A Time Domain Method for Calculating Motions of a Ship Sailing on Random Waves

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ABSTRACT

When a ship is sailing on given waves, no matter how rudder acts, the ship heading may change arbitrarily under the action of waves. Considering the wave excitation depending on the heading, it's very important to determine the ship heading together with the wave excitation at the same time. In this paper a time domain calculation method was proposed where the wave excitation in all motion modes can be obtained according to the impulse response functions and the wave trace. During the simulation, the impulse response functions were different depending on the heading at each instant. The details were described in the paper. An example was given.

Key words: *time domain; ship sailing; random waves; impulse response functions; memory effect*

1. INTRODUCTION

In ordinary sea-keeping calculation the wave heading is considered to be constant (Salvesen, Tuck and Faltinsen, 1970). But in a real situation, as a ship is sailing on a given wave, the ship heading may be varied due to the action of wave no matter how the rudder is applied. Because the magnitude of wave excitations in different motion modes depends on wave direction, the wave excitations should be determined instantly according to the wave direction at that instant.

As to ship sailing in waves, or in other words, maneuvering in waves, many researchers have investigated on this topic, such as Kose & Hinata (1985); de Kat & Paulling (1989); Ottosson & Bystrom (1991); Shen, Ma, & Sun, et al. (2000); Lee (2000); Zhu, Pang & Xu (2004) ; Fang, Luo & Lee (2005). They studied the maneuvering motions of a ship in regular waves. But they have not considered the effect of heading changing on the simulation of ship maneuvering in waves.

In this paper, we try to solve the sailing in waves considering the heading change in each instant and the wave excitation change as well. To wave exciting force, we use impulse response functions. The wave excitation force can be calculated in time domain by the convolution integral with the impulse response function of wave excitation and input wave trace. The method itself, using impulse response functions, is not new (Cummins, 1962, King, Beck, Magee 1988, Bingham, Korsmeyer, et al., 1993, etc), it still present a good try in solving the problem of sailing in waves.

For each instant, the impulse response functions are different depending on the heading. The only problem remains in the convolution integral is the integral is along the time axis from the past to the future for a given instant, such impulse response function should be changed if time departs from that instant and the heading angle varies. This problem is difficult to be solved rigorously at present. Considering the main part of the impulse response function is concentrated at the origin and reduced rapidly. It is possible to use this function in the convolution integral along the whole time axis approximately. We use such method in our calculation.

2. THEORY

2.1 Equations of Ship Sailing in Waves in Time Domain

Generally the motions of ships under irregular waves can be solved either in time domain or in frequency domain. To ships sailing in waves, the motions are nonlinear and complicate. Time domain method is used to investigate the motion of ships sailing on random waves.

Before presenting the motion equations, there are some assumptions:

a) forward speed is low, omit the effect of speed in free surface condition
b) omit the nonlinear effect of heading changing rate in determining the hydrodynamic coefficients
c) omit some cross-couple items in the motion equation
d) effect of forward speed is reflected in hydrodynamic coefficients

By maneuvering equations (Spyrou 1996) combined with seakeeping equations in time domain, we can get the six degree motion equations of a ship sailing on random waves including the memory effect of hydrodynamic force, which follows the idea in Chung & Bernitsas (1997):

$$\begin{aligned}&(M_{11}+m_{11})\ddot{x}_1(t)+\int_{-\infty}^{t}K_{11}(t-\tau)\dot{x}_1(\tau)d\tau+\int_{-\infty}^{t}K_{12}(t-\tau)\dot{x}_2(\tau)d\tau\\&-(M_{11}+m_{11})\dot{x}_2(t)\dot{x}_6(t)=-Y_r(\dot{x}_6(t))^2-\mathrm{Re}\,s(u)\\&+F_{W1}(t)+X_P(t)+X_R(t)\end{aligned}\tag{1}$$

$$\begin{aligned}&(M_{22}+m_{22})\ddot{x}_2(t)+\int_{-\infty}^{t}K_{22}(t-\tau)\dot{x}_2(\tau)d\tau+Y_v\dot{x}_2(t)+Y_r\dot{x}_6(t)\\&+Y_{vv}\dot{x}_2(t)\left|\dot{x}_2(t)\right|+(M_{22}+m_{22})\dot{x}_2(t)\dot{x}_6=F_{W2}(1)+Y_P+Y_R\end{aligned}\tag{2}$$

