Effect of Sway and Yaw Coupling on the Prediction of Resonant Roll Motions

Nuno Fonseca, *Unit of Marine Technology and Engineering, Technical University of Lisbon*

Carlos Guedes Soares, *Unit of Marine Technology and Engineering, Technical University of Lisbon*

ABSTRACT

Large roll motions are dominated by nonlinear effects arising from different sources, which results into a highly complex problem to calculate numerically. For this reason many researchers chose to uncouple the roll from the other modes of motion, while representing as accurately as possible the nonlinear effects on the restoring and damping terms. This paper investigates the consequences of neglecting, or of simplifying, the linear coupling of roll with the horizontal motions. The coupling is related to inertial and hydrodynamic effects that are felt when the line of action of the related forces in sway and yaw do not pass through the origin of the coordinate system.

Keywords: *Roll motion, coupling effects, horizontal motions.*

1. INTRODUCTION

Rolling is often the critical motion response of the ship in waves. This is because the damping factor in roll is usually very small, thus large dynamic amplifications occur when the ship encounters waves at a frequency close to the natural frequency. It is not unusual to obtain a roll transfer function with an amplification factor of around 10, meaning that for the natural frequency the ship will roll with an amplitude 10 times larger than the wave steepness.

When large roll amplitudes are achieved, then the problem is dominated by nonlinear effects arising from different sources. The most important nonlinear effects are related to the hydrostatic restoring moment and viscous roll damping. Although depending on the hull form, one may say that the roll restoring moment is approximately linear up to around 10 degrees of roll. For large amplitude angles, the restoring moment may either be fitted to a polynomial curve, or calculated by direct integration of hydrostatic pressure.

Regarding roll damping, inviscid wave making damping is relatively small and usually the viscous damping dominates. For this reason the damping moment is inherently nonlinear. One possibility is to represent it by a linear plus a quadratic term on the roll velocity. Anyway, since the damping coefficient is small, it is very important to be correctly estimated in order to obtain accurate response amplitude in resonant conditions. Presently it is not possible to determine analytically the viscous roll damping, but a few semi-empirical methods have been suggested to overcome this problem. Probably the most referred is the one proposed by IIkeda et al. (1978).

Linearization of the damping coefficient is possible by assuming that the linearized damping dissipates the same amount of energy as the nonlinear damping over one cycle of the motion (Loyd, 1989). However, besides being a simplification of the problem, the damping coefficient will depend on the roll amplitude, which leaves open the question of choosing the appropriate value in irregular seas.

The consistent way to calculate the large amplitude roll motion is to solve the equations of motion including the appropriate nonlinear terms. However this is a highly complex problem to solve numerically. For this reason several researchers chose to uncouple the roll from the other modes of motion, while representing as accurately as possible the nonlinear effects on the restoring and damping terms. A few examples are described below.

Peyton Jones and Çankaya (1997) applied the harmonic balance method to the uncoupled roll motion equation with cubic stiffness and angle dependent cubic damping to efficiently calculate the maximum roll amplitudes in regular beam waves. The case of sinusoidal beam waves is also investigated by Taylan (1999) who applied an asymptotic method in the time domain to solve the nonlinear roll motion equation.

Nonlinear rolling in random seas has also investigated assuming the uncoupled roll equation. Armand and Duthoit (1991) applied the linearize-and-match method to obtain response statistics for the nonlinear roll of ships. Jiang et al. (1996) studied the probability of ship capsizing due to excessive roll motion in random beam waves, including the effects of a bias on the vessel roll.

The uncoupled analysis permits to enhance the understanding of the roll motion and has certainly led to the identification of various peculiar features associated to the nonlinear behaviour, such as the jump phenomenon, multiple solutions, sub-harmonic and super-harmonic resonance, chaotic behaviour, etc. However, in many cases the coupling effects with the horizontal motions have significant effects and should not be neglected or simplified.

Some authors, acknowledging that the coupling effects should be considered use a procedure to decouple roll from sway which is based on choosing the proper origin for the coordinate system. This point would be the so called “roll centre”. The procedure has been used, for example, by Jons et al. (1987) and Hutchison (1991).

This paper investigates the consequences of simplifying the roll coupling with the sway and yaw modes of motion. Coupling with the horizontal motions is related to inertial effects, if the origin of the centre of the coordinate system is not at the centre of gravity, and it is related also to hydrodynamic effects since the line of action of the hydrodynamic forces in sway and yaw does not pass, in general, through the origin of the coordinate system. These effects are assessed by comparing motion responses of the coupled and of the uncoupled roll.

This study is an initial phase of a wider investigation on the effects of roll motion on the prediction of wave induced loads on ship structures.

2. COUPLED EQUATIONS OF SWAY-ROLL-YAW MOTIONS

The problem of ship motions in waves is very complex and therefore, with the objective of obtaining practical numerical solutions, formulations have been developed which are based on several levels of simplifications. The first hypothesis is of an ideal fluid and therefore the flow is represented by a velocity potential function.

Then further assumptions include: the ship is slender, the amplitudes of waves and of motions are small, and the speed of the ship is relatively small. One of the consequences of the former assumptions is that the linear velocity potential can be decomposed into several independent components that, through application of linear Bernoulli equation, lead to independent hydrodynamic forces components. The hydrodynamic forces are: exciting forces due to the wave field incident on the hull, radiation forces associated to each of the forced modes of rigid body motion, and hydrostatic forces due to hydrostatic pressure.

