An Investigation on Roll Parametric Resonance in Regular Waves

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ABSTRACT

Parametric resonance in regular waves is discussed. A set of non-linear equations is employed to describe the coupling between heave-roll-pitch modes. Limits of stability are the main area of interest of the paper. The present paper explores the influence of third order nonlinearities as well as the relevance of coupling between the vertical modes and the roll motion in the limits of stability.

The influence of initial conditions on the development of roll amplifications is investigated and the effect of coupled or uncoupled modelling of the roll motion is addressed.

Keywords: *parametric resonance, limits of stability, ship stability, nonlinear coupling*

1. INTRODUCTION

Lately, parametric resonance has become a source of great attention for the marine scientific community, classification societies and several other institutions involved in the development of ship's safety rules. Fishing, cruise and container vessels are known types of hulls that are often subjected to strong roll amplifications due to parametric instability (Neves et al., 2002, Luth & Dallinga, 1999, France et al., 2003). Experimental and numerical investigations have contributed to enlarge the knowledge of this phenomenon. However, there are not closed criteria that could let us measure or predict parametric roll motion with enough confidence for all kind of vessels, yet.

For many of these ships the simulation models available are capable of reproducing with confidence the amplification resulting from parametric resonance. But, unfortunately, there are some known cases of strong parametric excitation where the numerical models based on Mathieu (or Mathieu-Duffing) equation tend to overpredict the resonant rolling motions observed in experiments (Umeda et al., 2003). Recent works (Neves & Rodríguez, 2004, 2005) have shown that a third order nonlinear analytical model equivalent to a kind of Hill equation can reproduce well such extreme situations. This new mathematical model may display some interesting dynamic features that are still open to investigation.

Neves & Rodríguez (2004) showed a comparison of the limits of stability for the second and third order models based on Hsu's approach for the roll variational equation. The present paper explores in depth the influence of third order nonlinearities as well as the relevance of coupling between the vertical modes and the roll motion in the limits of stability. The above effects are investigated through the analysis of the time domain numerical responses obtained by systematic variation of encounter frequency and wave amplitude. This new way of obtaining the limits of stability is a more realistic procedure of assessing parametric resonance and also has an additional feature: a color map that identifies the magnitude of the steady roll parametric amplitude.

Other nonlinear characteristics of parametric roll behaviour, such as dependence on initial conditions are preliminarily investigated.

2. NONLINEAR MATHEMATICAL MODEL

As stated above, the mathematical model proposed by Neves & Rodríguez (2004, 2005) has demonstrated good capability in reproducing parametric resonance especially when strong roll amplifications take place. This mathematical model couples the equations of motions in heave, roll and pitch and contemplates nonlinearities up to the third order in the restoring actions as well as in the roll damping:

$$
\begin{aligned}
&(m+Z_{\ddot z})\ddot z + Z_{\dot z}\dot z + Z_{\ddot\theta}\ddot\theta + Z_{\dot\theta}\dot\theta + Z_z z + Z_\theta\theta + \\
&\frac{1}{2}Z_{zz}z^2 + \frac{1}{2}Z_{\phi\phi}\phi^2 + \frac{1}{2}Z_{\theta\theta}\theta^2 + Z_{z\theta}z\theta + \frac{1}{6}Z_{zzz}z^3 \\
&+\frac{1}{2}Z_{zz\theta}z^2\theta + \frac{1}{2}Z_{\phi\phi z}\phi^2 z + \frac{1}{2}Z_{\phi\phi\theta}\phi^2\theta + \frac{1}{2}Z_{\theta\theta z}\theta^2 z \\
&+\frac{1}{6}Z_{\theta\theta\theta}\theta^3 + Z_{\zeta z}(t)z + Z_{\zeta\theta}(t)\theta + Z_{\zeta\zeta z}(t)z + Z_{\zeta zz}(t)z^2 \\
&+Z_{\zeta\zeta\theta}(t)\theta + Z_{\zeta z\theta}(t)z\theta + Z_{\phi\phi\zeta}(t)\phi^2 + Z_{\theta\theta\zeta}\theta^2 = Z_w(t) \\
\\
&(J_{xx}+K_{\ddot\phi})\ddot\phi + K_{\dot\phi}\dot\phi + K_{\dot\phi|\dot\phi|}\dot\phi|\dot\phi| + K_\phi\phi + K_{z\phi}z\phi + K_{\phi\theta}\phi\theta \\
&+\frac{1}{2}K_{zz\phi}z^2\phi + \frac{1}{6}K_{\phi\phi\phi}\phi^3 + \frac{1}{2}K_{\theta\theta\phi}\theta^2\phi + K_{z\phi\theta}z\phi\theta \\
&+K_{\zeta\phi}(t)\phi + K_{\zeta\zeta\phi}(t)\phi + K_{\zeta z\phi}(t)z\phi + K_{\zeta\phi\theta}(t)\phi\theta = K_w(t) \\
\\
&(J_{yy}+M_{\ddot\theta})\ddot\theta + M_{\dot\theta}\dot\theta + M_{\ddot z}\ddot z + M_{\dot z}\dot z + M_z z + M_\theta\theta + \frac{1}{2}M_{zz}z^2 \\
&+\frac{1}{2}M_{\phi\phi}\phi^2 + \frac{1}{2}M_{\theta\theta}\theta^2 + M_{z\theta}z\theta + \frac{1}{6}M_{zzz}z^3 + \frac{1}{2}M_{zz\theta}z^2\theta \\
&+\frac{1}{2}M_{\phi\phi z}\phi^2 z + \frac{1}{2}M_{\phi\phi\theta}\phi^2\theta + \frac{1}{2}M_{\theta\theta z}\theta^2 z + \frac{1}{6}M_{\theta\theta\theta}\theta^3 \\
&+M_{\zeta z}(t)z + M_{\zeta\theta}(t)\theta + M_{\zeta\zeta z}(t)z + M_{\zeta zz}(t)z^2 + M_{\zeta\zeta\theta}(t)\theta \\
&+M_{\zeta z\theta}(t)z\theta + M_{\phi\phi\zeta}(t)\phi^2 + M_{\theta\theta\zeta}\theta^2 = M_w(t)
\end{aligned} \tag{1}
$$

