Theoretical Prediction of Broaching in the Light of Local and Global Bifurcation Analysis

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ABSTRACT

For developing design and operational criteria to be used at the International Maritime Organization (IMO), critical conditions for broaching are explored in the light of bifurcation analysis. Since surf-riding, which is prerequisite to broaching, can be regarded as a heteroclinic bifurcation, one of global bifurcations, of a surge-sway-yaw-roll model in quartering waves, the relevant bifurcation condition was mathematically formulated and then a numerical procedure for obtaining its solution was presented with successful example. This identified bifurcation condition was compared with direct numerical simulation in time domain. As a result, it was confirmed that the heteroclinic bifurcation provides a boundary between motions periodically overtaken by waves and non-periodic motions such as surf-riding, broaching and so on. Then a local bifurcation analysis was applied to the surf-riding equilibria. This results could explain a boundary between stable surf-riding and oscillatory surf-riding as a Hopf bifurcation. Furthermore, comparison with free-running model experiments shows some discrepancies and an improvement with an aid of nonlinearity in wave-induced surge force is presented.

Keywords: *broaching, surf-riding, capsizing, heteroclinic bifurcation, Hopf bifurcation, intact stability*

1. INTRODUCTION

Broaching is one of the three major capsizing scenarios that the new performance-oriented stability criteria to be added to the Intact Stability Code at the International Maritime Organization (IMO) are requested to cover. (Germany, 2005) This is a phenomenon that a ship cannot keep a constant course despite the maximum steering effort and the centrifugal force due to this uncontrollable yaw motion could result in capsizing. This phenomenon often occurs when a ship runs in following and quartering seas with relatively high forward speed, especially when a ship is surf-ridden. Thus, this phenomenon is relevant to ships having their Froude number of 0.3 or above, such as destroyers, high-speed RoPax ferries, fishing vessels and so on.

For avoiding this phenomenon, currently the guidance to the master for avoiding danger in following and quartering seas (MSC/Circ. 707) provide an operational criterion for preventing from surf-riding, which is a prerequisite to broaching. This criterion was developed with a phase plane analysis of an uncoupled surge model in pure following seas. (Umeda, 1990) For accurately determining the surf-riding threshold, numerical simulation for obtaining a global picture of surf-riding should be systematically repeated. This is because the occurrence of surf-riding can be regarded as a heteroclinic bifurcation of a nonlinear mathematical model (Umeda, 1999).

It is important to reduce such computational efforts for developing operational or design criterion applicable to individual ships. For this purpose, Ananiev (1966) developed an approximated analytical method, Spyrou (2001) did an exact analytical method of a simplified model and Umeda et al. (2004) did a geometric method, which can identify the heteroclinic bifurcation point with the Newton method.

On the other hand, once broaching occurs, a ship has heading angle from wave direction. This means coupling with a manoeuvring motion in quartering waves is essential. Thus, a surge-sway-yaw-roll model is required to identify the threshold. For this purpose, the geometric method was applied to the manoeuvring mathematical model. So far the authors had already developed a manoeuvring model with linear wave forces and qualitatively validated it with model experiments. (Umeda & Hashimoto, 2002). The major difficulty arises here is the increase of dimensions of state vector describing this four degrees-of-freedom (DOF) model. This requires us to upgrade bifurcation analysis in a phase plane to that of a vector field. Therefore, the authors attempted to develop such a new methodology, as briefly introduced by Umeda et al. (2005). In this paper, more details are described and the numerical example here demonstrates its applicability and limitation and then an improvement is provided.

2. MATHEMATICAL MODEL

The mathematical model used in this paper is a manoeuvring model of the surge-sway-yaw-roll motion developed for prediction of broaching associated with surf-riding in following and quartering waves. (Umeda, 1999) In cases of ship runs with higher forward velocity in following and quartering waves, the encounter frequency becomes much smaller than the natural frequencies in heave and pitch. Therefore these motions were estimated by simply tracing their stable equilibrium.

