Physics and Statistics of Extreme and Freak Waves

Leonid Lopatoukhin, *St. Petersburg State University*

Alexander Boukhanovsky, *St. Petersburg State University*

ABSTRACT

Problem of estimation of high waves is not as critical as 10-20 years ago. Now it is possible to make continues hindcasting (30 and longer years duration) of directional spectra for any region and to estimate probability of high waves with n-years return period. For ship navigations, transport operations, offshore supplying etc spatiotemporal variability of wave fields may be also estimated. One of the most interesting extreme phenomena is freak (or rogue) waves. Freak wave is unusual not only by there height, but by their form. Results of hindcasting do not display any suspicion to a freak wave as this is a single wave. Classical statistical approach to analysis of time series do not allows estimating the probabilities of freak waves. Proposed approach to statistics of freak waves is based on probabilistic treatment of a wave field as random model with contaminated distribution

In order to efficiently use calculated statistics in practice it is important to focus on the measure of risk, which is taken as acceptable limit. This is very delicate balance of factors in which consequence of damage, cost of construction, and cost of mitigation of possible accident are major players.

Keywords: *wind waves, extremes, freak*

1. INTRODUCTION

Presently the main source of wave climate information is based on the results of hydrodynamic simulation (in other words hindcasting). Reanalysis data are the input to hindcasting are. The NCEP/NCAR, ERA40 and Sweden reanalysed wind fields were used in present report. For extreme wave estimation reanalysis data have to be improved by assimilation of additional ship observation and synoptic data. Regression and Kalman filtration approaches are used. (For details see, e.g. (Lopatoukhin et al, 2004)). Nested models Wave Watch (versions 1.18, 2.22) and SWAN (versions 40.11, 40.31) are applied. Barents, Caspian, Baltic, North, Okhotsk, Azov seas and Ladoga Lake used as deep and shallow water basins. Metocean fields, like ocean waves, have a complex spatial and temporal variability (synoptic, annual, year-to-year). Traditionally, the approach for statistical formalization of such phenomena has been based on a multiscale hypothesis and modelling the total variability by means of a set of stochastic models for each temporal scale.

2. EXTREME WAVES

2.1 Extremes at a Point

There are a lot of approaches to calculations of extreme wave heights in a point. The main are IDM (Initial Distribution Method), AMS (Annual Maxima Series), POT (Peak Over Threshold) and BOLIVAR. Short resume with the example is presented at fig. 3. IDM method estimates the extreme wave height h_{max} of certain return period as quantile h_p of wave height distribution F(h) with probability p. For log-normal long-term wave height distribution, the quantile with probability p can be computed as follows:

$$h_p = h_{0.5} \exp\left(\frac{U_p}{s}\right) \tag{1}$$

U_p is quantile of the standard normal distribution. Here quantile h_p should be understood as wave height, which is likely to be observed once (at the standard synoptic observation times) in T years. In applied studies the period *T* is called “return period”, and the corresponding probability is defined as

$$p = \frac{\Delta t}{24 \cdot 365 \cdot T} \tag{2}$$

Where Δt is interval (in hours) between subsequent observations (say, 6 hours). Then we get $p = 0.000684/T$. For $\Delta t = 3(hr)$, we get $p=0.000342/T$.

AMS approach defines h_{max} as the last term of the ranked independent series of wave heights h. The AMS method has the most solid theoretical background with Gumbel distribution

$$F(x) = \exp(-\exp(-a(x-b))) \tag{3}$$

Where a, b – parameters. The POT approach uses k strongest storms with the heights greater than selected threshold. Thus, the POT method estimates depend on the choice of threshold and approximations for corresponding distributions. Unlike other methods, in the POT approach the uncertainty is connected both with the wave height h_p^* and return period. BOLIVAR approach (Rozhkov et al, 1999; Lopatoukhin et al, 2000) excludes the limitations of the POT method and take into account the asymptotic characteristics of AMS. BOLIVAR approach considered *n* samples, consisting of heights h_{ij}^+ of the largest waves in the *k* the strongest storms in year number *i*,(i=1,..,n; j=1,…,k). The BOLIVAR method represents its further development that includes into consideration the second, third and, potentially, other maximums in a year. Each of the considered methods has its advantages and disadvantages and has to be used accordingly (Lopatoukhin et al, 2000).