On the right hand side of these equations, $Z_w(t)$, $K_w(t)$, $M_w(t)$ describe wave external excitations in the heave, roll and pitch modes, respectively. In the left hand side of the equations, nonlinear restoring terms include dependence on all body modes (z, ϕ, θ) and wave passage (ζ). Dots refer to velocities; double dots to accelerations. In all modes, coefficients with dotted and double dotted subscripts are damping and added masses coefficients, respectively.

In the numerical implementation of this mathematical model, added masses, vertical motion damping, and wave external excitations are assumed linear and are computed using the strip method theory. Roll damping is computed based on Ikeda's method as described by Himeno (1981); and restoring actions are taken into account considering couplings (up to the third order) among the heave, roll and pitch motions, and the wave profile passing along the ship. As demonstrated in previous works (Neves & Rodríguez, 2004, 2005) for certain hull forms and motions of moderate amplitude, nonlinear restoring actions of higher order play an important role in the dynamic behaviour.

Figure 1 illustrates the good agreement obtained in the comparisons between experimental results and numerical simulations for a typical transom stern fishing vessel (denominated TS), and also shows how a second order model fails to follow the trends.

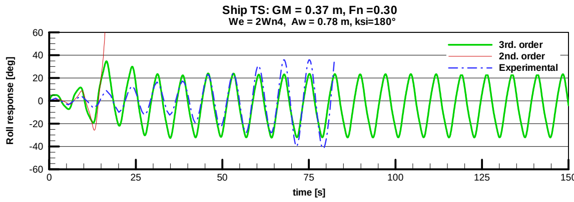

Figure 1 Roll response, Fn=0.30, A_w=0.78 m.

3. STABILITY ANALYSIS OF ROLL PARAMETRIC RESONANCE

3.1 Analytical Approach – Hsu's Limits

Stability of motion may be assessed by means of the variational system. In its linear form it may be derived by assuming that the nonlinear motions can be expressed as the sum of steady oscillatory solutions plus some small perturbations:

$$z(t)=\hat{z}(t)+\xi(t)=A_w\eta_3 Cos(\omega_e t+\alpha_z)+\xi(t)$$
$$\phi(t)=\hat{\phi}(t)+\varphi(t)=A_w\eta_4 Cos(\omega_e t+\alpha_\phi)+\varphi(t)$$
$$\theta(t)=\hat{\theta}(t)+\vartheta(t)=A_w\eta_5 Cos(\omega_e t+\alpha_\theta)+\vartheta(t)$$

where $\hat{z}(t)$, $\hat{\phi}(t)$, and $\hat{\theta}(t)$ correspond to the heave, roll and pitch well known linear solutions (steady), and η_3, η_4, η_5 are the corresponding transfer functions. Perturbations in the heave, roll and pitch modes are defined as $\xi(t)$, $\varphi(t)$, and $\vartheta(t)$, respectively.

The linear variational equation in roll is then derived as:

$$\begin{aligned}&(J_{xx}+K_{\ddot{\phi}})\ddot{\varphi}+B_e\dot{\varphi}+K_\phi\varphi+\left(K_{z\phi}\hat{\phi}+K_{zz\phi}\hat{z}\hat{\phi}\right.\\&\left.+K_{z\phi\theta}\hat{\phi}\hat{\theta}\right)\xi+\left(K_{z\phi}\hat{z}+K_{\phi\theta}\hat{\theta}+\frac{1}{2}K_{\phi\phi\phi}\hat{\phi}^2+\frac{1}{2}K_{zz\phi}\hat{z}^2\right.\\&\left.+\frac{1}{2}K_{\theta\theta\phi}\hat{\theta}^2+K_{z\phi\theta}\hat{z}\hat{\theta}\right)\varphi+\left(K_{\phi\theta}\hat{\phi}+K_{\theta\theta\phi}\hat{\phi}\hat{\theta}\right.\\&\left.+K_{z\phi\theta}\hat{z}\hat{\phi}\right)\vartheta+K_{\zeta\phi}(t)\varphi+K_{\zeta z\phi}(t)\hat{\phi}\xi+K_{\zeta z\phi}(t)\hat{z}\varphi\\&+K_{\zeta\phi\theta}(t)\hat{\theta}\varphi+K_{\zeta\phi\theta}(t)\hat{\phi}\vartheta+K_{\zeta\zeta\phi}(t)\varphi=0\end{aligned} \tag{2}$$