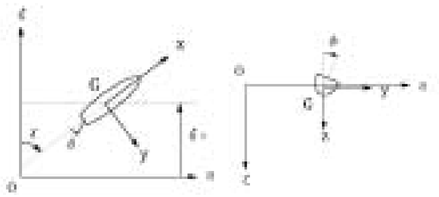

Figure 1 Cooordinate systems

As can be seen in Fig.1, two coordinate systems are used: (1) a wave fixed with its origin at a wave trough, the ξ axis in the direction of wave travel; and (2) an upright body fixed with its origin at the centre of ship gravity, with the x axis pointing toward the bow, the y axis to starboard, and the z axis downward. The state vector, $\mathbf{x}$, and control vector, $\mathbf{b}$, of this system are defined as follows:

$$\mathbf{x} = (x_1, x_2, \cdots, x_8)^T = \{\xi_G / \lambda, u, v, \chi, r, \phi, p, \delta\}^T \quad (1)$$

$$\mathbf{b} = \{n, \chi_c\}^T \quad (2)$$

The dynamical system can be represented by the following state equation:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x};\mathbf{b}) = \{f_1(\mathbf{x};\mathbf{b}), f_2(\mathbf{x};\mathbf{b}), \cdots, f_8(\mathbf{x};\mathbf{b})\}^T \quad (3)$$

where

$$f_1(\mathbf{x};\mathbf{b}) = \{u\cos\chi - v\sin\chi - c\}/\lambda \quad (4)$$

$$f_2(\mathbf{x};\mathbf{b}) = (T(u;n) - R(u) + X_w(\xi_G/\lambda, \chi)) / (m + m_x) \quad (5)$$

$$f_3(\mathbf{x};\mathbf{b}) = (-(m + m_x)ur + Y_v(u;n)v + Y_r(u;n)r + Y_\phi(u)\phi + Y_\delta(u;n)\delta + Y_w(\xi_G/\lambda, u, \chi; n))/(m + m_y) \quad (6)$$

$$f_4(\mathbf{x};\mathbf{b}) = r \quad (7)$$

$$f_5(\mathbf{x;b}) = (N_v(u;n)v + N_r(u;n)r + N_\phi(u)\phi + N_\delta(u;n)\delta + N_w(\xi_G/\lambda, u, \chi; n) \,/(I_{ZZ} + J_{ZZ}) \tag{8}$$

$$f_6(\mathbf{x;b}) = p \tag{9}$$

$$f_7(\mathbf{x;b}) = (m_x z_H ur + K_v(u;n)v + K_r(u;n)r + K_P(u)p + K_\phi(u)\phi + K_\delta(u;n)\delta + K_w(\xi_G/\lambda, u, \chi; n) + mgGZ(\phi)) \,/(I_{xx} + J_{xx}) \tag{10}$$

$$f_8(\mathbf{x;b}) = \{-\delta - K_R(\chi - \chi_C) - K_R T_D r\}/T_E \tag{11}$$

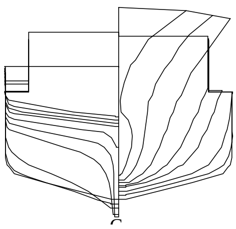

Figure 2 Body plan of the subject ship

Table.1 Principal particulars of the ship

Items	Values
length: L_{BP}	34.5 m
breadth: B	7.60 m
depth: D	3.07 m
draught at FP: d_f	2.50 m
mean draught: d_m	2.65 m
draught at AP: d_a	2.80 m
block coefficient: C_b	0.597
metacentric height: GM	1.00 m
pitch radius of gyration: κ_{yy}/L_{BP}	0.302
l.c.b. (aft)	1.31 m
rudder aspect ratio	1.84
time constant for steering gear: T_E	0.63 s
rudder gain: K_R	1.0
time constant for differential control: T_D	0.0 s

Based on the above-mentioned mathematical model, numerical calculations were carried out for a 135GT Japanese purse seiner used as a subject ship of the ITTC benchmark testing. (Umeda et al., 2001) Principal particulars and body plan are shown in Table1 and Figure 2, respectively. Hydrodynamic coefficients and other relating coefficients can be found in the literature. (Umeda and Hashimoto., 2002)

3. HETEROCLINIC BIFURCATION

A nonlinear dynamical system described by Eq. (3) could have fixed points,

$$\bar{\mathbf{x}} = (\bar{\xi}_G/\lambda, \bar{u}, \bar{v}, \bar{\chi}, \bar{r}, \bar{\phi}, \bar{p}, \bar{\delta}) \tag{12}$$

where

$$\mathbf{F}(\bar{\mathbf{x}} : \mathbf{b}) = \mathbf{0} \tag{13}$$

These fixed points correspond to surf-riding, under which a ship runs with a regular wave train. $\mathbf{F}(\mathbf{x};\mathbf{b})$ is linearised at $\bar{\mathbf{x}}$ 、 putting $\mathbf{x} = \bar{\mathbf{x}} + \mathbf{y}$ to obtained following equation:

$$\dot{\mathbf{y}} = \mathrm{D}\mathbf{F}(\bar{\mathbf{x}};\mathbf{b})\mathbf{y} \tag{14}$$

where

$$\mathrm{D}\mathbf{F}(\mathbf{x} : \mathbf{b}) = \partial/\partial x_j \, f_i(\mathbf{x} : \mathbf{b}) \tag{15}$$

