

PARAMETRIC EXCITATION OF FLOATING OFFSHORE PLATFORMS

Jeffrey Falzarano, Jun Cheng, and Samrat Das
University of New Orleans
School of Naval Architecture and Marine Engineering
(United States)

Abstract

Parametric excitation and the resulting Mathieu instability has recently received new interest with regards to ships[1] but this problem also exists for other types of vessels or floating bodies specifically floating offshore platforms such as semi-submersibles in transit and SPAR floating production platforms. In this paper, we intend upon reviewing some of our previous work and presenting some new work in order to draw attention to these important problems. Moreover, since the geometry of floating offshore platforms is quite diverse, a so-called first principles or physics based analysis technique seems warranted. In addition, a first principles approach such as suggested herein would also be useful to analyse advanced ship designs with unique hull forms where existing methods of static stability analysis may not be appropriate.

1. INTRODUCTION

The two specific problems to be analysed in this paper are 1) the transit draft dynamics and stability of a large semi-submersible such as typically used for drilling but can also be used for as a production platform and 2) the coupled heave and roll motion of a deep draft caisson vessel called a SPAR in head seas. Although both problems involve the Mathieu instability the first is due to the variation of the water-plane area and the other is due to a heave roll coupling.

2. PREVIOUS AND RELATED WORK

The motion of vessels under parametric excitation has been studied in the past by a number of investigators. Esparza and Falzarano[2] have investigated the effect of

parametric excitation on fishing vessels both with and without wind bias. In an experimental and analytic study of parametric excitation, Francescutto[3] has investigated the stability characteristics of a destroyer. Investigations into parametrically excited roll motion using a prescribed heave or pitch motion was carried out in the past by Kerwin[4], Paulling and Rosenberg[5] and Blocki[6].

A generalized analysis of saturation induced ship rolling motion both with and without parametric excitation has been accomplished by Falzarano, Holappa, and Taz-Ul-Mulk[7]. Spyrou[8] investigated the parametric rolling of ships where the coupling of surge on roll dynamics has been considered. Previous work, related to this investigation, on the stability analysis of the Mobile Offshore Base includes papers by Falzarano, et al[9]. Previous work on time-varying hydrostatics of the US Navy

MOB platform was done in Das and Falzarano[10].

3. TRANSIT DRAFT DYNAMICS OF A LARGE SEMI-SUBMERSIBLE

In this section we describe our previous work on the dynamics and stability of the US Navy's Mobile Offshore Base (MOB). Although transit draft is an important issue in all Mobile Offshore Drilling Units (MODU) it seems to have been generally ignored by the research and design community. In this paper we describe the analysis of the transit draft behaviour of the US Navy's MOB.

3.1. Physical System Modeling

Any vessel motion by nature is a coupled motion; hence some or all the other degrees of freedom influence the other degrees of freedom. For a completely accurate analysis of the motions of any vessel it is necessary to model all the six degrees of freedom in the equations of motion.

Considering the linearized equations of motion it is seen that no inertial coupling exists between the symmetric and the asymmetric modes of motion. Hence for linear roll motion analysis only sway, roll and yaw motion need to be considered. These motions can then be analysed without the consideration of heave, surge and pitch motions. Furthermore, in this analysis only head seas have been considered which precludes the consideration of sway and yaw motion. Essentially, in head seas, the directly forced heave motion is coupled to the roll motion due to the parametric variation of the righting arm with draft.

Hence for the purpose of the analysis carried out in this work, only the most significant motion (pertaining to stability and capsizing), namely roll has been considered. This

simplification can be justified by the reasoning that vessel capsize is most strongly influenced by the roll motion. In addition, amongst the three transverse coupled motions, only roll has restoring forces and exhibits strong resonant motions. Hence, roll motion can be considered the most important ship motion in any stability analysis of a vessel.

Based on this simplification, stability of the Mobile Offshore Base has been defined by its ability to recover from a given angle of roll. After the roll equation of motion has been decoupled from the other degrees of freedom the equation becomes that of a simple single degree of freedom oscillator with a time varying restoring force. The uncoupled roll motion equation is similar to that of a simple single degree of freedom oscillator with a time varying restoring force. Hence the results pertaining to parametric excitation of such systems can be applied to the dynamic stability analysis of the Mobile Offshore Base.