Regarding the radiation forces, since they are decomposed into contributions from each of the six degrees of freedom, this means that the force along any of the six directions of the coordinate system (3 translation and 3 rotations) is made of six components. Each radiation force is coupled with all modes of motion. However, because forced motions are of small amplitude, the hull sides are assumed nearly vertical around the water line, and ship hulls are symmetric about a longitudinal vertical plane passing through the centre of gravity, then most of the radiation force couplings vanish. Finally radiation forces become represented in terms of added masses and damping coefficients which multiply respectively by accelerations and velocities of the forced motions.

Hydrostatic forces combine with ship weight forces resulting into the restoring forces, which are proportional to restoring coefficients and to the displacements. Reasoning similar to the one done for the radiation forces can be done leading to the conclusion that the only nonzero restoring forces are for heave, roll and pitch, and that couplings exist only between heave and pitch.

The differential equations of motion combine hydrodynamic forces with ship mass inertial forces. If the angular motions are of small amplitude, then the inertial forces are represented in terms of mass coefficients that multiply accelerations. If the weight distribution is symmetric with respect to the longitudinal vertical plane of geometric symmetry, then the mass inertial coupling is restricted to roll-yaw only, and the related values are usually very small. Additional terms may appear if the origin of the coordinate system is not coincident with the centre of gravity of the ship.

One of the consequences of all the simplifications presented before is that the resulting six degrees of freedom equations of motions can be separated into two groups of three coupled equations. One of groups include sway, roll and yaw equations given by:

$$\begin{cases} [M + A_{22}(\omega)]\ddot{\xi}_2 + B_{22}(\omega)\dot{\xi}_2 \\ + [A_{24}(\omega) - Mz_G]\ddot{\xi}_4 + B_{24}\dot{\xi}_4 \\ + A_{26}(\omega)\ddot{\xi}_6 + B_{26}(\omega)\dot{\xi}_6 = F_2^E(t) \\ \\ [A_{24}(\omega) - Mz_G]\ddot{\xi}_2 + B_{42}(\omega)\dot{\xi}_2 \\ + [A_{24}(\omega) + I_{44}]\ddot{\xi}_4 + B_{44}\dot{\xi}_4 + C_{44}\xi_4 \\ + [A_{46}(\omega) - I_{46}]\ddot{\xi}_6 + B_{46}(\omega)\dot{\xi}_6 = F_4^E(t) \\ \\ A_{62}\ddot{\xi}_2 + B_{62}(\omega)\dot{\xi}_2 + [A_{64}(\omega) - I_{64}]\ddot{\xi}_4 \\ + B_{64}\dot{\xi}_4 + [A_{66}(\omega) + I_{66}]\ddot{\xi}_6 \\ + B_{66}(\omega)\dot{\xi}_6 = F_6^E(t) \end{cases} \tag{1}$$

In equations (1): ξ_j, $j = 2,4,6$ represent the sway, roll and yaw motions, the dots over the symbols stand for differentiation with respect to time, A_{kj} and B_{kj} are the added mass and damping coefficients, M is the ship mass, Z_G is distance of the centre of gravity with respect to the origin of the coordinate system (both points are on the same vertical), I_{kj} are the ship mass moments of inertia and F_k^E are the exciting forces induced by the waves.

The seakeeping calculations performed here are based on a frequency domain strip method (Salvesen et al., 1970), which means that all potential flow hydrodynamic coefficients and exciting forces were obtained applying strip theory formulas. The exception is the roll damping coefficients, whose values have been estimated from model tests.

3. RESULTS IN REGULAR WAVES

3.1 Characteristics of Tested Ships

The importance of the coupling effects of sway and yaw motions into roll are analysed for two different ships. The first is the well known S175 containership, previously investigated by several researchers and institutions (see for example ITTC, 1987). The second ship is a Roll On Roll Off passenger ferry, which was used for a comparative study carried out by the 24th ITTC Stability Committee.