If an eigenvalue of $\mathrm{D}\mathbf{F}(\bar{\mathbf{x}};\mathbf{b})$, λ_i, which is obtained by

$$[\mathrm{D}\mathbf{F}(\mathbf{x};\mathbf{b}) - \lambda_i]\mathbf{y} = 0 \tag{16}$$

has a positive real part, local asymptotic behaviour at $\bar{\mathbf{x}}$ is unstable.

Hartman's theorem and the stable manifold theorem (Guckenheimer & Holmes, 1983) enable us to investigate the local topological structure of the system by Eq. (3). That is, there exist local stable and unstable maniufolds, $W^S_{loc}(\bar{\mathbf{x}} : \mathbf{b})$ and $W^U_{loc}(\bar{\mathbf{x}} : \mathbf{b})$, tangent to eigenspaces, spanned by $\mathrm{D}\mathbf{F}(\bar{\mathbf{x}};\mathbf{b})$ at $\bar{\mathbf{x}}$. Then the global stable and unstable manifolds W^S and W^U are obtained by letting points in $W_{loc}{}^S$ flow backward in time and those in W_{loc}^U flow forward.

The numerical survey for the system described by Eq. (3) applied to the subject ship (Umeda, 1999) indicates that there is normally one fixed point having only one eigenvalue having a positive real part, λ_1, if a fixed point exists. Thus, such fixed point has a 1-dimensional unstable invariant manifold and a 7-dimensional stable invariant manifold. A heteroclinic bifurcation requires that W^U of a fixed point is connected to W^S of other fixed point. Although calculation of W^S is not easy, W^U is easily calculated as a trajectory, which is obtained by numerically integrating Eq. (3) from the fixed point with small perturbation, δ_1, for the direction of eigenvector as follows:

$$\mathbf{x}(t) = \varphi(t, \mathbf{x}_\alpha; b) \tag{17}$$

where

$$[\mathbf{x}_\alpha - \bar{\mathbf{x}}]^T [\mathbf{x}_\alpha - \bar{\mathbf{x}}] = \delta_1^2 \tag{18}$$

Then , if we find $\mathbf{b_0}$ satisfying the following relationship (Kawakami et al., 1997):

$$[\mathrm{D}\mathbf{F}^T(\bar{\mathbf{x}}^*;\mathbf{b_0}) - \lambda_1 \mathbf{I}]h = 0 \tag{19}$$

$$\mathbf{h}^T[\mathbf{x}_\omega{}^* - \bar{\mathbf{x}}^*] = 0 \tag{20}$$

$$[\mathbf{x}_\omega{}^* - \bar{\mathbf{x}}^*]^T [\mathbf{x}_\omega{}^* - \bar{\mathbf{x}}^*] - \delta_2^2 = 0 \tag{21}$$

$$\bar{\mathbf{x}}^* = \{(\bar{\xi}_G / \lambda - 1), \bar{u}, \bar{v}, \bar{\chi}, \bar{r}, \bar{\phi}, \bar{p}, \bar{\delta}\} \tag{22}$$

$$\varphi(T, \mathbf{x}_\alpha; \mathbf{b_0}) - \varphi(-T, \mathbf{x}_\omega{}^*; \mathbf{b_0}) = \mathbf{0} \tag{23}$$

$$\delta_2 << 1 \tag{24}$$

this is a heteroclinic bifurcation point.