where, for simplicity, B_e is adopted as an equivalent damping moment. In the particular case of longitudinal waves, the roll linear solution is zero. That is, $\hat{\phi}\equiv 0$. Hence:

$$\begin{aligned}&(J_{xx}+K_{\ddot{\phi}})\ddot{\varphi}+B_e\dot{\varphi}+\left[K_\phi+K_{z\phi}\hat{z}+K_{\phi\theta}\hat{\theta}\right.\\&+\frac{1}{2}K_{zz\phi}\hat{z}^2+\frac{1}{2}K_{\theta\theta\phi}\hat{\theta}^2+K_{z\phi\theta}\hat{z}\hat{\theta}+K_{\zeta\phi}(t)\\&\left.+\ K_{\zeta z\phi}(t)\hat{z}+K_{\zeta\phi\theta}(t)\hat{\theta}+K_{\zeta\zeta\phi}(t)\right]\varphi=0\end{aligned} \tag{3}$$

Substituting: $\hat{z}(t)=A_w\eta_3 Cos(\omega_e t+\alpha_z)$ and $\hat{\theta}(t)=A_w\eta_5 Cos(\omega_e t+\alpha_\theta)$ and decomposing the wave coefficients in their sine and cosine terms, we arrive to an expression of the following type for the roll variational equation (see Neves & Rodríguez, 2004):

$$\begin{aligned}&\left(J_{xx}+K_{\ddot{\phi}}\right)\ddot{\varphi}+B_e\dot{\varphi}+\left[K_\phi+R_0\right.\\&\left.+R_1 Cos(\omega_e t+\tau_1)+R_2 Cos(2\omega_e t+\tau_2)\right]\varphi=0\end{aligned} \tag{4}$$

where R_0, R_1, and R_2 are time-independent restoring coefficients, and τ_1 and τ_2, are the phases of the periodic restoring moments relative to the wave. In contrast to the second order model, where the resultant roll variational equation is a Mathieu type, in the third order model we obtain a Hill type equation. As can be seen from eq. (4), in addition to the Mathieu terms two additional contributions appear. These new terms, as explained in Authors' previous works, rise interesting nonlinear features in the stability analysis of roll variational equation: nonlinear stiffness and biharmonic parametric excitation.

The stability analysis of Hill equation is presented by Hsu (1963), and when applied to equation (4), two instability regions appear defined by:

- First Region of Stability (s = 1):

$$\begin{aligned}&2\omega_4+\frac{A_w}{2\omega_4}\left[(d_{44}{}^{(1)})^2+(e_{44}{}^{(1)})^2\right.\\&\qquad\left.-4\omega_4{}^2(f_{44}{}^{(0)})^2\right]^{\frac{1}{2}}>\omega_e\\&>2\omega_4-\frac{A_w}{2\omega_4}\left[(d_{44}{}^{(1)})^2+(e_{44}{}^{(1)})^2\right.\\&\qquad\left.-4\omega_4{}^2(f_{44}{}^{(0)})^2\right]^{\frac{1}{2}}\end{aligned} \tag{5}$$

- Second Region of Stability (s = 2):

$$\begin{aligned}&\omega_4+\frac{A_w}{4\omega_4}\left[(d_{44}{}^{(1)})^2+(e_{44}{}^{(1)})^2\right.\\&\qquad\left.-4\omega_4{}^2(f_{44}{}^{(0)})^2\right]^{\frac{1}{2}}>\omega_e\\&>\omega_4-\frac{A_w}{4\omega_4}\left[(d_{44}{}^{(1)})^2+(e_{44}{}^{(1)})^2\right.\\&\qquad\left.-4\omega_4{}^2(f_{44}{}^{(0)})^2\right]^{\frac{1}{2}}\end{aligned} \tag{6}$$

where:

$$d_{44}^{(1)} = \frac{\left[K_{z\phi}\eta_3 Cos(\alpha_z) + K_{\phi\theta}\eta_5 Cos(\alpha_\theta) + K_{\zeta\phi c}\right]}{(J_{xx} + K_{\ddot{\phi}})}$$

$$e_{44}^{(1)} = \frac{\left[-K_{z\phi}\eta_3 Sin(\alpha_z) - K_{\phi\theta}\eta_5 Sin(\alpha_\theta) + K_{\zeta\phi s}\right]}{(J_{xx} + K_{\ddot{\phi}})}$$