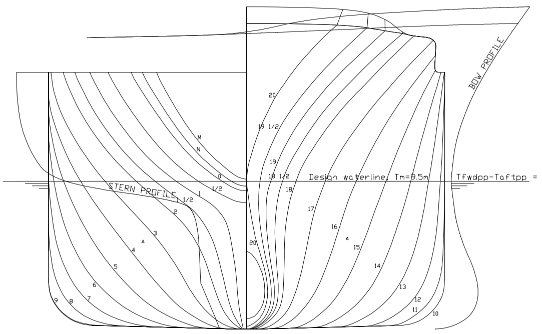

Figure 1 Bodylines of the S-175

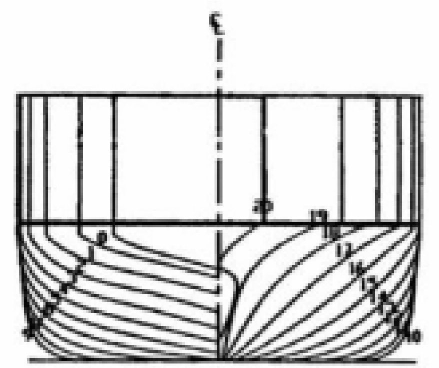

Figure 2 Bodylines of the RoRo

Table 1. Main particulars of the two ships

		S175	*RoRo*
Length betw. perp.	$L_{pp}(m)$	175.0	170.0
Beam	$B(m)$	25.40	27.80
Depth	$D(m)$	15.40	9.40
Draught	$T(m)$	9.50	6.25
Displacement	$\Delta(ton)$	24742	16645
Block coefficient	C_b	0.572	0.550
Long. posit. of CG	$LCG\ (m)$	-2.43	-0.70
Vert. posit. of CG	$VCG\ (m)$	0.0	7.55
Metacentric height	$GM(m)$	1.00	1.70
Roll radius of gyration	K_{xx}/B	0.328	0.363
Pitch & yaw rad. gyr.	K_{yy}/L_{pp}	0.24	0.25
Roll-yaw coup. inert.	I_{46}	0.	0.
Roll damping factor	ζ	0.075	0.090

Figures 1 and 2 present the bodylines of both ships, while table 1 includes their main characteristics. The longitudinal position of the centre of gravity is with respect to midship and positive forward, while the vertical position of the same point is with respect to the waterline and positive upwards. The inertial coupling between roll and yaw, usually a very small quantity, is neglected here. Calculations for the containership correspond to the service speed, meaning a Froude number of 0.275, and for the RoRo a zero speed condition was assumed.

3.2 Transfer Functions for the Containership

The first results presented are for the containership advancing in harmonic waves with a constant speed corresponding to a Froude number of 0.275. Figure 3 presents the amplitudes of the transfer functions of sway, roll and yaw for five headings between bow waves (150º) and quartering waves (30º). The roll graphs include two type of results: the amplitudes calculated by the coupled sway-roll-yaw equations of motion (square symbols), and the amplitudes calculated by the uncoupled roll equation (line). Sway amplitudes are made nondimensionalised by the wave amplitude, ζ_a, and roll and yaw amplitudes by the wave steepness, $k\zeta_a$, where k represents the wave number. All moments in the uncoupled model are calculated about an horizontal axis passing through the centre of gravity of the ship.

The graphs clearly show that the coupling effects of the horizontal motions into roll are important. Comparing the roll transfer function amplitudes of the coupled and uncoupled models one concludes that the low frequency uncoupled results are much smaller than coupled ones. Furthermore the nondimensional uncoupled amplitudes tend asymptotically for wrong values at the low frequency range. The correct values are given by: $k\zeta_a \cos\beta$, being β the heading of the ship with respect to the waves.