The physical situation that was modelled herein is that of a vessel exhibiting roll motion in head seas. While this concept seems somewhat counter intuitive it is the focus of the study undertaken here. As has been mentioned earlier, the Mobile Offshore Base has a very sharp change in water plane area at a draft of 50 feet. Hence the righting arm of the vessel decreases dramatically whenever the instantaneous draft of the vessel exceeds 50 feet at any location along the pontoon. This increase in draft (pontoon submergence) can occur due to wave crests or vertical motion and is a function of the initial transit draft. Figure 1 shows the column wetting in head seas and Figure 2 shows the sudden change in the water plane at the pontoon tops

When a system is parametrically excited the response frequency is typically a sub-harmonic of the excitation frequency. Hence a parametrically excited system may exhibit large resonant motion response amplitudes

when the excitation frequencies, is twice the natural frequency. In this study, both factors; namely wave excitation frequencies and amplitudes have been systematically varied. The single degree of freedom roll equation of motion that has been used to model the behaviour of the Mobile Offshore Base is given in Equation (1).

$$(I_{44} + A_{44})\ddot{\phi}(t) + B_{44}\dot{\phi}(t) + \Delta GZ(\phi, t) = 0 \quad (1)$$

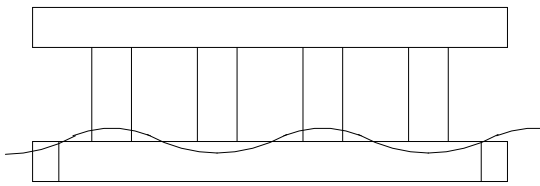


Figure 1 Mobile Offshore Base in head seas

Table 1 Principal Characteristics of MOB

Length	900ft
Beam	400ft
Height	230ft
Column Diameter	75 ft
Pontoon Width	100 ft
Pontoon Height	50ft
Pontoon C-L distance	300ft
Transit Drafts	40ft & 45ft

As can be observed from Equation (1), no external forcing on the system is considered. Additionally, the righting arm is a function of both the time and the angle of roll. The time variation of the GZ curve is due to the progression of a wave along the vessel's length, which causes a change in the water plane area of the vessel. This change in the water plane area subsequently changes the meta-centric radius (BM) (Equation (2)) while the volume stays relatively constant and consequently the meta-centric height (GM) of

the vessel also changes according to Equation (3).

$$BM(t) = \frac{I(t)}{V} \quad (2)$$

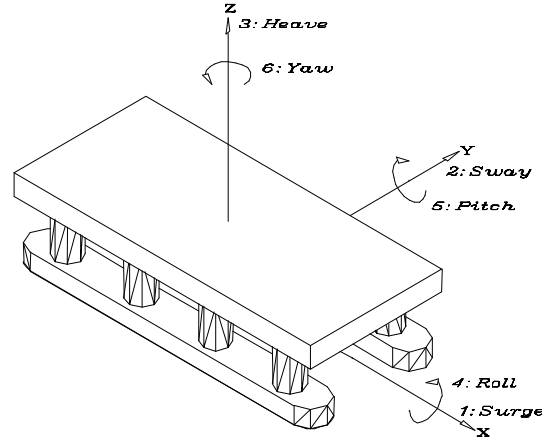


Figure 2 Mobile Offshore Base Isometric View

$I(t)$ is the time variation of the area moment of inertia of the water plane area of the vessel. V is the total immersed volume or displacement volume of the vessel. The relation between $GM(t)$ and $BM(t)$ is given as follows:

$$GM(t) = BM(t) + KB - KG \quad (3)$$

Any naval architecture textbook will explain the various terms used in these equations for a single hull displacement ship in more detail.

For the linear analysis carried out in this work, Equation (1) has been modified such that the variation of the $GM(t)$ is modelled instead of the variation of $GZ(\phi, t)$. For small angles, the relation between these two quantities can be linearly approximated as in Equation (4).

$$GZ(\phi, t) \cong GM(t)\phi \quad (4)$$

By carrying out the changes detailed above the equation of motion can now be expressed as follows:

$$(I_{44} + A_{44})\ddot{\phi} + B_{44}\dot{\phi} + \Delta GM(t)\phi = 0 \quad (5)$$

Another factor, which is important in a stability analysis under parametric analysis, is the wave encounter frequency of the vessel. This is due to the fact that the excitation frequency has to be a super harmonic of the roll natural frequency of the body. For a vessel in following seas, i.e., with both the ship and the wave headings in the same direction, the relation between the incident wave frequency and the encounter frequency is given in Equation (6).

$$\omega_e = \omega_0 - v_s K \quad (6)$$

In Equation (6), ω_0 is the incident wave frequency, ω_e is the wave encounter frequency, K is the wave number and equals $2\pi/\lambda$, λ is the wavelength and V_s is the velocity of the vessel.