2.2 Extremes at a Field

Storm evolution in any basin may be presented as an impulse random field

$$\zeta(\vec{r},t) = \sum_k W_k^{z(\vec{r})}(\vec{r},t \mid X) \tag{4}$$

Where $W_k^{z(\vec{r})}(\bullet)$ – spatiotemporal impulse with respect level $z(\vec{r})$. At any time $W_k^{z(\vec{r})}(\bullet)$ can be presented as an elliptic cone. The size of the storm $\{r_0(t), h^+(t), S_\Omega(t)\}$ is equal to the fraction of total area of the region, where wave heights larger than z.

The behavior of the extreme wave in a single storm in a fixed point is known [Boukhanovsky et al 1998]. For spatial region this problem more complex, because unique enumeration available only for two-dimensional waves.

In the simplest case, with a narrow angular spreading of sea waves, the generalized distribution of maximal wave in a spatial storm region is

$$F_m(h) = \exp\left[2\pi \int_0^L \left(-\exp\left(-\frac{\pi}{4}\left(\frac{h}{\bar{h}(r)}\right)^2\right)\right)\frac{rdr}{\lambda(r)}\right] \tag{5}$$

Here 2L is the equivalent diameter of the storm, where $L = 2\sqrt{S_\Omega(t)/\pi}$, $S_\Omega(t) = \int_{\Omega(t)} d\vec{r}$. For small-amplitude waves $\lambda(r) \approx 36\bar{h}(r)$.

The storm impulse $\bar{h}(r)$ is approximated by expression $\bar{h}(r) = h^+ - (h^+ - z)(r/L)^m$, where m is the shape parameter of storm impulse (m=1 – cone, m=2 – parabolic etc).

Fig. 1 shows extreme wave heights (0.1% probability) with return period 100 years. This figure (as a lot of similar, published in different papers, handbooks and atlases) is a result of calculation at separate points and driving isolines. Data of such figure represent the wave heights estimates that are possible at any point, *but not in all points simultaneously*. In the last case the return period of such events will be highly more, than 100 years.

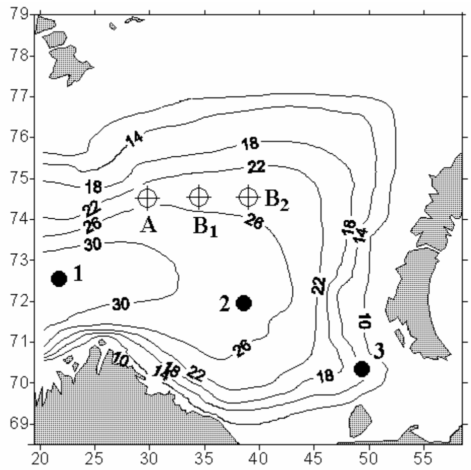

Fig. 1. Spatial estimates of waves (0.1%) with return period 100 years. Barents Sea.

This became more clear from the Fig. 2. There are shown annual maxima $h_s^{(A)}$ in the point "A" and conditional values $h_s^{(B/A)}$ in the points B_1 and B_2. They are at distance 120 and 240 km from point "A". It is seen, that in spite of significant distance between points, the values of wave heights of rare probability are dependent. This also means that the same return period may appropriate to different combinations of waves. E.g. (fig. 2), 100 years significant wave in the point "A" is 14.4m, but wave height in the point "B_1" is 13.7m (i.e. with return period 50 years). Moreover 100-year event may result from a set of events each of it is less than 100 year. E.g., 100 years case will be, when:

- in the point "A" $h_s^A = 12.1m$ (10 years return period);
- in the point "B_1" $h_s^{B_1} = 13.2m$ (30 years return period);
- in the point "B_2" $h_s^{B_2} = 13.8m$ (60 years return period).