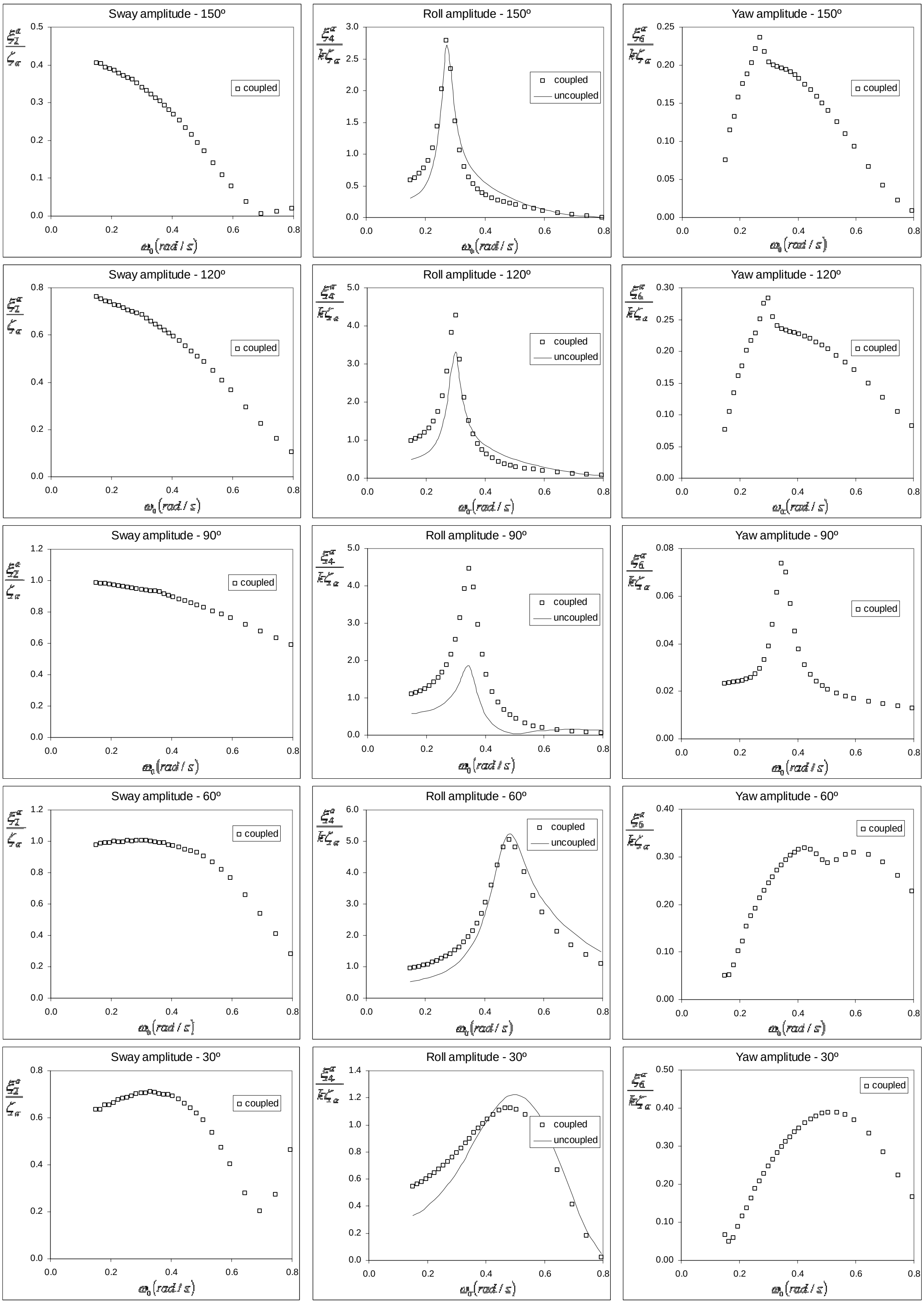

Figure 3 Amplitudes of the transfer functions of sway, roll and yaw for the containership.

Perhaps the most important differences between the two roll curves occur around the roll natural frequency. The natural frequency is 0.35rad/s, corresponding to a natural period of around 18 seconds. The natural frequency is well predicted by the uncoupled model, indicating that, in this respect, the coupling effects are negligible. However large differences are observed in the roll resonance amplitudes, especially for beam waves. One may say the uncoupled equation completely fails to predict the roll transfer function, thus the coupling effects with sway and yaw cannot be neglected.

The influence of roll is noticed in the yaw motion, especially for frequencies around the roll natural frequency, but the same influence is not observed in the sway amplitudes

3.3 Analysis of Coupling Effects

With the objective of understanding how the horizontal motions are coupled with roll, transfer functions of the various roll coupling moments have been computed. The results are presented in the graphs of figure 4 to 6 as function of the wave frequency, and they correspond to the coordinate system origin at the centre of gravity. Each graph includes the coupling effects for several headings between the ship and the regular waves. All moments are nondimensionalised by $Mg\zeta_a$, where the symbols represent respectively the ship mass, acceleration of gravity and wave amplitude.

Figures 4 and 5 present the coupling effects separated in four groups of moments, namely: damping coupling, inertia coupling, sway coupling and yaw coupling. These moments are calculated respectively by the following formulas:

$$M_c^{dam} = B_{42}(\omega)\dot{\xi}_2 + B_{46}(\omega)\dot{\xi}_6 \tag{2}$$

$$M_c^{iner} = [A_{24}(\omega) - Mz_G]\ddot{\xi}_2 + [A_{46}(\omega) - I_{46}]\ddot{\xi}_6 \tag{3}$$

$$M_c^{sway} = [A_{24}(\omega) - Mz_G]\ddot{\xi}_2 + B_{42}(\omega)\dot{\xi}_2 \tag{4}$$

$$M_c^{yaw} = [A_{46}(\omega) - I_{46}]\ddot{\xi}_6 + B_{46}(\omega)\dot{\xi}_6 \tag{5}$$

Equations (2) to (5) are taken directly from the coupled roll equation in (1). It should be noted that, since the roll motion is calculated about the centre of gravity, MzG is zero in equations (3) and (4), therefore the inertia coupling is due to hydrodynamic effects only.

In the former graphs, the wave frequency ranges from 0.1 to 0.8 rad/s, which corresponds to the range where the roll motion is amplified. To have an idea of the magnitude of the coupling moments, within this frequency range the maximum nondimensional exciting moments vary between 0.010 for beam waves and 0.050 for bow waves (see figures 7 to 10). Therefore one concludes that the various coupling moments reach important values.

Figure 4 shows the amplitudes of the damping and inertia coupling moment into roll. The coupling due to damping is very small for the frequencies where roll is amplified, while most of the coupling effects are due to hydrodynamic inertia.

A comparison between sway coupling and yaw coupling is shown in figure 5. Analysing again the frequency range of interest for roll motion, one concludes that the coupling effects induced by sway are more important then those of yaw, but not by a large margin. As expected, yaw coupling is very small for beam waves, because the hull is nearly symmetric fore-aft with respect to the centre of gravity, however the same coupling effects are important for bow waves.

Figure 6 presents the total coupling moments, together with the roll exciting moments, for several headings. The total coupling moment is calculated by:

$$\begin{aligned} M_c^{total} &= [A_{24}(\omega) - Mz_G]\ddot{\xi}_2 + B_{42}(\omega)\dot{\xi}_2 \\ &\quad + [A_{46}(\omega) - I_{46}]\ddot{\xi}_6 + B_{46}(\omega)\dot{\xi}_6 \end{aligned} \tag{6}$$

These graphs are interesting because they show that the coupling effects are similar in magnitude to the exciting moments induced by the waves for bow waves, and they are larger for beam waves.

