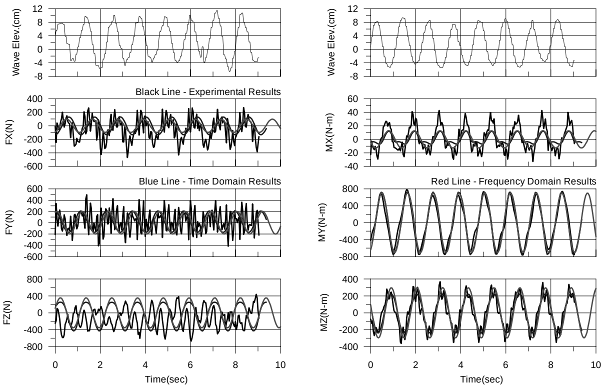

Figure 12 Comparisons of experiments, calculation in time domain, frequency domain. (Fn=0.235, Lw/L=1, H/Lw=1/30, head.=135º)

Because that the model test is captive, so the assumptions in frequency domain analysis are fulfilled, therefore it was expected that the results of frequency domain would agree more. The results of frequency domain agree with the experiments in large part of conditions but there are also a large part of conditions in which the agreement is poor. This seems because of linear assumption of frequency domain analysis, and there may be experimental errors. The result of method introduced in this paper was expected to have some errors, because there is no frequency dependent term. But results are not poorer than those of frequency domain results, and the agreements are rather good. The introduced formulation seems to be useful practically because the agreements with experiments are good although there is no limitation in speed and heading.

5. CONCLUSIONS

In frequency domain, the wave diffraction problem is well-defined, but its application is limited to the case of constant speed and heading. In real situation, a ship can change the speed and heading, so the incoming wave changes the encounter frequency constantly with respect to a ship. The wave diffraction force is difficult to be formulated in time domain for arbitrary speed and heading. Therefore, it is needed the calculation method for the wave diffraction forces in time domain whether theoretically consistent or practically applicable.

In this paper, the wave force modelling was introduced for a time domain application that has been used in authors institute, and new modelling was derived for a speed effect on the wave force. This modelling has no explicit frequency dependent term, so it can be applied in time domain easily. Even if there are some assumptions, the model derived in this paper is practically useful for the case of arbitrary speed and heading in waves. The model was verified by the comparisons with the experiments and the results in frequency domain.

6. ACKNOWLEDMENTS

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