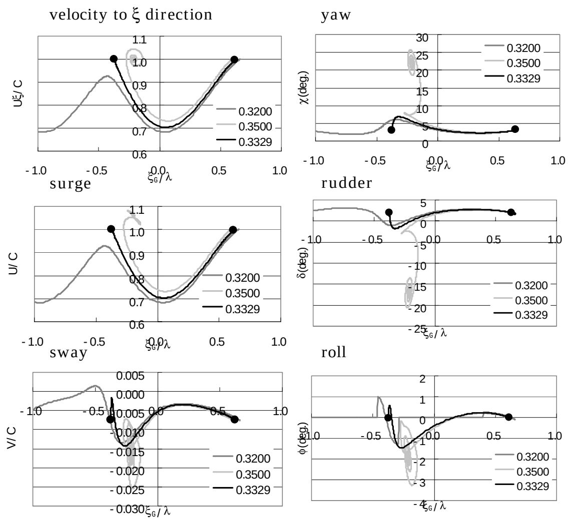

Figure 3 An example of the heteroclinic bifurcation under the wave steepness of 0.05, the wave length to ship length ratio of 1.0 and the auto pilot course of 5 degrees.

In this paper, the above equation set was numerically solved by the Newton method. A numerical example is shown in Figure 3. here the wave steepness is 0.05 and the wave length to the ship length ratio is 1.0. In this case the obtained heteroclinic bifurcation point is the nominal Froude number, Fn, of 0.3329 for the autopilot course of 5 degrees from the wave direction. Below this value the ship is overtaken by waves and above this value the ship is captured by a wave downslope.

This method was applied to different auto pilot courses and wave conditions and then the results are compared with numerical results obtained from time series based on sudden change concept as shown in Figure 4. Here the initial state for the sudden change concept is fixed with a periodic state under Fn=0.1 and χ_c=0 degrees and its computational time is 1000 seconds. The time series were categorised into periodic motions, surf-riding, broaching