Finally the graphs of figure 7 compare the roll exciting moments (left vertical axis) with the roll response amplitudes for the coupled and uncoupled models (right vertical axis). Besides the “true” exciting moment curves represented by the squares, the triangles represent the exciting moments added to the total coupling moments. The later is obtained if the roll equation in (1) is modified by passing all coupling terms to the right side and saying that the new right side is the modified exciting moment.

The difference between the exciting moment (squares) and the modified exciting moment (triangles) is proportional to the differences between the coupled and uncoupled roll amplitudes. This can be observed in the graphs of figure 7, where, for example, for beam waves one observes that the coupling effects are very important.

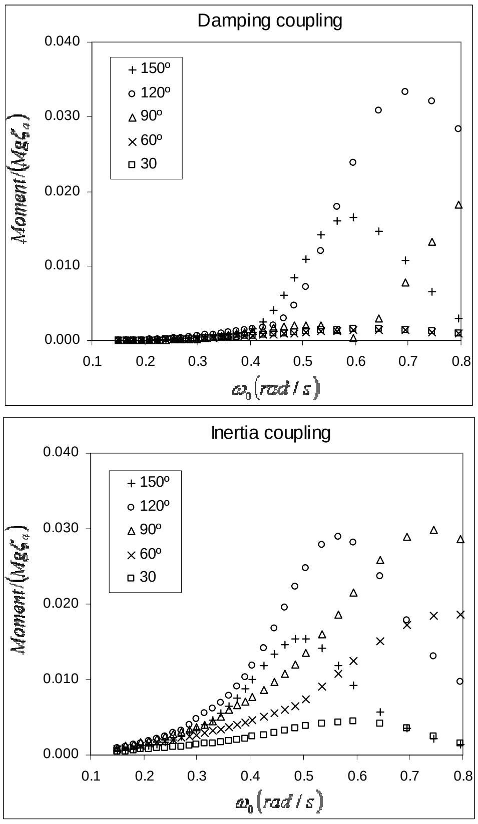

Figure 4 Transfer function amplitudes of the damping and inertia coupling moments into roll.

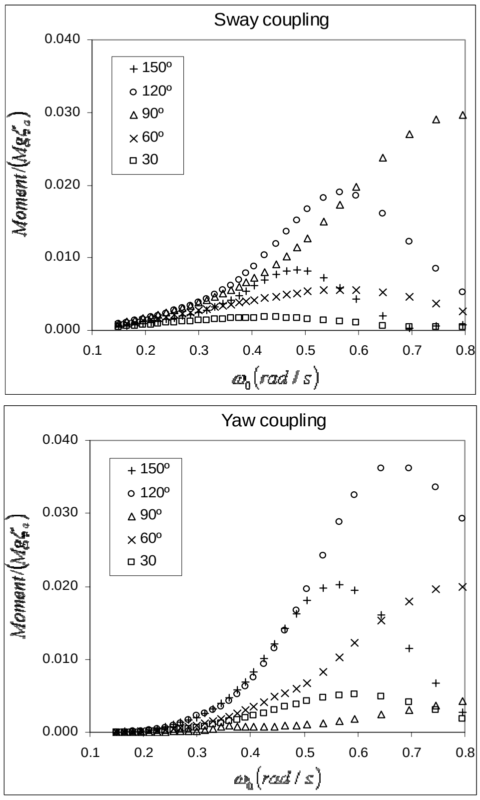

Figure 5 Transfer function amplitudes of the sway and yaw coupling moments into roll.

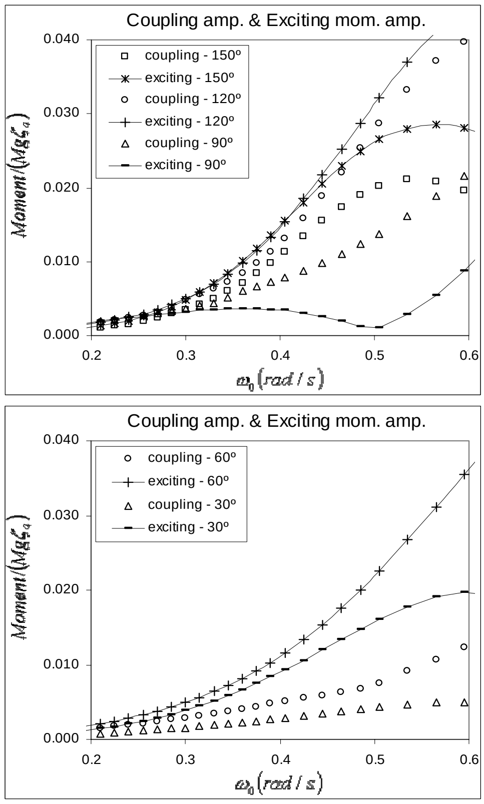

Figure 6 Transfer function amplitudes of the roll exciting moments and total coupling moments.