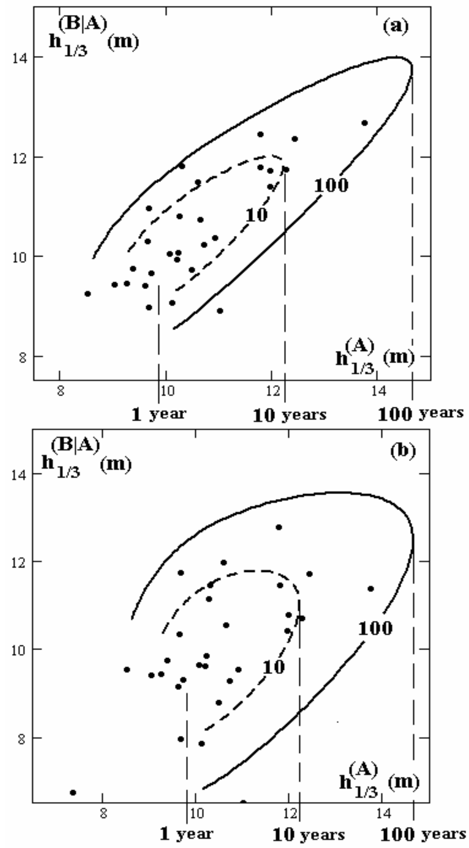

Fig. 2. Points and curves of return periods of annual maxima. Significant wave heights $h_{1/3}^{A}$ in the point "A" and wave heights $h_{1/3}^{B|A}$ in the points « B1» (a) and «B2» (b), at the same time.

3. FREAK WAVES

One of the most interesting extreme phenomena is freak waves (Mallory 1974, 1984; Provision, 1998; Proceedings, 2000, 2002; Rogue 2000, 2004), – as anomaly steep and high waves. Today a lot of hypotheses try to explain freak wave generation mechanism. All the reasons may be separated on *external* or *internal* (Lopatoukhin, Boukhanovsky, 2003, 2004, 2005). The external reasons include the opposing wave-current interaction, refraction around shoals or from inclined seabed, wave caustics from diffraction at coastlines, and crossing wave systems. The internal reasons are mainly the frequency and (or) amplitude wave modulation in a random sea, cooperative effect of four- and five-wave interactions, the high-order nonlinearties and nonlinear focusing (Rogue, 2000, 2004; Lopatoukhin, Boukhanovsky, 2003). Some definitions of freak waves as set of parameters $\Xi = \{h, \tau, c, ...\}$, characterizing the shape of the wave and the steps of it selection from a record are presented in the fig. 3. Really, hypotheses of freak wave generation allow their arising in any place of the Ocean, and not only in the well-known dangerous regions, such as South shore of Africa etc. **Any metocean event described by a system of nonlinear thermo hydrodynamic equations, possesses their own freaks.** Hindcasting of waves do not allow revealing freak wave (Lopatoukhin et al, 2005). Freak wave had been recorded in such "calm" region as the NE part of Black sea (Lopatoukhin et al, 2003, 2005). There were three such waves recorded during six years of measurements, i.e. three waves from more than million recorded.

3.1 Probabilistic scenarios for freak waves generation

There are two ways to formulate the conditions of freak waves generation in the Ocean. The first way considers the arising of the different external conditions, leading to possibility of freak wave generation, and computation the joint probability of these conditions (e.g. combinations the severe waves and opposite currents etc.). But the real input of this approach is not obvious, because it is hard to take into account all the driving factors. Another way considers the ensemble of all waves (their heights h, periods τ, crests c etc.) and estimate occurrence of its crucial combinations, leads to freak wave arising. This approach seems more reliable in practice, because it is based on the consideration of freak waves as the elements of the same ensemble, as all the waves. But, it requires the sophisticated statistical techniques for rare events analysis, because the extreme combinations of the waves parameters belong to the tails of its joint probability function.

The problem of freak wave occurrence, and associated scenarios, include the procedures of statistical analysis and synthesis of huge data samples, because freak wave is very rare event. Moreover, due to multiscale and spatio-temporal variability of sea waves, the numerical simulation here is very resource-consuming procedure. It requires the development of special approach for stochastic simulation, that allows investigating the freak waves occurrence efficiently and precisely.