$$\begin{aligned}&(M_{33}+m_{33})\ddot{x}_3(t)+\int_{-\infty}^{t}K_{33}(t-\tau)\dot{x}_3(\tau)d\tau+\int_{-\infty}^{t}K_{35}(t-\tau)\dot{x}_5(\tau)d\tau\\&+(M_{35}+m_{35})\ddot{x}_5(t)+C_{33}x_3(t)+C_{35}x_5(t)+D_{33}\dot{x}_3(t)=F_{w3}(t)\end{aligned}\tag{3}$$

$$\begin{aligned}&(M_{44}+m_{44})\ddot{x}_4(t)+\int_{-\infty}^{t}K_{41}(t-\tau)\dot{x}_1(\tau)d\tau+\int_{-\infty}^{t}K_{42}(t-\tau)\dot{x}_2(\tau)d\tau\\&+\int_{-\infty}^{t}K_{44}(t-\tau)\dot{x}_4(\tau)d\tau+\int_{-\infty}^{t}K_{46}(t-\tau)\dot{x}_6(\tau)d\tau+C_{44}x_4(t)\\&+C_4'x_4(t)^3+D_{44}\dot{x}_4(t)-mz_G(\ddot{x}_2(t)+\dot{x}_6(t)\dot{x}_1(t))=\\&F_{W4}(t)+K_P+K_R\end{aligned}\tag{4}$$

$$\begin{aligned}&(M_{55}+m_{55})\ddot{x}_5(t)+\int_{-\infty}^{t}K_{53}(t-\tau)\dot{x}_3(\tau)d\tau+\int_{-\infty}^{t}K_{55}(t-\tau)\dot{x}_5(\tau)d\tau\\&+(M_{53}+m_{53})\ddot{x}_3(t)+C_{55}x_5(t)+C_{53}x_3(t)+D_{55}\dot{x}_5(t)=F_{W5}(t)\end{aligned}\tag{5}$$

$$\begin{aligned}&(M_{66}+m_{66})\ddot{x}_6(t)+\int_{-\infty}^{t}K_{61}(t-\tau)\dot{x}_1(\tau)d\tau+\int_{-\infty}^{t}K_{62}(t-\tau)\dot{x}_2(\tau)d\tau\\&+\int_{-\infty}^{t}K_{64}(t-\tau)\dot{x}_4(\tau)d\tau+\int_{-\infty}^{t}K_{66}(t-\tau)\dot{x}_6(\tau)d\tau+\\&mx_G(\ddot{x}_2(t)+\dot{x}_6(t)\dot{x}_1(t))=F_{W6}(t)+N_P+N_R+\\&N_{\dot{v}}\ddot{x}_2(t)+N_rU\dot{x}_6(t)+N_vU\dot{x}_2(t)+N_{rr}\dot{x}_6(t)\left|\dot{x}_6(t)\right|\\&+N_{rrv}\dot{x}_6(t)\dot{x}_6(t)\dot{x}_2(t)+N_{vvr}\dot{x}_2(t)\dot{x}_2(t)\dot{x}_6(t)\end{aligned}\tag{6}$$

Where：

M_{kj} mass matrix

X_P X -direction propulsion force

Y_P Y - direction propulsion force

K_P X - direction propulsion moment

N_P Z - direction propulsion moment

$\mathrm{Re}\,s(u)$ ship resistance (we assume: $X_P=\mathrm{Re}\,s(u)$)

X_R X - direction rudder force

Y_R Y - direction rudder force

K_R x - direction rudder moment

N_R z - direction rudder moment

$K_{ij}(t)$ Retardation functions representing memory effect of hydrodynamic forces

C_{ij} restoring coefficients. In case of large amplitude motions, it should be a function of displacement

D_{ij} viscous damping

C_4' nonlinear restoring coefficient

U forward speed of ship

$F_{Wi}(t)$ wave excitation force in i-th motion mode

The above equations (1)~(6) consist of inertia terms, memory effect terms, viscous terms, restoring terms (including nonlinear restoring terms in roll equation), external excitation terms.