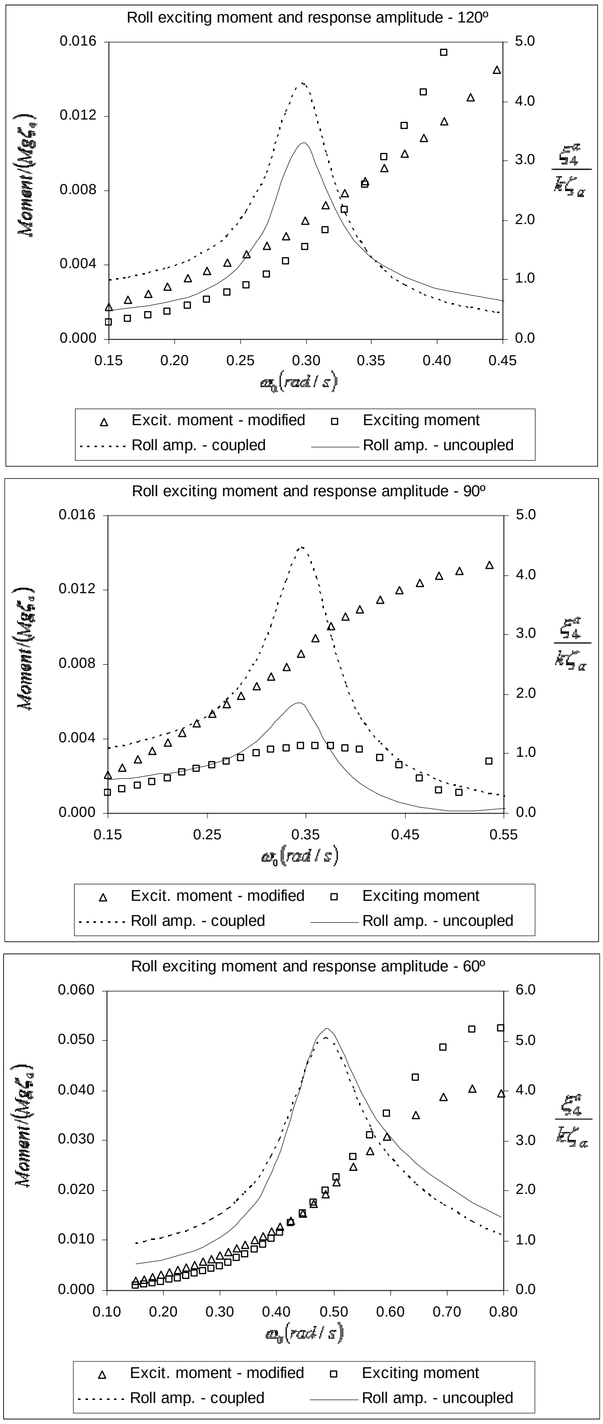

Figure 7 Comparison between roll exciting moments and roll response amplitudes for three headings (120º, 90º and 60º).

3.4 Roll Axis

Some authors, acknowledging that the coupling effects should not be completely ignored, use a procedure to decouple roll from sway which is based on choosing the proper origin for the coordinate system. This point would be the so called “roll center”, thus the coordinate system is subjected to a translation to the new origin and the ship now rolls about the roll axis. The procedure has been used by Jons et al. (1987) and Hutchison (1991). The height of roll center with respect to the waterline is given by:

$$H_O = \frac{\left(MZ_G - A_{42}\right)}{\left(M + A_{22}\right)} \qquad (7)$$

where M, Z_g, A_{42} and A_{22} are respectively the ship mass, height of the centre of gravity with respect to the waterline, the sway into roll added mass coupling coefficient and the sway added mass. Expression (7) is valid for A_{42} calculated on a reference system with origin at the calm waterline.

It is assumed that the yaw motion is lightly coupled with roll and their effects can be neglected, which may be reasonable for beam seas and nearly fore-aft symmetrical ships. Furthermore, this decoupling is with respect to the inertial effects in sway only, thus neglecting all damping effects. Finally, only the free undamped motion is decoupled, not the forced motion.

Even with the former restrictions, it is worthwhile to analyse how the results are improved when roll is decoupled from sway in this way. The added masses in equation (7) are frequency dependent, as is the decoupling. However, one more simplification is assumed here, namely that the added masses change slowly around the roll natural frequency and constant values corresponding to that frequency can be used. To test the validity of this assumption the roll axis has been calculated for several frequencies of oscillation and the results are plotted in figure 8. The height of the roll axis remains nearly constant for the frequency range between 0.20 and 0.45 rad/s where the roll dynamic amplification in beam waves occur. This conclusion should not be generalised.

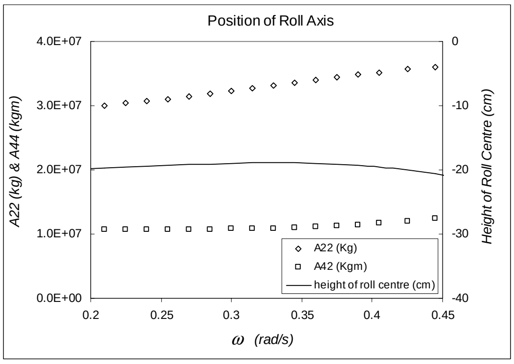

Figure 8 Height of the roll centre as function of the frequency.

Table 2 presents the heights of the centre of gravity, *Zg*, and of the roll centre, *Zo*, with respect to the waterline and positive upwards. The added mass effects do not change much the position of the roll centre with respect to the centre of gravity, but the consequences on the prediction of uncoupled roll amplitudes are very important as will be seen in the following graphs.