3.2 Problem Analysis Technique

The stability analysis carried out in this work has been done in the time domain. Hence a quasi-static time domain analysis technique has been employed. For this analysis, quasi-static methods are applicable due to the low frequencies at which the Mobile Offshore Base encounters different waves. At these low frequencies, the hydrostatic properties of the vessel are the single largest influence on the motion of the Mobile Offshore Base. The procedure employed in the analysis is detailed below.

The righting arm curves for the Mobile Offshore Base have been generated numerically using standard offshore analysis software. The righting arm curves have been generated for a range of time instants starting from zero time and ending with one wave

period. This has been done by positioning the wave crests at various positions along the hull of the vessel corresponding to different instants of time. From the righting arm curves for each time instant or wave position, the GM curves have been obtained. After the GM values were calculated for a number of time instants, the time history of the GM is plotted out. Since the $GM(t)$ curves cannot always be represented to the required degree of accuracy by a single sinusoidal wave (see [10]), a Fourier analysis of the time history is carried out. This allows us to represent any GM time history as the sum of a series of regular components of different harmonics.

Solution using Mathieu's Equation

The equation of motion as defined by Equation (1) Substituting equation (5) into equation (1) yields equation (5). Dividing through by $(I_{44} + A_{44})$, we obtain,

$$\ddot{\phi} + \frac{B_{44}}{(I_{44} + A_{44})}\dot{\phi} + \frac{\Delta}{(I_{44} + A_{44})}GM(t)\phi = 0 \quad (7)$$

For the Mathieu's analysis, Equation (7) is further simplified by using the expression in equation (8).

$$GM(t) = GM_m + \delta GM \cos(\omega t) \quad (8)$$

On further simplification this gives:

$$\ddot{\phi} + \frac{B_{44}}{(I_{44} + A_{44})}\dot{\phi} + \omega_0^2 \left[1 + \frac{\delta GM}{GM_m} \cos(\omega t)\right]\phi = 0 \quad (9)$$

Transforming $\omega t = 2\tau$, we get the familiar form of the Mathieu equation in Equation (10)[3].

$$\ddot{\phi} + \frac{B_{44}}{(I_{44} + A_{44})}\dot{\phi} + 4\frac{\omega_0^2}{\omega^2} \left[1 + \frac{\delta GM}{GM_m} \cos(2\tau)\right]\phi = 0 \quad (10)$$

In Equation (10), the damping term has been included into the equation of motion. In initial

studies an undamped equation of motion was used to define the boundaries of the instability zone on the Strutt diagram. Equation (11) was the working equation for this.

$$\frac{d^2\phi}{dt^2} + (a + 16q \cos 2t)\phi = 0 \quad (11)$$

Later when the damped case was studied, a unique damping ratio was introduced to yield Equation (12)

$$\frac{d^2\phi}{dt^2} + 2\mu \frac{d\phi}{dt} + (a + 16q \cos 2t)\phi = 0 \quad (12)$$

The various terms of Equation (12) are defined as follows:

$$2\mu = \frac{B_{44}}{(I_{44} + A_{44})} \quad (12a)$$

$$a = 4 \left(\frac{\omega_n^2}{\omega^2} \right) \quad (12b)$$

$$16q = \left(\frac{\delta GM}{GM_m} \right) \left(\frac{\omega_n^2}{\omega^2} \right) \quad (12c)$$

For the parametric excitation instability analysis carried out in this work, only the first instability zone has been investigated as it presents the highest probability of influencing the behaviour of the vessel. This higher probability is due to two factors, namely the wide detuning exhibited by the first instability zone and the frequency ratio (the ratio of the natural frequency to the excitation frequency).

From the inspection of a general Strutt diagram, it is observed that the first instability region covers the widest variation in frequency ratios. Therefore, the possibility of the vessel exhibiting large motions in response to an excitation, which is slightly detuned (not an exact integer multiple of the natural frequency), is the highest in this zone. This implies that, an

analysis of vessel stability in this zone is of the utmost importance and must be considered.