$$\begin{aligned} d_{44}^{(2)} = \frac{A_w}{\left(J_{xx} + K_{\ddot{\phi}}\right)} \Bigg[& \frac{1}{4} K_{zz\phi}\eta_3^2 Cos(2\alpha_z) \\ & + \frac{1}{4} K_{\theta\theta\phi}\eta_5{}^2 Cos(2\alpha_\theta) \\ & + \frac{1}{2} K_{z\phi\theta}\eta_3\eta_5 Cos(\alpha_z + \alpha_\theta) \\ & + \frac{\eta_3}{2}\left[K_{\zeta z\phi c} Cos(\alpha_z) + K_{\zeta z\phi s} Sin(\alpha_z)\right] \\ & + \frac{\eta_5}{2}\left[K_{\zeta\theta\phi c} Cos(\alpha_\theta) + K_{\zeta\theta\phi s} Sin(\alpha_\theta)\right] \\ & + K_{\zeta\zeta\phi c} \Bigg] \end{aligned}$$

$$\begin{aligned} e_{44}^{(2)} = \frac{A_w}{\left(J_{xx} + K_{\ddot{\phi}}\right)} \Bigg[& -\frac{1}{4} K_{zz\phi}\eta_3^2 Sin(2\alpha_z) \\ & - \frac{1}{4} K_{\theta\theta\phi}\eta_5{}^2 Sin(2\alpha_\theta) \\ & - \frac{1}{2} K_{z\phi\theta}\eta_3\eta_5 Sin(\alpha_z + \alpha_\theta) \\ & + \frac{\eta_3}{2}\left[-K_{\zeta z\phi c} Sin(\alpha_z) + K_{\zeta z\phi s} Cos(\alpha_z)\right] \\ & + \frac{\eta_5}{2}\left[-K_{\zeta\theta\phi c} Sin(\alpha_\theta) + K_{\zeta\theta\phi s} Cos(\alpha_\theta)\right] \\ & + K_{\zeta\zeta\phi s} \Bigg] \end{aligned}$$

$$f_{44}^{(0)} = \frac{B_e}{A_w\left(J_{xx} + K_{\ddot{\phi}}\right)}$$

$$\omega_4 = \left[\omega_{n4}^2 + \omega_{m4}^2\right]^{1/2}$$

$$\omega_{n4}^2 = \frac{K_\phi}{\left(J_{xx} + K_{\ddot{\phi}}\right)}$$

$$\begin{aligned} \omega_{m4}^2 = \frac{A_w^2}{\left(J_{xx} + K_{\ddot{\phi}}\right)} \Bigg[& \frac{1}{4} K_{zz\phi}\eta_3^2 + \frac{1}{4} K_{\theta\theta\phi}\eta_5^2 \\ & + \frac{1}{2} K_{z\phi\theta}\eta_3\eta_5 Cos(\alpha_z - \alpha_\theta) \\ & + \frac{\eta_3}{2}\left[K_{\zeta z\phi c} Cos(\alpha_z) - K_{\zeta z\phi s} Sin(\alpha_z)\right] \\ & + \frac{\eta_5}{2}\left[K_{\zeta\theta\phi c} Cos(\alpha_\theta) - K_{\zeta\theta\phi s} Sin(\alpha_\theta)\right] \\ & + K_{\zeta\zeta\phi 0} \Bigg] \end{aligned}$$

The numerical implementation of Hsu's analytical approach for the TS ship in head seas under different speeds and for a metacentric height (*GM*) of 0.37 m gives the following stability limits (see Figures 2 to 5). Different from past Authors' works, in these cases damping is considered, although for simplicity only the linear damping was taken into account.

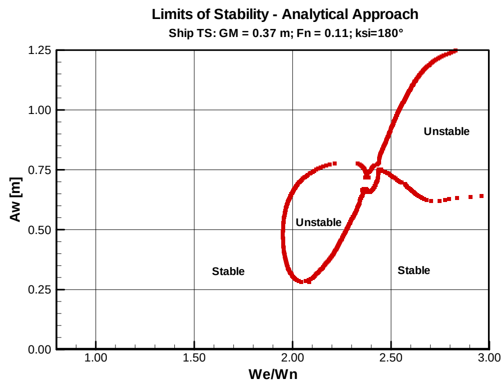

Figure 2 Analytical approach, Fn=0.11

As stated in Neves & Rodríguez (2004) and in Rodríguez (2004), the introduction of damping shortens the instability regions and displaces to the right the tuning frequency of the minimum threshold wave amplitude. Other main features of the Hsu's instability regions have already been discussed in Neves & Rodríguez (2004).