Table 2. Height of the centre of gravity and of roll centre

	Zg (m)	*Zo (m)*
Containership	0.0	-0.19
RoRo	7.55	7.22

Calculations with the uncoupled roll equation were repeated, but now using the new position of the roll centre. Roll transfer function amplitudes are presented in figure 9 for the containership and figure 10 for the RoRo. The graphs include coupled results, uncoupled using the centre of gravity axis (continuous lines) and uncoupled using the roll axis (dashed lines).

The analysis of results shows that small variations on the vertical position of the origin of the coordinate system, reflects on large differences on the uncoupled roll motion amplitudes. Furthermore, one may say that the translation of the coordinate system according to equation (7) successfully uncouples roll from the horizontal motions in beam waves. In fact, for both ships, the uncoupled roll axis transfer function is almost coincident with the coupled results. This is because in beam waves the hydrodynamic inertial sway coupling dominates over all other coupling effects. As seen before, damping coupling is negligible over the frequency range of interest for this heading, as is the yaw inertia coupling.

The other graphs show that yaw couplings cannot be neglected for the headings other than 90º. In several cases, computation of the roll motion about the roll axis worsens the results compared with the calculations about the centre of gravity axis. The solution would obviously be to decouple also roll from yaw by a rotation of the coordinate system. The problem is the burden in the whole numerical procedure which would be increased. Anyway this is a topic for further analysis in the future.

3.5 Roll Exciting Moment

The previous section has shown that decoupling roll with respect to sway is possible and it might be possible and practical to decouple roll also with respect to yaw. The decoupling is with respect to inertial effects only, therefore damping coupling effects will need to be better investigated. However the result is consistent for the free roll motion only. The fact is that the exciting moment is very dependent of the origin of the coordinate system.

To demonstrate the former aspect, the exciting moment for the containership, in beam waves and at the natural roll frequency of 0.35 (rad/s), has been calculated for several vertical positions of the origin of the coordinate system. The resulting amplitudes and phase angles of the roll exciting moment are presented in figure 11 as function of the vertical position of the origin which ranges from 0.00 to 0.25m (Zg = 0.0m for this ship). The vertical position is with respect to the waterline.

The graph shows that over a small range of 25cm for the variation of the vertical position of the origin, the exciting moment reduces from the actual value, to zero, and than changes signal and increases again in the opposite direction. This result shows that much care is needed when deciding on the vertical position for the origin of the coordinate system.

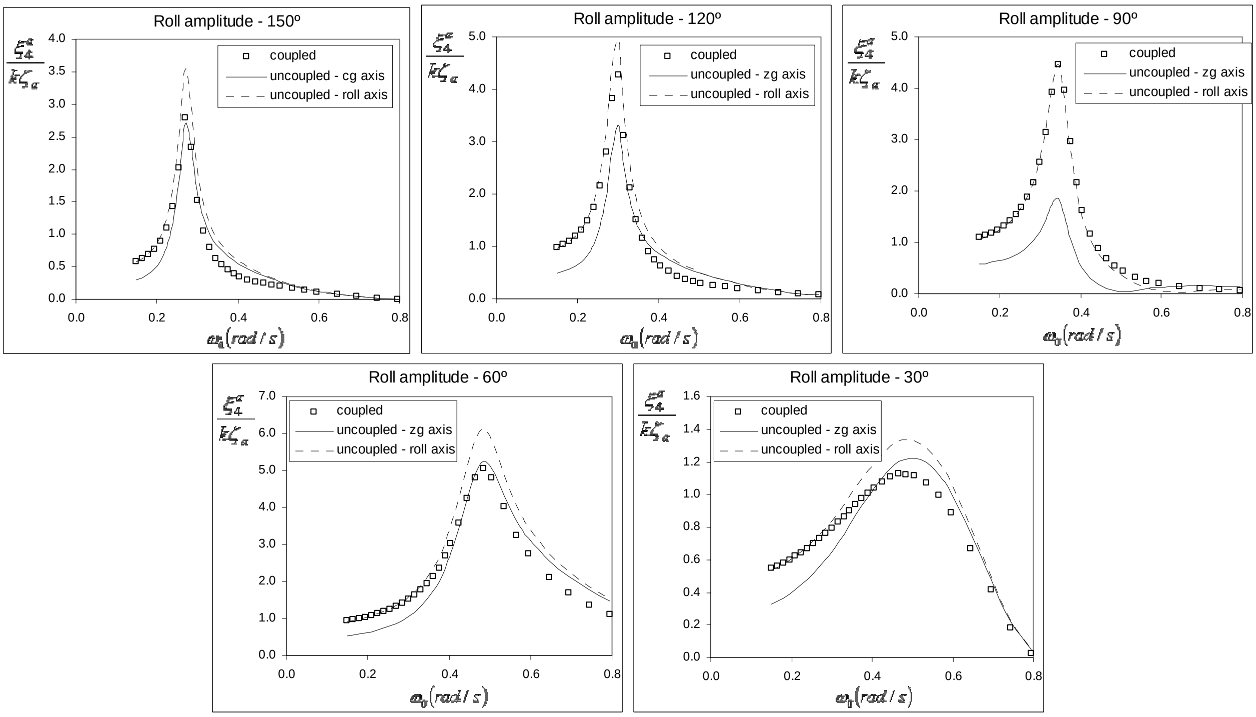

Figure 9 Amplitudes of the transfer function of roll for the S175. Comparison of coupled responses with uncoupled with origin of coordinate system at ZG and uncoupled with origin at the roll axis.