The rudder forces are calculated according to the following formulas (Spyrou 1996):

$$\left.\begin{aligned} X_R &= -F_N \sin\delta \\ Y_R &= -(1+\alpha_H)F_N \cos\delta \\ K_R &= -(1+\alpha_H) z_R F_N \cos\delta \\ N_R &= -(1+\alpha_H(x_H/x_R)) x_R F_N \cos\delta \end{aligned}\right\} \tag{7}$$

where F_N is the rudder normal force which can be expressed by:

$$F_N = \frac{1}{2}\rho A_R U_R^2 f(\lambda)\sin\alpha_R \tag{8}$$

α_R is the effective rudder inflow angle

α_H is the rudder-to-hull interaction coefficient

δ is the rudder angle

U_R is the mean rudder inflow velocity

A_R is the rudder area

x_R longitudinal position of rudder

x_H x-coordinate of rudder-to-hull interaction force

$f(\lambda)$ open-water normal rudder force coefficient

2.2 Time Dependent Functions in Time Domain

In equations(1)~(6), besides the parameters such as the inertia matrix and frequency independent added mass and viscous damping, there are several time dependent functions. One is retardation function. We can get them from damping coefficients in frequency domain (Chen, 1996) by Fourier cosine transform:

$$Y_j(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty} H_j(\omega)e^{i\omega\tau}d\omega \tag{9}$$

b_{kj} is the damping coefficient, which is a function of frequency.

The other one is wave excitation. When solving motions of a ship sailing in time domain on given random waves, the time trace of wave exciting forces in each motion mode should be provided.

To calculate the wave forces, we use the method of impulse response function. With the frequency response function of wave excitation in frequency domain in some heading, the corresponding impulse response function $Y_j(\tau)$ can be obtained by Fourier inverse transform (Price & Bishop, 1974):

$$Y_j(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_j(\omega)e^{i\omega\tau}d\omega \tag{10}$$

where $j = 1,\cdots 6$, $Y_j(\tau)$ is the j-th mode impulse response function, $H_j(\omega)$ is the corresponding frequency response function of the wave excitation in frequency domain.

Apart from the impulse response function, the wave trace should also be known. As to random waves, according to the given wave spectrum $S_{\varsigma}(\omega)$, the trace of wave elevation $\varsigma(t)$ can be calculated using Longuet-Higgins model:

$$\varsigma(t) = \sum_{i=1}^{N} H_i \cos(\omega_i t + \delta_i) \tag{11}$$

where $\varsigma(t)$ is the time history of wave elevation, $H_i = \sqrt{2S_{\varsigma}(\omega_i)d\omega}$ is the amplitude of component wave, δ_i is random phase angle.

With impulse response function and wave trace, the time history of wave excitation can be obtained by convolution integral:

$$F_j(t) = \int_{-\infty}^{\infty} \varsigma(t-\tau) Y_j(\tau) d\tau \tag{12}$$

It must be pointed out that the convolution integral are proceeded in both positive and negative directions, which means that the whole time trace of wave must be known in advance, not only the time trace of wave before that instant is known.

3. NUMERICAL CALCULATION

Based on the theory mentioned above, the motion of a ship with forward speed was calculated in time domain as an example. The principal particulars of the ship are listed in table1. The water depth is 10m. The forward speed of the ship is 5m/s and the rudder is in the mid-ship section.

Table 1. particulars of the ship

Length over all	53.45m
Breadth	11.60m
Mean draft	3.6 m
Displacement	1758t
Transverse metacentric height	1.7 m
long metacentric height	40.25m
Roll radius of gyration	3.84 m
Pitch radius of gyration	14.5 m
Yaw radius of gyration	14.5 m
Centre of gravity in x direction	-3.77 m
Centre of gravity in y direction	0.00 m
Centre of gravity above keel	3.25 m

The following figure is grids of the submerged portion of the ship.