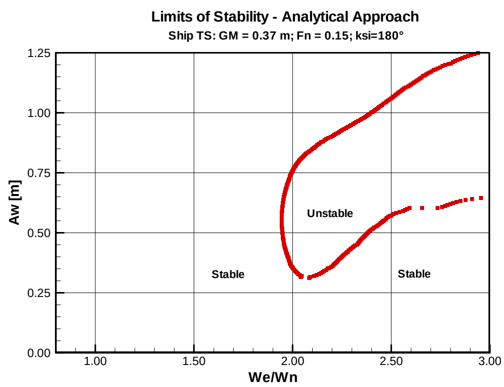

Figure 3 Analytical approach, Fn=0.15

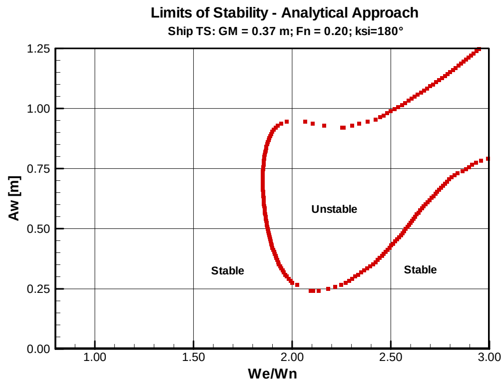

Figure 4 Analytical approach, Fn=0.20

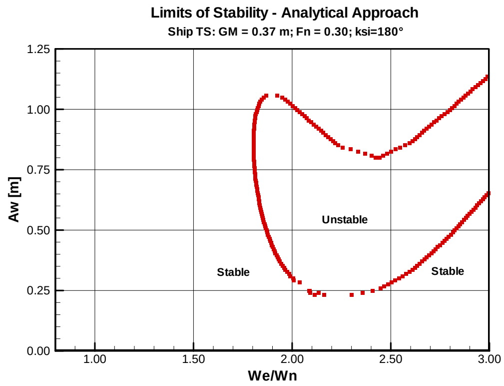

Figure 5 Analytical approach, Fn=0.30

Although the limits of stability using the analytical approach is a valuable and easy tool in the ship design stage as a rough indicator of the extension and location of the instability regions, they do NOT provide quantitative information, i.e., the steady amplitude of parametric rolling.

An alternative way of computing the instability regions is obtained by solving the motion equations for a large set of waves amplitudes and tuning factors (encounter frequency/natural roll frequency), which will be varied systematically. Then, each time instabilization takes place (roll amplification), a point will be plotted in the corresponding plane (A_w vs. tuning factor). Depending on the magnitude of the steady roll amplitude, these points will have an identifying color.

Yet this procedure is much more time consuming for computation, it has the advantage of letting us know the instability regions not only qualitative, but also quantitatively.

Under this alternative method, two different numerical approaches could be used: one assuming an uncoupled nonlinear roll motion equation, and the other using the 3 DOF motion equations coupling the heave, roll and pitch modes.

3.2 Numerical Approach – Uncoupled Roll Motion

Here, the vertical modes are assumed linear while the rolling motion keeps all its nonlinear terms, resulting in an uncoupled roll equation. When the analytical approach was used, we analyzed the behaviour of the roll variational equation, which, as seen above, is linear. So, by comparing the analytical and the numerical uncoupled roll approach is possible to identify the effects of pure roll nonlinearities, which in the case of the former approach cancel due to the null linear roll response. Figures 6 to 9 show the instability regions obtained integrating the nonlinear roll equation of motion in the time-domain for an initial condition in roll of 2°.

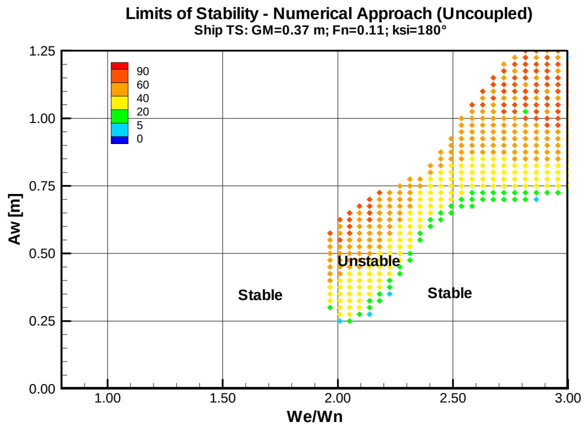

Figure 6 Num. uncoupled approach, Fn=0.11

As can be noted from figures 6 to 9, the shape and location of the regions of stability agree well with the analytical approach. This would indicate that pure roll nonlinearities have little or NO influence on these characteristics of the limits.