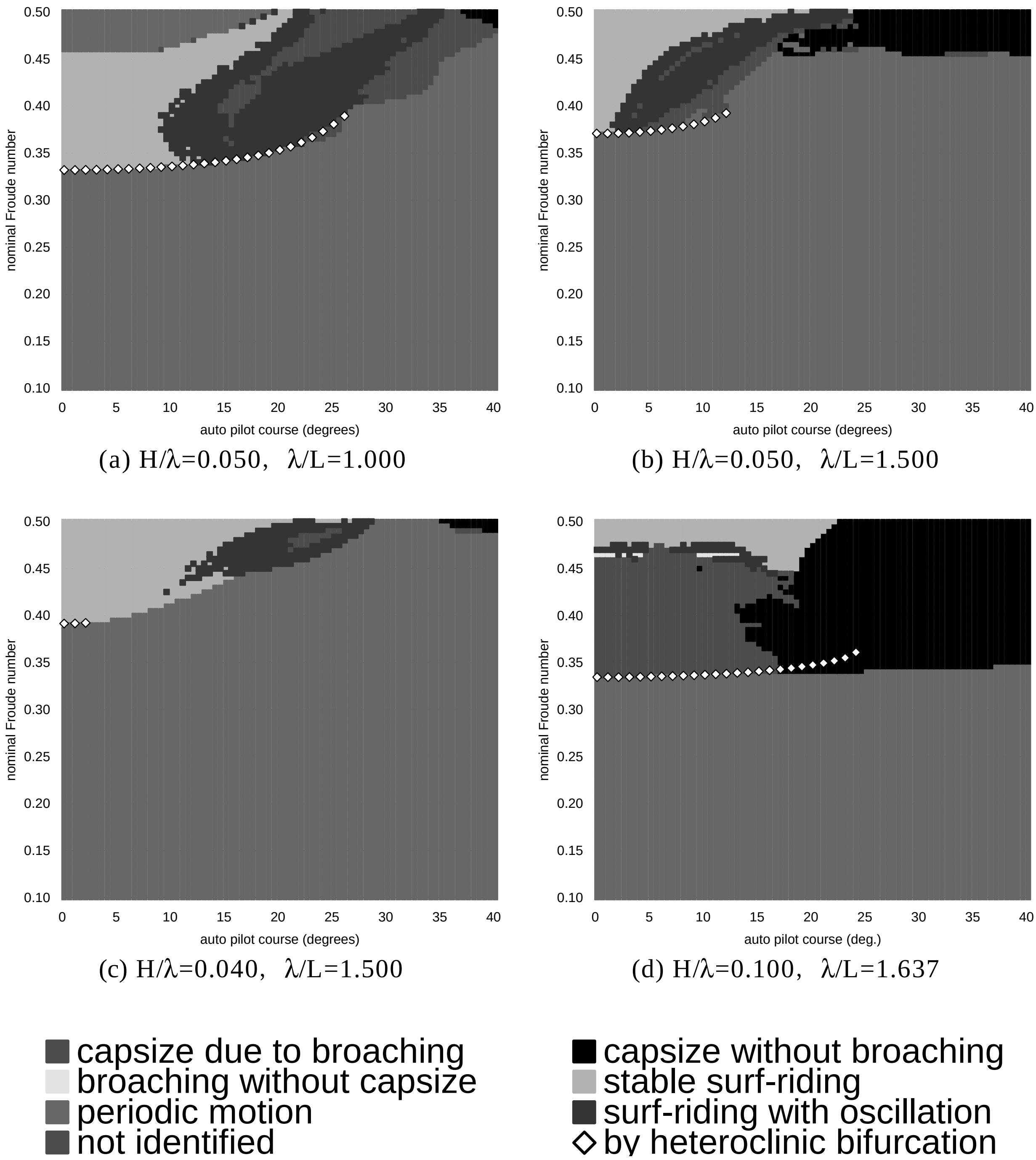

Figure 4 Numerical simulation and Hetero-clinic bifurcations

and capsizing with judging criteria (Umeda & Hashimoto, 2002). The heteroclinic bifurcation points obtained by the present method reasonably well predict the boundary between the periodic motions overtaken by waves and other motions such as surf-riding at least for smaller auto pilot course from the wave direction. Therefore, the present method for identifying a surf-riding threshold can be used as an alternative to time-consuming numerical simulation. When the auto pilot course increases, the surf-riding threshold also increases. This is comparable to the current MSC/Circ. 707. When the wave steepness increases, the surf-riding threshold decreases because of the increase of wave-induced surge force. For the wave steepness is 0.05 or below, stable or oscillatory surf-riding occurs above the heteroclinic bifurcation points. For much larger wave steepness, such as 0.1, broaching and/or capsizing occur. This is because broaching could occur once surf-riding happens under such wave condition. Thus, the heteroclinic bifurcation can be used as a threshold for broaching. However, it is noteworthy that the heteroclic bifurcation does not distinguish broaching from surf-riding.

Although the sudden change concept used here for the numerical simulation is designed to minimise the initial-value dependence, small disagreement between the bifurcation and the surf-riding threshold could be explained as the initial-value dependence. (Umeda, 1999) In the case of larger wave steepness, a heteroclinic connection could occur beyond more than one wave length in a special case. And periodic broaching could occur in the very limited region. In addition, the existing range of heteroclinic bifurcation may depend on a sweeping direction for providing the initial value of the Newton method. Thus, these should be further investigated in future.

4. LOCAL BIFURCATION

To investigate the ship behaviour above the heteroclinic bifurcation, local bifurcation analysis on fixed points were carried out. Here the eigenvalues of locally-linearised system at all fixed points were calculated as shown in Figures 5-6.

For smaller wave steepness, the region where fixed points exist is slightly larger than the region above the heteroclinic bifurcation. When the auto pilot course increases, eigenvalues having non-zero imaginary part appear. This can be regarded as the Hopf bifurcation. The numerical simulation also

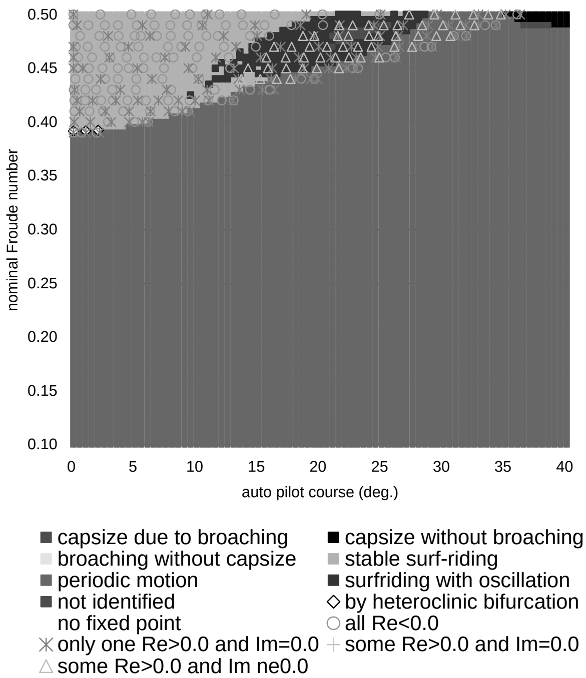

Figure 5 Numerical simulation and Eigenvalues of fixed points with H/λ=0.040, λ/L=1.500