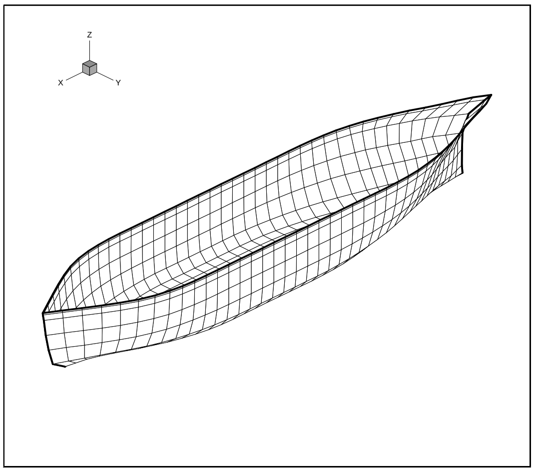

Fig 1 Numerical grids of the ship

3.1 Damping Coefficients

In the following figures, theoretical results of damping coefficients in surge, sway, pitch and yaw are presented. In the calculation, there are irregular frequencies in high frequency range (van Oortmerssen, 1976). Here we almost get rid of them using a method presented by Bao & Kinoshita (1992).

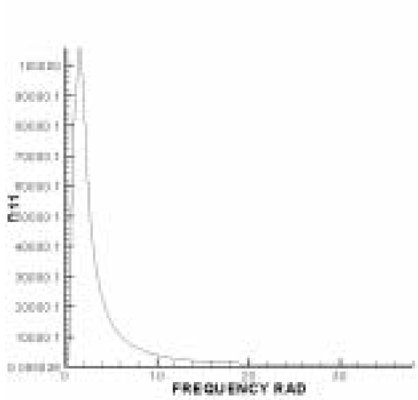

Figure 2 Damping coefficient of surge

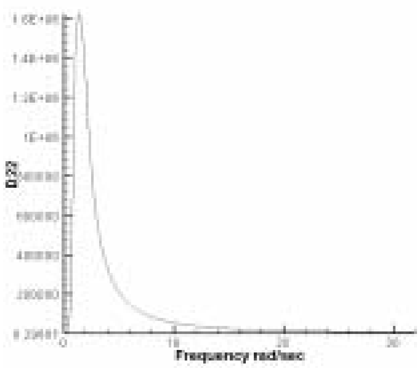

Figure 3 Damping coefficient of sway

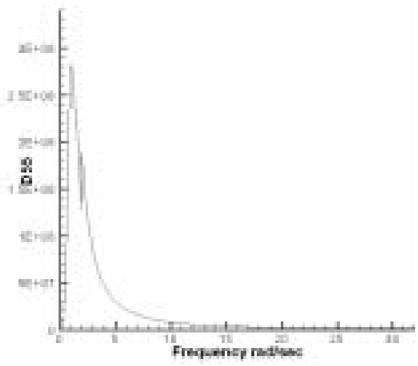

Figure 4 Damping coefficient of pitch

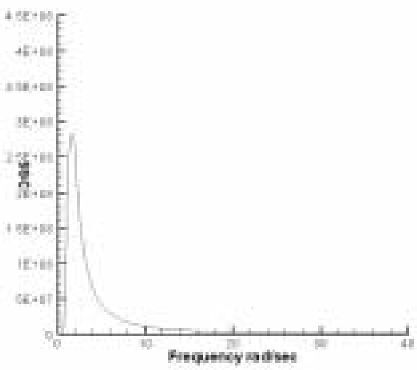

Figure 5 Damping coefficient of yaw

3.2 Retardation Function

The Fig 6~fig 9 are the corresponding retardation functions by using Fourier cosine transform from the damping coefficients in frequency domain.