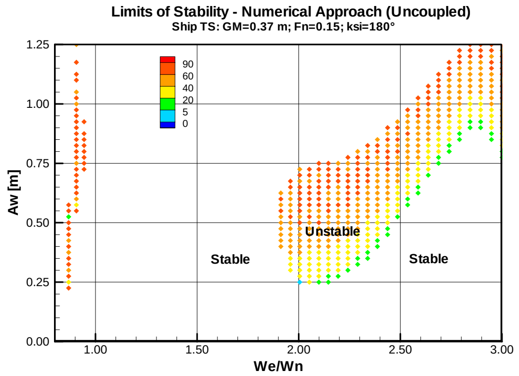

Figure 7 Num. uncoupled approach, Fn=0.15

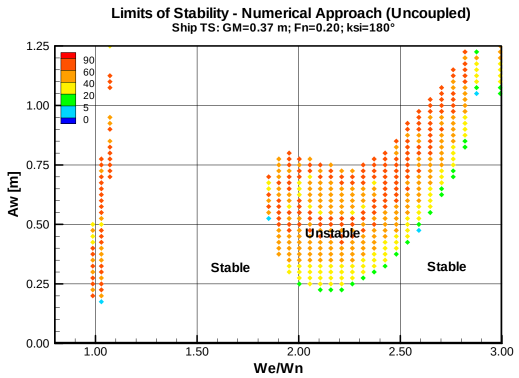

Figure 8 Num. uncoupled approach, Fn=0.20

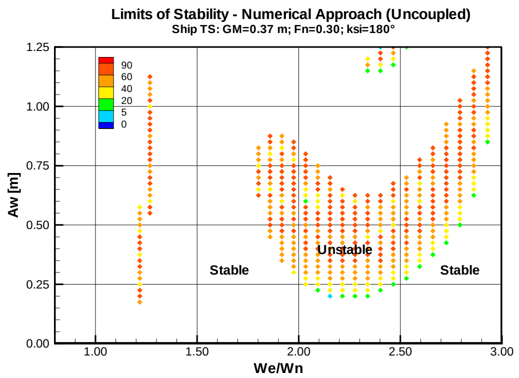

Figure 9 Num. uncoupled approach, Fn=0.30

Concerning the amplitudes of parametric rolling, as expected, the greater the wave amplitudes the greater the responses within the instability regions. At low speeds parametric roll amplitudes grow gradually as wave amplitude increases (up the top limit), however, at high speeds (Fn = 0.20 and Fn = 0.30), parametric rolling is violent starting at angles of 20° and rapidly reaching capsizing angles for little increases in wave amplitudes (see fig. 9). Another characteristic observed at high speeds is the appearance and accentuation of a concavity in the top limit, to the right of the Mathieu *exact* tuning ($\omega_e/\omega_{n4} = 2.0$).

3.3 Numerical Approach – 3 DOF Motion

A more refined and reliable way of getting the limits of stability for parametric resonance is to solve numerically the three-degrees-of freedom (DOF) ship motion equations shown in section 2 of the present work, and plot the responses, as explained in the previous section. This more complete approach when applied to the same conditions tested above for TS ship resulted in the limits of stability shown in figures 10 to 13.

In general, the shape and location of first instability regions ($\omega_e = 2.0\omega_{n4}$) obtained with the 3 DOF numerical approach agrees well with the previous approaches. However, when comparing the limits of figures 10 to 13 with its corresponding ones of figures 6 to 9, relevant differences can be observed in the amplitudes of parametric rolling for all speeds cases. Such differences reflect the influence of nonlinearities of heave and pitch, which in general tend to control the magnitude of roll amplifications.

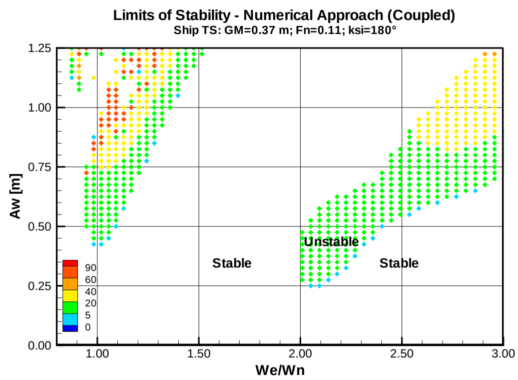

Figure 10 Num. 3 DOF approach, Fn=0.11 (ϕ_0 = 2°)

Another significant and surprising feature is the notoriety that the second region of instability ($\omega_e = \omega_{n4}$) gain in comparison to the other approaches: not only the wider area, but also the trend and the magnitude of unstable roll. Concerning the concavity of the instability regions at high speeds pointed out in the previous section, we confirm here its existence and also call the attention of the readers for the risk of getting more critical responses at frequencies higher than the Mathieu tunings, even for quite low wave amplitudes.

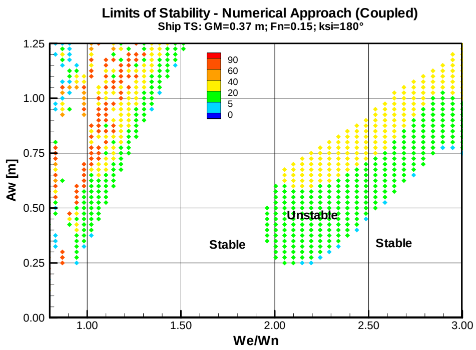

Figure 11 Num. 3 DOF approach, Fn=0.15 (ϕ_0 = 2º)

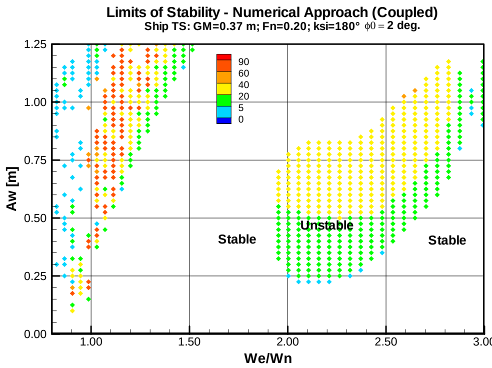

Figure 12 Num. 3 DOF approach, Fn=0.20 (ϕ_0 = 2º)