The other aspect, which makes it imperative that vessel motions in the first instability zone be considered, is the fact that this zone represents excitation frequencies, which are double the natural frequency of the vessel. Physically this translates into a situation, where the vessel exhibits large amplitude motions even at low speeds. Hence from a stability point of view it becomes necessary to consider this case, which is reflected in the first instability zone on the Strutt diagram.

Figure 5 shows the boundaries of the first instability zone on the stability diagram, which is referred to as the Strutt diagram. It can be seen that there is a wide range of slightly detuned frequencies for this zone.

Stability analysis using diagrams

The method of analysing stability, using Strutt diagrams has many advantages. It allows us to easily visualize the change in stability characteristics as variables are changed. For a better comparison of different physical situations it also allows us to view multiple solutions simultaneously. Since the stability diagrams play a very important role in the analysis carried out in this work, a brief explanation of how to generate and interpret these diagrams is given in the following paragraphs.

An instability diagram is plotted on an axes system where ordinate values are the frequency ratios and the abscissa values are some function of the ratios in the time varying coefficients.

In our case, for the Mathieu's equation analysis, $\delta GM/GM_m$ is plotted on the vertical axis. A very important part of these diagrams that must be considered in this analysis is the phase lag of the response. In some cases this

phase angle may determine whether or not the structure is stable or unstable for a given value of damping. As mentioned in the previous section, the phase lag is represented from 0 degrees to 90 degrees on the diagram and corresponds physically to a phase lag varying from 0 degrees to 180 degrees.

The phase lag in the response of the system has been determined by time simulating the response, using a numerical simulation code which includes both nonlinear, $GZ(\phi, t)$ and memory effect in the hydrodynamics[11].

In this work, both the added mass and the damping coefficients have been considered invariant. This is a simplification of the actual physical phenomenon, where the response frequency changes with time. Actually the added mass and damping coefficients are both dependent on the time due to the frequency variation. To better model this variation, a time domain analysis using impulse response functions should be used[11].

Non-linear damping is an important part of the vessel dynamics. This problem is still being researched extensively, and at present there are not enough methods to numerically simulate this effect accurately. In this work this aspect of the equation of motion has not been investigated.

The series of iso-damping ratio curves are plotted in the instability diagrams to represent the instability zones for the physical system with different damping ratios. These plots are generated by taking different values of the phase lag in the response and then solving backwards for the corresponding damping ratio. It is seen that with increasing damping ratios the instability zone constricts and is lifted up above the zero excitation horizontal axis. Physically this means that with increasing damping, the system exhibits more stable behaviour for increasing values of excitation.

Finally, points that correspond to the physical states of the Mobile Offshore Base are superimposed on the diagram to evaluate the stability of the system for a given damping ratio. Figure 5a shows a typical instability zone with curves corresponding to different damping ratios. The legend shows the values of the different damping ratios and also the boundaries of the instability zone. From the figure it is seen the boundaries of the instability zone constrict with increasing values of damping ratios.

3.3 MOB Transit Draft Results

The results presented in this study are exclusively for the first instability zone (excitation frequency twice the natural frequency). As mentioned earlier, two transit drafts 40 feet and 45 feet have been considered in the model tests but only the 40' draft results are described herein. A number of waves with wavelengths ranging from 720 feet to 1080 feet have been considered for the analysis. The reason for choosing these wavelengths is that the Mobile Offshore Base exhibits greatest response to excitation from waves in this range. The results are from a numerical simulation using an offshore analysis computer program.

Time histories to determine excitation

The results of the numerical simulations used to get the different time variation of the metacentric height as the wave progresses along the pontoons of the Mobile Offshore Base are included in the next two sections.

The results are arranged according to the two transit drafts considered in the USNA model tests. The results for the 40 feet draft are given in Figure 3. For a particular transit draft the results are arranged in increasing order of wavelength. The time histories for the 900-foot wavelength have been shown on two plots for

ease of representation. The legend at the bottom of each plot refers to the wave heights for which the time histories are plotted out.

Mathieu analysis

As stated earlier, the Mathieu analysis considers only the first harmonic of the time history while solving the parametrically excited roll equation of motion. Hence the time variation of the GM in the Mathieu equation is expressed as Equation (7). Using Fourier analysis techniques, the harmonics are obtained. In this work we consider only the effect of the principle harmonic and ignore higher harmonics; clearly this would not be appropriate for higher wave amplitudes.