Freak wave is unusual not only by there height, but by their form. This uncommonness specified by means:

- Set of parameters, e.g. h>2.4hs, crest>0.65h, unusual steepness δ of a wave and (or) it front or back slope, deep trough, twice as greater than preceding and subsequent waves, etc. Not all of these parameters are realized simultaneously, but as a rule at least three can be achieved.
- Governs by nonlinear Schrödinger equation.
- Suddenness of arising in some point of a wave field.

One of the main objectives of investigation is a probabilistic treatment of a wave field $\zeta(\vec{r}, t)$ as probabilistic contaminated distribution.

$$\Phi_\varsigma(\vec{x}, \vec{r}, t) = (1-\varepsilon)F_\Xi(\vec{x}) + \varepsilon\hat{F}_\Xi(\vec{x}). \qquad (6)$$

Where $F(\vec{x})$ - joint distribution of wave parameters (e.g., height, crest, steepness), $\hat{F}(x)$ - asymptotic distribution of this parameters, $\varepsilon(\vec{r}, t)$ - probability of freak wave arising in specific place at a moment t. Ξ - multidimensional system of random values (h,c,δ…).

The first term in (6) describes "background" distribution of Ξ in short-term domain. It is approximated as,

$$F_\Xi(X) = F_h(x_1)F_{c|h}(x_2|x_1)F_{\delta|h}(x_3|x_1) \qquad (7)$$

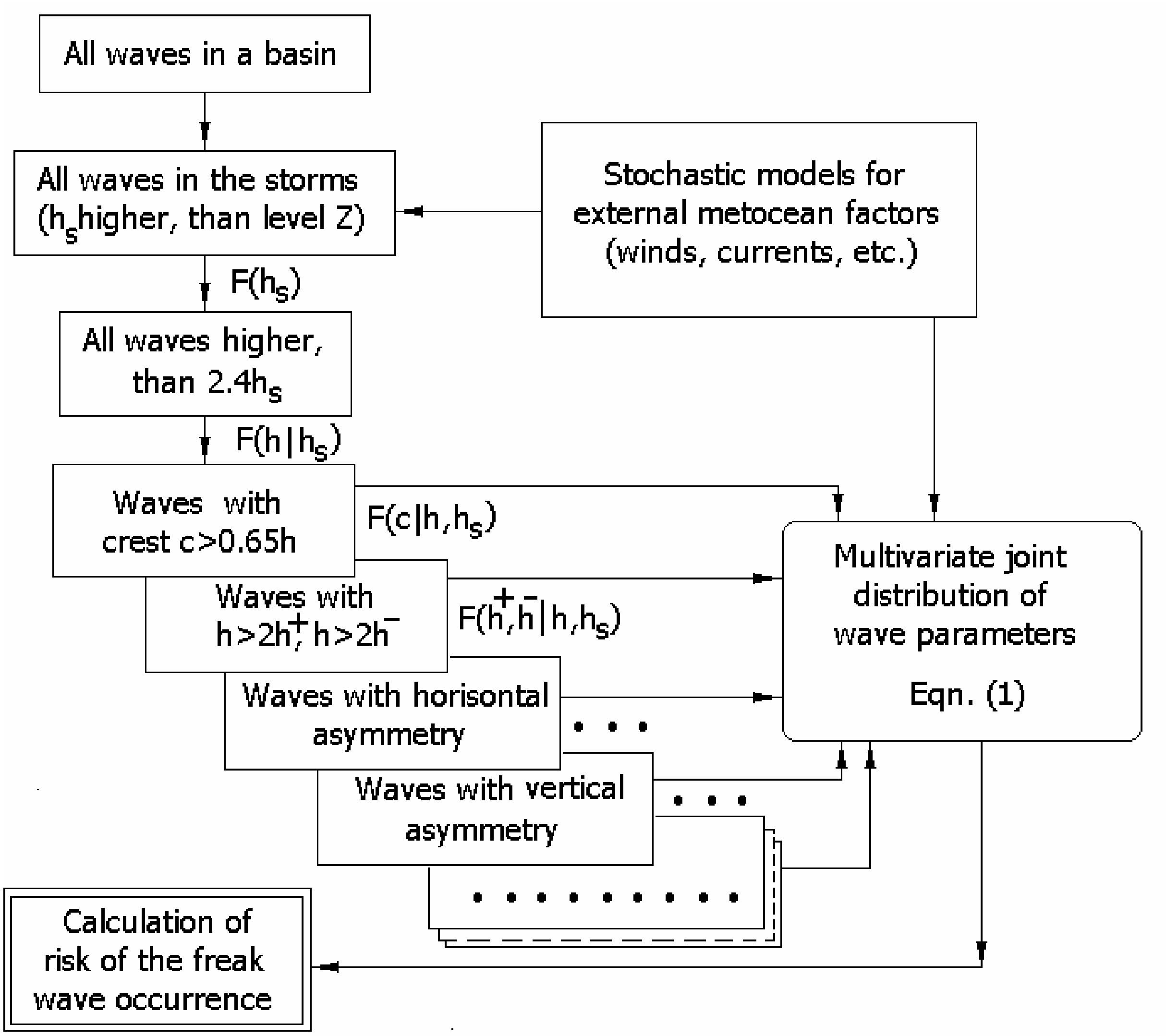

Fig. 3. General scheme of freak wave generation scenarios

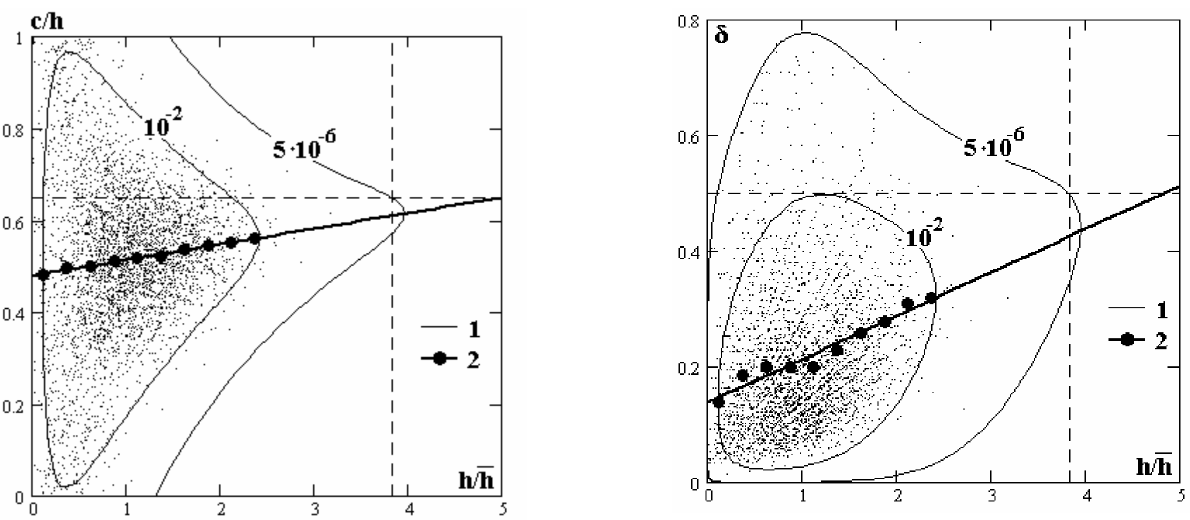

Fig. 4. Joint distribution of parameters $\{h, c/h\}$ (a) and $\{h, \delta\}$ (б). (1) – Lines of equal probability; (2) – Regression.

In short-term scale distribution (6) is a set of Weibull distributions with different shape parameters. The second term in (6) incorporate contamination (litters) of a “background” distribution by freak wave. Asymptotic distribution $F(x_1,x_2,x_3)$ may be used. Some definitions of freak waves as set of parameters $\Xi = \{h, \tau, c, ...\}$, characterizing the shape of the wave and the steps of it selection from a record are presented in the fig. 3.