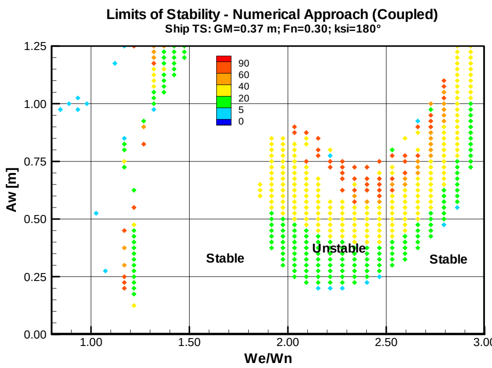

Figure 13 Num. 3 DOF approach, Fn=0.30 (ϕ_0 = 2º)

4. OTHER NONLINEAR CHARACTERISTICS OF PARAMETRIC ROLL BEHAVIOUR

Based on the procedure outlined in the previous sub-section, it is possible to compute the limits of stability for different initial conditions, and identify the conditions that bring different steady parametric roll amplitudes. This phenomenon would indicate the possibility of occurrence of jump effect, bifurcation or even chaos. Figure 14 illustrates the limits of stability for the same conditions showed in figure 12, but considering an initial condition for roll of 20º.

It is obvious that the first influence of initial conditions is to modify the size of the instability regions. In the case illustrated in figure 14, the first region of instability became greater (growing upwards and to the left side). This additional instability region denotes a zone of initial condition susceptibility. Then, a good beginning for the study of typical nonlinear behaviour would be the exploration of this zone.

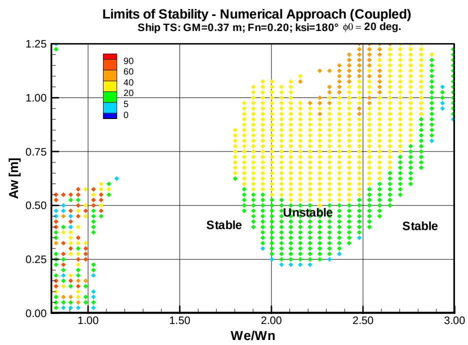

Figure 14 Num. 3 DOF approach, Fn=0.20 (ϕ_0 = 20º)

Time simulations are one of the most used tools for exploring such behaviour. Figures 15 to 17 show time series for three different initial conditions in roll for the case of TS ship at Fn=0.20, A_w=0.95 m and ω_e = 2.158 ω_{n4}.

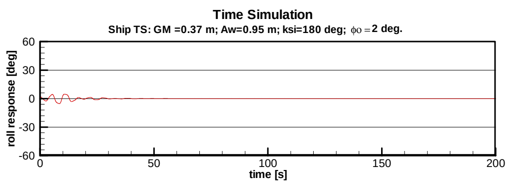

Figure 15 Time series, Fn=0.20; ϕ_0 = 2º

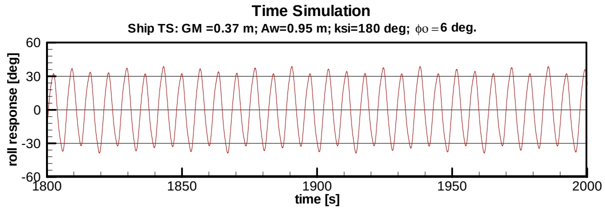

Figure 16 Time series, Fn=0.20; $\phi_0 = 6^o$

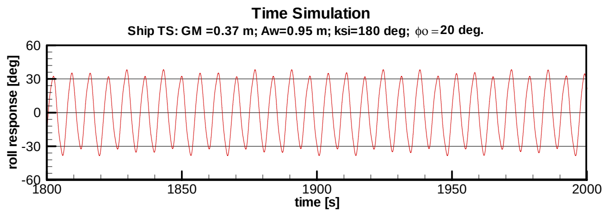

Figure 17 Time series, Fn=0.20; $\phi_0 = 20^o$

As expected, for these parameters different initial conditions resulted in different roll responses. For the smaller initial conditions no parametric resonance was developed, as had been noted in the stability map of figure 12 (2^o of roll initial condition). For the initial condition larger than 6^o, parametric rolling develops in an erratic way, no single amplitude is observed. Then, it becomes necessary to look at phase diagrams in order to identify if there is a multiperiod response or the possibility of occurrence of chaos. Figures 18 to 20 show the phase diagrams for the respective conditions shown in the time series, initial conditions $\phi_0 = 2^o$, $\phi_0 = 6^o$ and $\phi_0 = 20^o$.