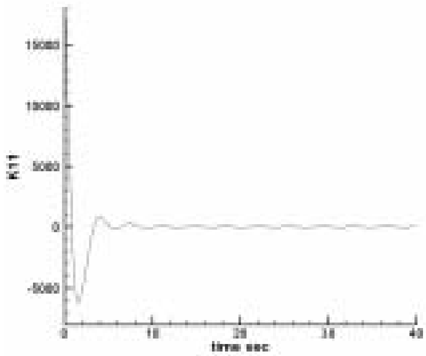

Figure 6 Retardation function in surge

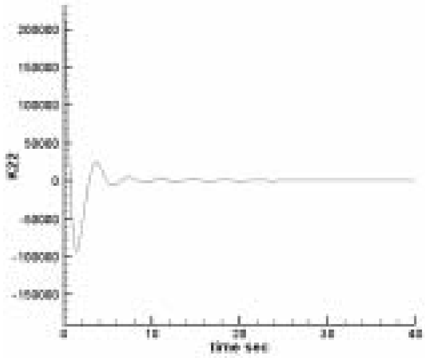

Figure 7 Retardation function in sway

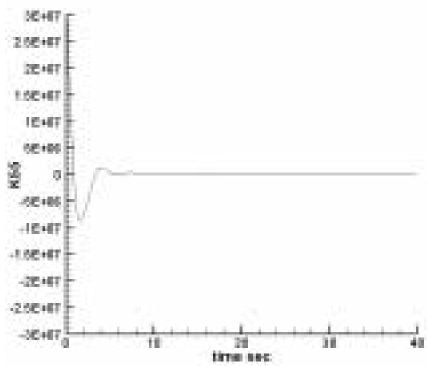

Figure 8 Retardation function in pitch

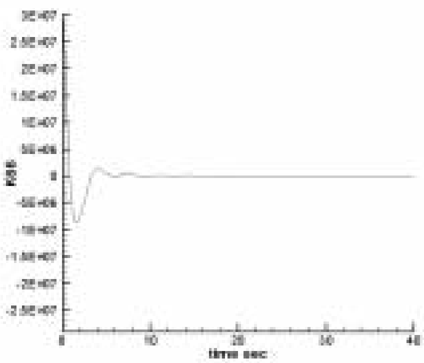

Figure 9 Retardation function in yaw

It can be seen that the lasting time of the memory effect is limited. In ordered to give an impression on the length of such memory effect, we use the following quantity to show the average time length of the memory effect:

$$\tau_{effect} = \int_0^\infty \tau|R(\tau)|d\tau \Big/ \int_0^\infty |R(\tau)|d\tau \tag{13}$$

where $R(\tau)$ is the retardation function, τ_{effect} is the characteristic time to express the memory effect. The following table presents the characteristic time length of memory effect about six motion modes.

Table 2. characteristic time length of the memory effect

Surge	4.916239 s
Sway	3.833640s
Heave	8.964086s
Roll	5.204061s
Pitch	5.296179s
Yaw	4.436357s

We can find that the length about this characteristic time is less than 10s, which is in the same order with all of the motion modes. Considering the maneuvering motion is slow, in this short time range, the sailing angle changes little.

3.3 Impulse Response of Excitation Force

The fig10~fig12 are the impulse response of excitation forces (moments) of the ship under 15 degree for an example.

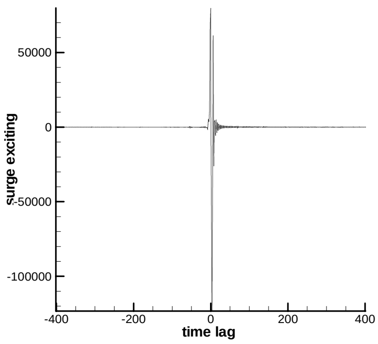

Figure 10 Impulse response of surge excitation

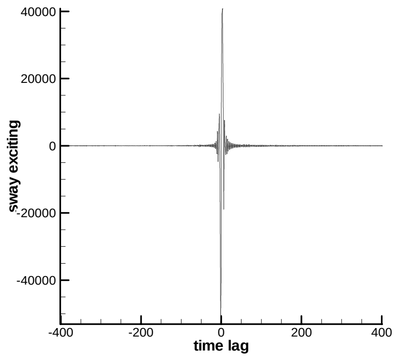

Figure 11 Impulse response of sway excitation

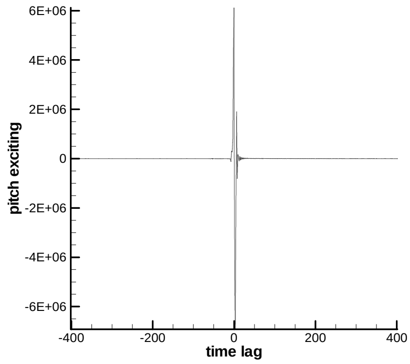

Figure 12 Impulse response of pitch excitation

3.4 Frequency Response Function

With the impulse response of wave excitation and the wave trace, the time history of wave excitation can be obtained by the convolution integral method mentioned above. In order to check the availability of the method, the correlation analysis, with the input trace of wave and output trace of wave excitation, is used to get the frequency response functions of the wave excitation.