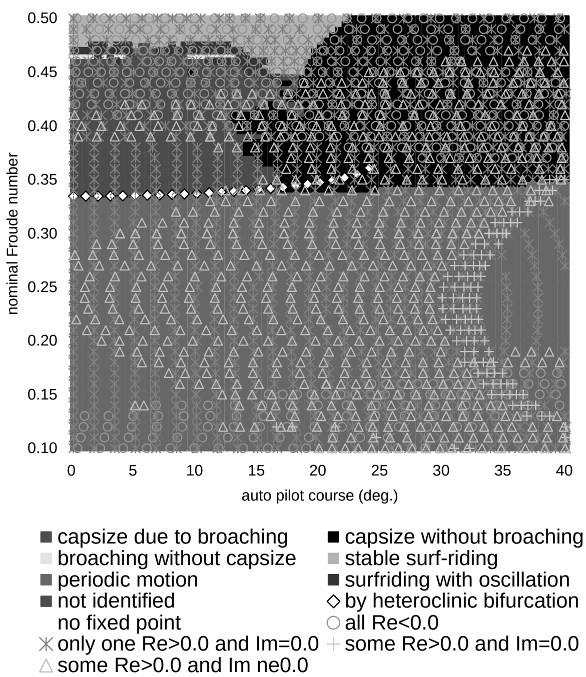

Figure 6 Numerical simulation and Eigenvalues of fixed points with H/λ=0.100, λ/L=1.637

provides the region of oscillatory surf-riding, which is almost inside the zone of the fixed points whose eigenvalues have non-zero imaginary parts. This suggests that the Hopf bifurcation results in limit cycle around unstable surf-riding. This phenomenon was discussed by Spyrou (1995).

For larger wave steepness, the region that fixed points exist is enlarged to cover all explored region. In the region of stable surf-riding, which is identified with the numerical simulation, fixed points whose eigenvalues have no positive real parts can be found so that stable surf-riding can exist. Regarding capsizing due to broaching or capsizing without broaching, a fixed point of which a eigenvalue have a positive real part can be found for each operational condition but it is not sufficient to distinguish these phenomena. In general, the local bifurcation analysis can provide prerequisite for dangerous phenomena but can identify their sufficient conditions. This is because a trajectory does not always approach to fixed points.

5. COMPARISON WITH EXPERIMENT AND IMPROVEMENT

So far the prediction of heteroclinic bifurcation was successfully validated with numerical simulation. As a next step, the calculated heteroclinic bifurcation was compared with existing free-running model experiments for the subject ship (Umeda et al., 1999) In the experiment periodic motions, stable surf-riding, broaching and capsizing were observed. The comparisons are shown in Figure 7. Here the heteroclinic bifurcation (A) indicates that from the above mentioned method, and overestimates danger. The measured periodic motions overtaken by waves can be found even above the heteroclinic bifurcation (A). It can be presumed that this is because the accuracy of mathematical modelling of the motions is insufficient. After proposing the mathematical model described in Eq. (3), the authors have continued their effort to improve it by utilising captive model tests and hydrodynamic modelling. As a result, Hashimoto et al. (2004B) proposed an improved mathematical model for

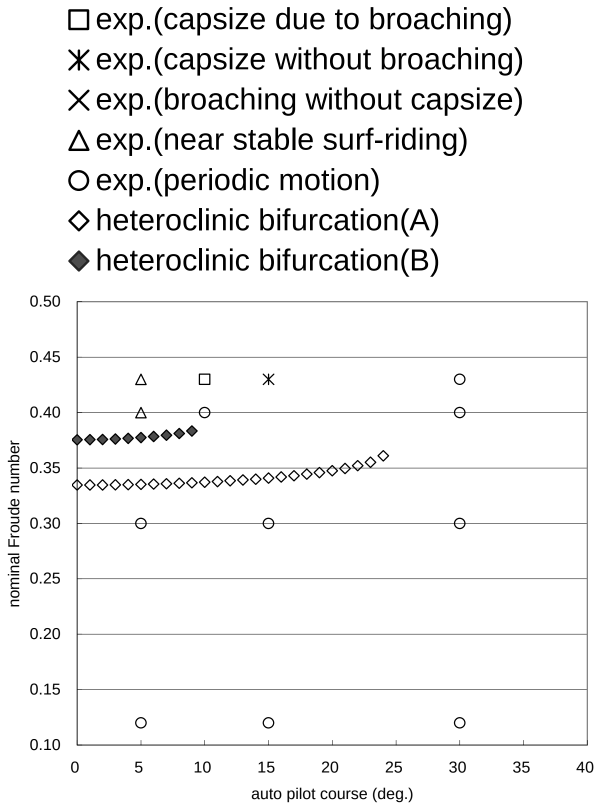

Figure 7 Comparison between the calculated heteroclinic bifurcation and free-running model experiment. Here the heteroclic bifurcations (A) and (B) are based on Eq. (3) and an improved mathematical model, respectively.