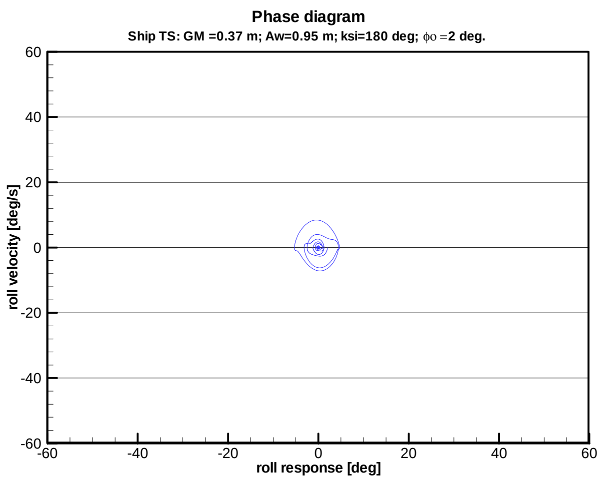

Figure 18 Phase diagram, Fn=0.20; $\phi_0 = 2^o$

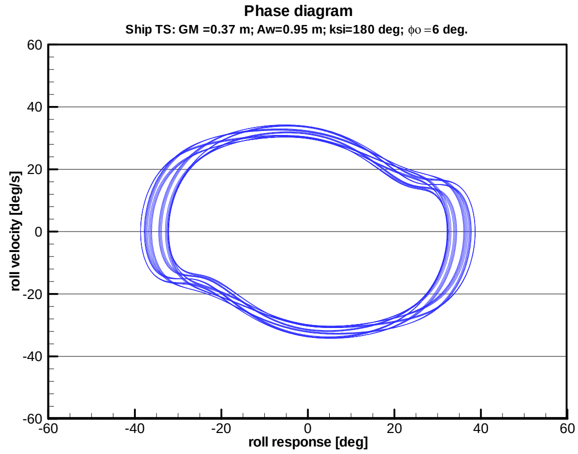

Figure 19 Phase diagram, Fn=0.20; $\phi_0 = 6^o$

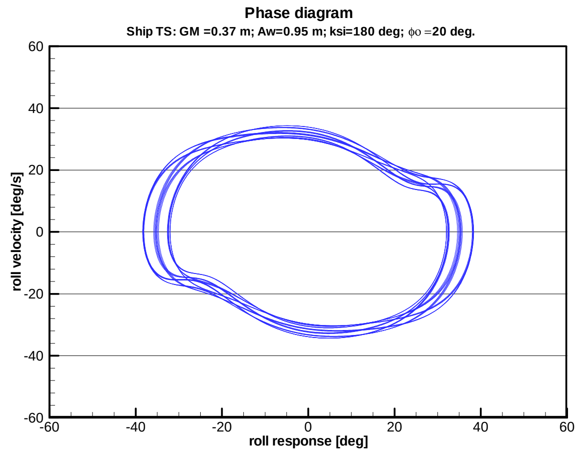

Figure 20 Phase diagram, Fn=0.20; $\phi_0 = 20^o$

Clearly, it is noted that the small initial condition response is stable and does NOT develop parametric rolling. In other words, the response is attracted by a single attractor, which in this case corresponds to the null parametric rolling attractor.

For the larger initial conditions, the responses are attracted by a "not-easy-to-identify attractor". We can just say that it is not a single point or a limit cycle attractor. Maybe a strange attractor or even a chaotic attractor, but a definite answer to this question should be given based on more specific nonlinear analysis tools such as bifurcation analysis, Poincaré mapping, Lyapunov exponents, etc. This kind of analysis is out of the scope of the present work, but literature on this subject is ample (Guckenheimer & Holmes, 1983, Seydel, 1988, Liaw et al., 1993).

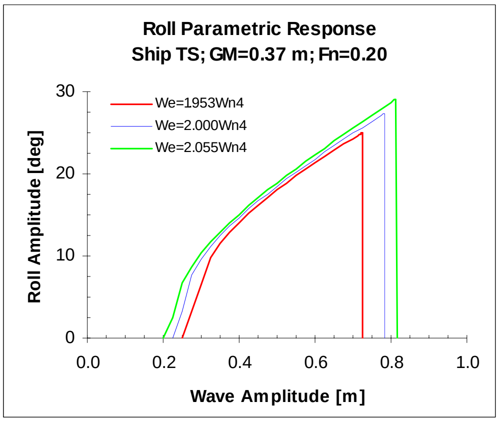

Figure 21 Jump in roll response at high wave amplitudes, Fn=0.20.

Figure 21 shows the roll amplitudes against wave amplitude for three tunings around the *exact* Mathieu tuning $\omega_e = 2.0\ \omega_{n4}$. Again, for high amplitudes (extreme right of the graph) abrupt changes in roll response are observed, indicating the occurrence of a bifurcation for changing wave amplitudes.

5. CONCLUSIONS

Based on a third order mathematical model for parametric rolling, three approaches for computing the limits of stability have been presented.

The analytical approach has shown good agreement with the numerical responses, and due to its relatively easy implementation, should find good applicability in the ship preliminary design stage. One limitation of the analytical approach is that it does not provide information on the magnitude of parametric rolling. To overcome this inconvenience, two numerical approaches have been proposed. One using the uncoupled roll equation, and the other applying the full nonlinear equations coupling heave, roll and pitch. Comparing the two latter approaches, a relevant conclusion can be drawn, i.e., the extreme importance of nonlinear couplings between the vertical modes and roll in the determination of parametric roll amplitudes. As can be noted in the respective figures, the uncoupled numerical approach can induce us to wrong predictions of parametric amplitudes.

Another contribution of the present investigation is the identification of initial conditions susceptibility zones within the instability regions, so that the analysis of typical nonlinear phenomena can be focused on these zones. Preliminary analysis of these zones has shown great influence of initial conditions on the development of parametric rolling.

6. ACKNOWLEDGMENTS

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