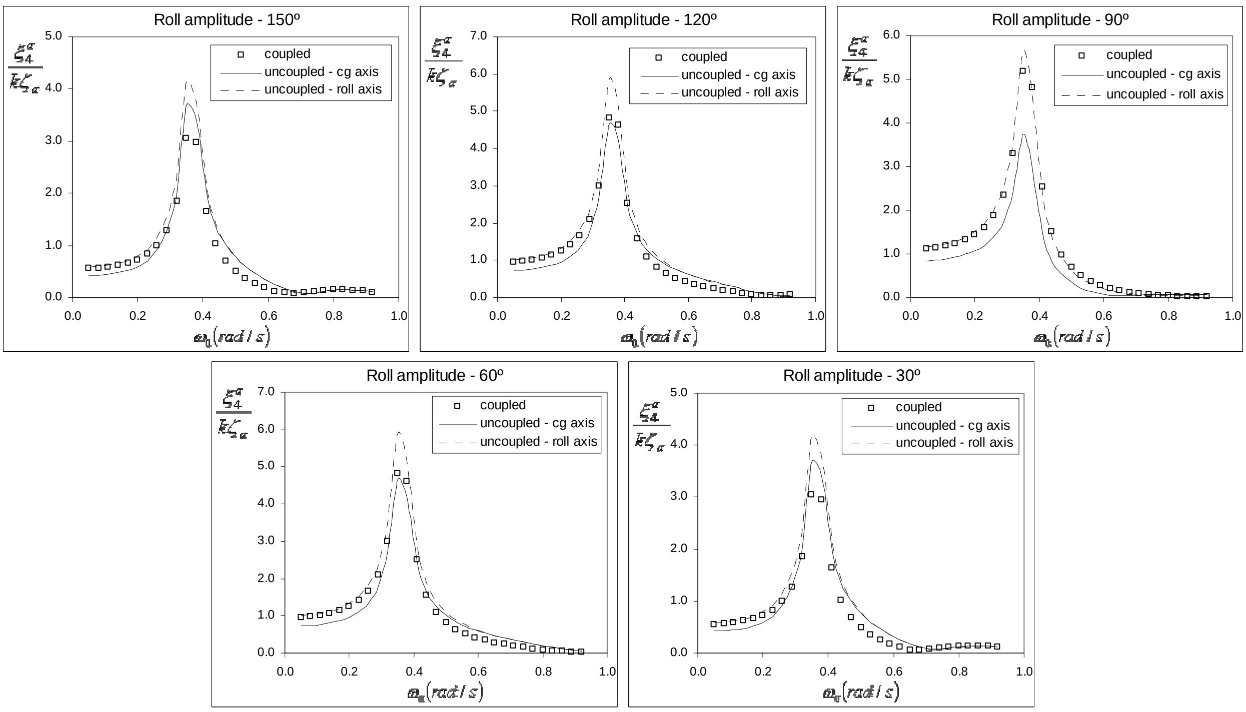

Figure 10 Amplitudes of the transfer function of roll for the RoRo. Comparison of coupled responses with uncoupled with origin of coordinate system at ZG and uncoupled with origin at the roll axis.

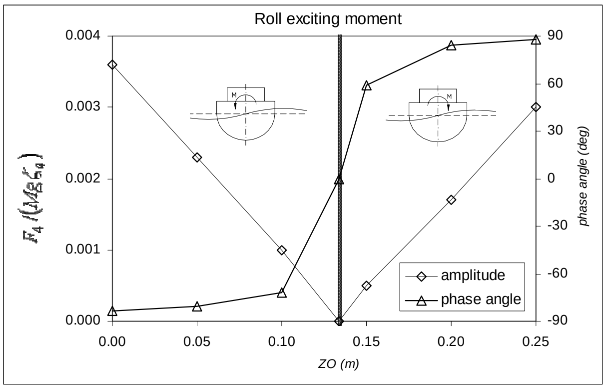

Figure 11 Amplitude and phase angle of the roll exciting moment for the containership as function of the vertical position of the coordinate system origin.

4. CONCLUSIONS

The paper investigates the consequences of neglecting, or of simplifying, the linear coupling of sway and yaw in the prediction of roll motion. It is concluded that the roll coupling moments are important and solving the uncoupled roll equation with respect to a horizontal axis passing through the centre of gravity may lead to completely wrong results.

It is possible remove the inertial sway coupling from the roll equation by a vertical translation of the origin of the coordinate system, which, for the ships investigated, results on a good prediction of the roll transfer functions in beam seas. For this heading yaw couplings and damping coupling can be neglected. However for other headings such decoupling of sway from roll does not improve the uncoupled model predictions. Finally it is shown that the choice of vertical position of the origin of the coordinate system may have a big effect on the calculation of the roll exciting moment.

5. ACKNOWLEDGMENTS

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