Fig13~15 are the corresponding comparison results under 15 degree between the ones got by correlation analysis and original ones got by potential theory.

In the above figures the solid line means the original frequency response function of wave excitation obtained by potential theory in frequency domain. The dots are frequency response function of wave excitation got by the correlation analysis method mentioned above. From the comparison, it can be found that the two results fit well.

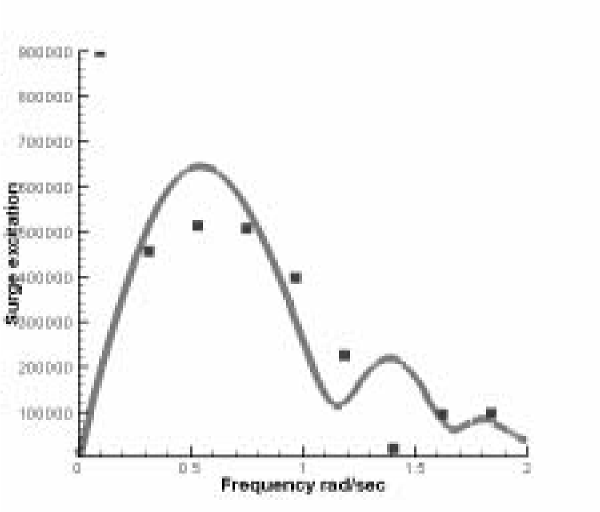

Figure 13 Wave excitation in surge

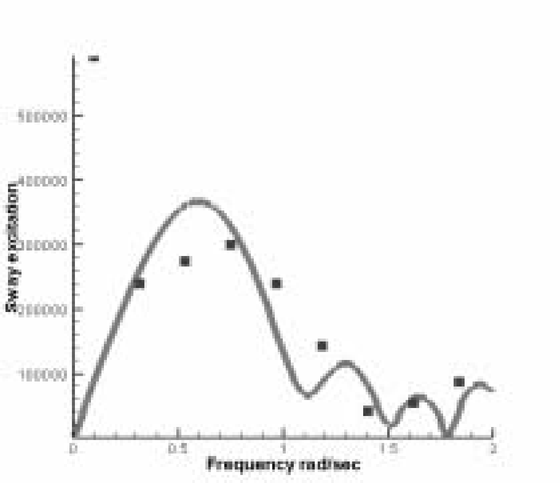

Figure 14 Wave force in sway

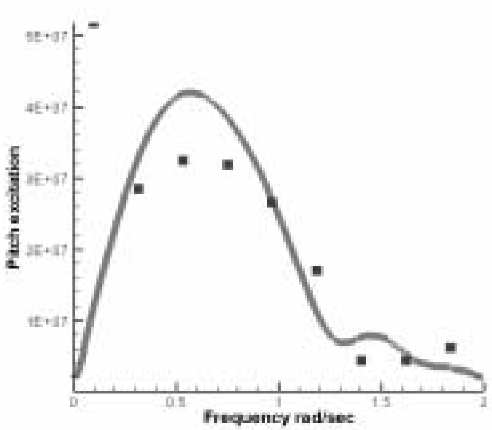

Figure 15 Wave force in pitch

3.5 Time Trace of the Motion Modes

With retardation coefficients and impulse responses of wave excitation, the motion of ship sailing can be simulated. Although our program can handle the case in irregular waves, considering that the results in regular waves may be easy to explain, at present we chose regular waves. There are three wave heights: 1m, 2m and 2.5m with 8 second wave period. The origin wave heading is head sea. In the calculation, some hydrodynamic derivatives are refer to (Matsumoto & Suemitsu 1984, Aoki, Kijima, Nakiri, et al. 2005). The calculated time history for yaw angle and roll angle are shown in fig16-21.