Short tem distributions (7) may be the following: Rayleigh distribution as marginal distribution $F_h(x_1)$, conditional distributions of wave crests c and steepness δ as Weibull distributions with scale parameter from 2 to 7. Joint distributions $F_h(x_1)F_{c|h}(x_2 \mid x_1)$ and $F_h(x_1)F_{\delta|h}(x_3 \mid x_1)$ are presented at fig. 4. This fig. is generalization of about 5000 wave records, but without freaks. The equal probability (p%) curves for values $\{h, c/h\}$ и $\{h, \delta\}$ are drawn. It is seen, that value $\{h/\bar{h} \geq 3.8, c/h \geq 0.65\}$ for any δ, or $\{h/\bar{h} \geq 3.8, \delta \geq 0.5\}$ for any c/h have the probability $5{\cdot}10^{-6}$. Probability defined from three-dimensional distribution $\mathrm{P}\{\delta \geq 0.5 \mid \mathrm{h}/\bar{\mathrm{h}} \geq 3.8 \cap \mathrm{c}/\mathrm{h} \geq 0.65\} = 0.12$ This means, that probability of three conditions simultaneously $\{h/\bar{h} \geq 3.8, c/h \geq 0.65, \delta \geq 0.5\}$, will be $5 \cdot 10^{-6} \cdot 0.12 = 6 \cdot 10^{-7}$. This means, that only one wave from 1.7 million will be with height greater, than $3.8\bar{h}$, crest greater than $0.65h$ and steepness $\delta > 0.5$. This value is the lower limit of probability ε, i.e. probability of freak wave in a specific point not greater than $6{\cdot}10^{-5}$ %.

For short term range with 1000 waves, freak wave may arise in one of 1660 time series. Relation (8) may be adopted as asymptotic distribution in (6).

$$F_{\Xi}(X) = F_h(x_1)F_{c,\delta|h}(x_2, x_3) \tag{8}$$

First limit distribution may be used.

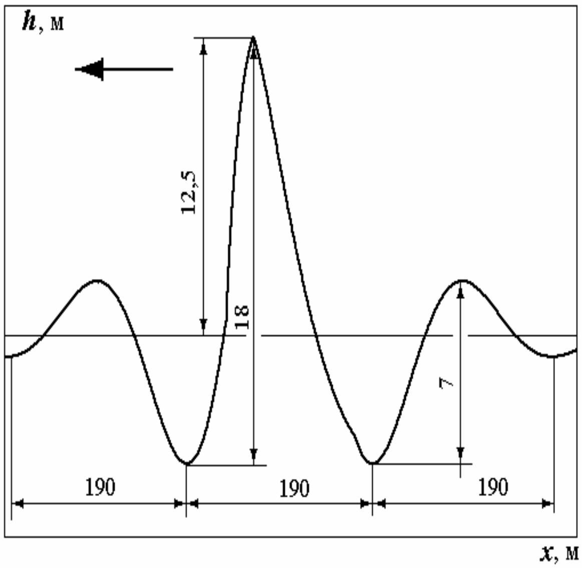

Fig. 5. Possible freak wave during loss of ship «Aurelia». February 2, 2005, N. Pacific.

Estimation of $\varepsilon = 6 \cdot 10^{-7}$ in short-term range may be increased due to specific of some metocean situations. As an example loss of ship “Aurelia” (Class of Russian Register of shipping, 34000tonn) in February 2005 in the N. Pacific took place during passing of atmospheric front with wind waves and swell. Fig. 4 shows possible parameter of freak wave during this case.

Wave measurements shows that in Black and North seas freak waves arise during transformation of wind wave spectra to waves with swell. In this case both wave spectrum and angular distribution became more broad (Lopatoukhin et al, 2005).

4. CONCLUSION

Classical statistical analysis of time series does not allow estimating the probabilities of freak waves occurrence and associated weather conditions. Results of hindcasting for any specific time also do not display any suspicion to such a wave. Directional spectrum of wave record does not reveal existence of freak wave. Occurrences of freak wave have to be regarded as multidimensional random event. This is the main difference between extreme and freak wave. General scheme of freak wave selection from the sample of measured waves is shown on the fig. 3. Special attention is paid to investigation of field conditions leading to freak wave generation. These include weather features, current effects and bottom bathymetry.

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