quantitatively predicting broaching. The crucial factors are nonlinear hull manoeuvring forces,wave effect on linear manoeuvring forces, wave effect on rudder forces, wave effect on restoring moment, wave effect on propeller thrust, nonlinearity of wave-induced surge force, nonlinear coupling effect between sway and roll, heel-dependent nonlinear hydrodynamic forces in calm water. Among them, nonlinearity of wave-induced surge force is identified as the main cause of discrepancy in surf-riding threshold, which can be obtained from a captive model test in following waves. (Hashimoto et al., 2004A) Therefore, in this paper, these experimental data were incorporated into the mathematical model and then the above mentioned technique for estimating heteroclinic bifurcation was applied. The calculated results are also plotted in Figure 7 as the heteroclinic bifurcation (B). This new results improves agreement between the experiment and calculation significantly. Because of nonlinear relationship between the wave-induced surge force and wave steepness, the wave-induced surge force becomes smaller than that from a linear theory. As a result, the nominal speed of heteroclinic bifurcation increases. Thus, it is also important to utilise accurate but still practical hydrodynamic modelling for correctly estimate broaching and capsizing. The authors (Hashimoto and Umeda, 2005) proposed a mathematical model as an candidate. It is desirable to incorporate it into the global bifurcation analysis, and is a future task.

6. CONCLUSIONS

This paper presents a numerical method for estimating the heteroclinic bifurcation of the surge-sway-yaw-roll model in quartering waves, which can be regarded as a threshold for surf-riding and/or broaching. Numerical examples are reasonably well compared with numerical simulation from some initial value sets. Hydrodynamic modelling in wave-induced surge force was improved with a captive model test data so that sufficient agreement with the free-running model experiments was realised. In addition, the existence of the Hopf bifurcation, which could result in oscillatory surf-riding, was confirmed with the bifurcation analysis on fixed points.

7. ACKNOWLEDGEMENTS

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9. NOMENCLATURE

c	wave celerity
F_n	nominal Froude number
g	gravitational acceleration
GZ	righting arm
H	wave height
I_{xx}	moment of inertia in roll
I_{zz}	moment of inertia in yaw
J_{xx}	added moment of inertia in roll
J_{zz}	added moment of inertia in yaw
K_p	derivative of roll moment with respect to roll rate
K_r	derivative of roll moment with respect to yaw rate
K_R	rudder gain
K_T	thrust coefficient of propeller
K_v	derivative of roll moment with respect to sway velocity
K_w	wave-induced roll moment
K_δ	derivative of roll moment with respect to rudder angle
K_ϕ	derivative of roll moment with respect to roll angle
L	ship length between perpendiculars
m	ship mass
m_x	added mass in surge
m_y	added mass in sway
n	propeller revolution number
N_r	derivative of yaw moment with respect to yaw rate
N_v	derivative of yaw moment with respect to sway velocity
N_w	wave-induced yaw rate
N_δ	derivative of yaw moment with respect to rudder angle
N_ϕ	derivative of yaw moment with respect to roll angle
p	roll rate
r	yaw rate
R	ship resistance
t	time
T	propeller thrust

T_D	time constant for differential control
T_E	time constant for steering gear
u	surge velocity
v	sway velocity
X_w	wave-induced surge force
Y_r	derivative of sway force with respect to yaw rate
Y_v	derivative of sway force with respect to sway velocity
Y_w	wave-induced sway force
Y_δ	derivative of sway force with respect to rudder angle
Y_ϕ	derivative of sway force with respect to roll angle
z_H	vertical position of centre of sway force due to lateral motions
δ	rudder angle
λ	wave length
ξ_G	longitudinal position of centre of gravity
ϕ	roll angle
χ	heading angle from wave direction
χ_c	desired heading angle for auto pilot