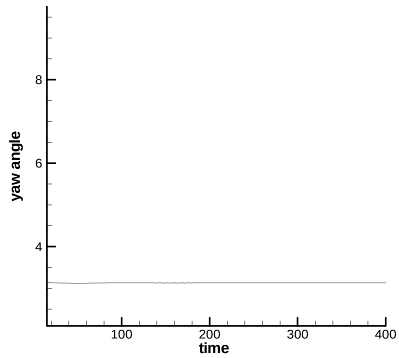

Figure 16 Time histories of yaw (1m)

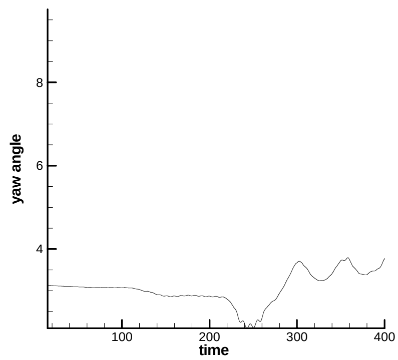

Figure 17 Time histories of yaw (2m)

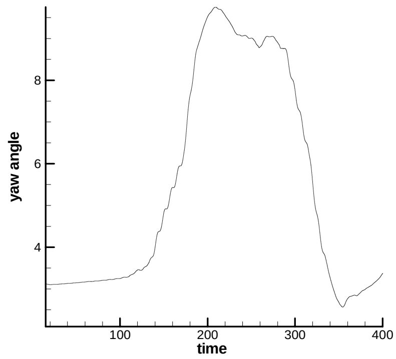

Figure 18 Time histories of yaw (2.5m)

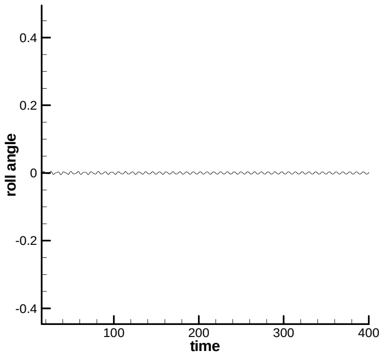

Figure 19 Time histories of roll (1m)

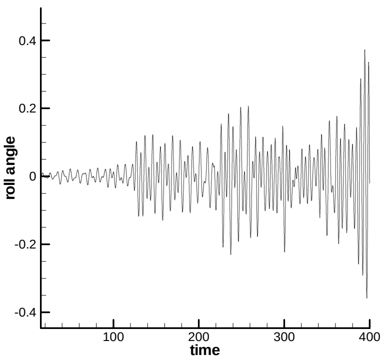

Figure 20 Time histories of roll (2m)

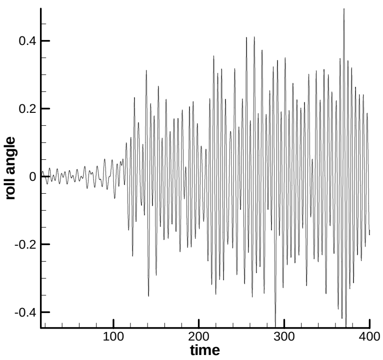

Figure 21 Time histories of roll (2.5m)

4. CONCLUDING REMARKS

In this paper, we do some investigation on the ship sailing on waves in time domain method. Based on the theoretical calculation results about time histories of roll and yaw, several points can be pointed out:

1. The changing of the characteristics of horizontal motion modes in turn will have some effect on roll motion. With the increasing of the wave height, variation of roll angle begins large sooner.

2. The yaw angle may increase dramatically if the wave height increases. When sailing on waves with high wave height, the variation of yaw angle should be noticed. It is possible to induce broaching, which is dangerous and maybe result in capsizing.

In our research, we try to solve the ship sailing in waves. In this paper we consider the variation of wave excitation with the heading changing in different instants. There are still lots of important points left need to be noticed. At present we have no model test results at hand. In the future work we need to at first do some efforts to verify the theoretical results with some corresponding model test.

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