Strutt Diagrams

The data from the previous section is used to generate the first instability zone of the Strutt diagrams. As has been stated earlier the Strutt diagram allows for the analysis of the stability under parametric excitation, if the system has been modelled using the Mathieu's equation. As will be shown, stability of the Mobile Offshore Base is seen to be highly sensitive to the wavelength, the draft and the phase lag of the response motion.

Description of Strutt Diagram plots

The first instability zone plots of the Strutt diagram show the different damping ratio curves and the points corresponding to the different wave heights encountered by the Mobile Offshore Base. Each point on the plot corresponds to the stability state of the Mobile Offshore Base in one particular combination of wavelength and wave height. A number of different wave heights, varying in magnitude from 15 feet to 55 feet have been considered. For the shorter wavelengths, only wave heights

up to 50 feet have been considered, while for the longest wavelength (1080 feet) wave heights up to 55 feet have been taken. Figure 5 is for the 40 draft. As has been stated earlier all the points which lie within an instability zone (defined by a particular damping ratio) indicates wave lengths and corresponding wave heights in which the Mobile Offshore Base is unstable for that particular level of damping. Viewing these results, one can see the importance of damping in the stability of the system. It should also be noticed here that only radiated wave damping has been calculated in this work. Unfortunately, the calculation of the non-ideal flow damping is a very difficult task, which has not been adequately investigated[12].

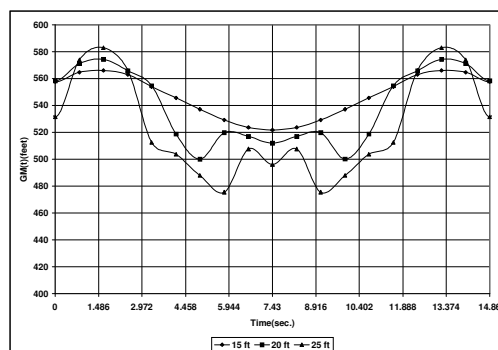


Figure 3a: GM Variation for wavelength 720 ft

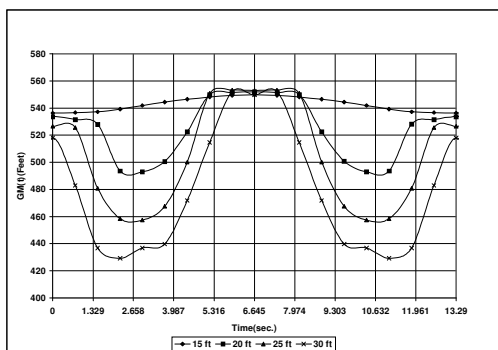


Figure 3b GM Variation for wavelength 900 ft

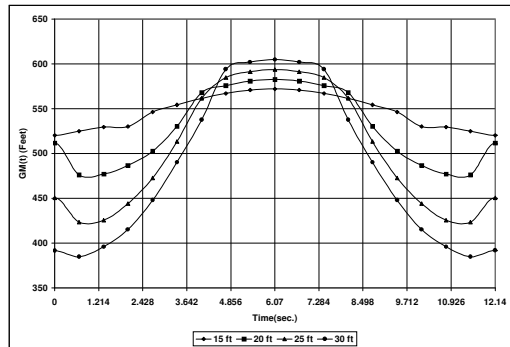


Figure 3c GM Variation for wavelength 1080 ft

3.4 Discussion of the MOB Results

A representative set of results is given for the 40' draft (10 ft freeboard) and wavelengths equal to the vessel length (900 feet) and 20% less (720 feet) and 20% greater (1080 feet). One can see how as the wave amplitude increases the variation in the GM becomes less sinusoidal and more or less like a step function. Clearly, for the lower wave height the one term harmonic approximation of the Mathieu equation may be adequate. For higher wave amplitudes a more complicated analysis may be required[10].

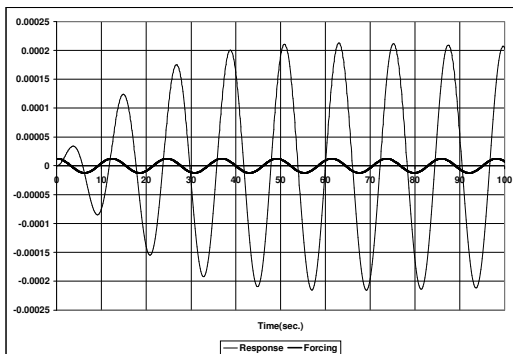


Figure 4 Forcing and response time history:

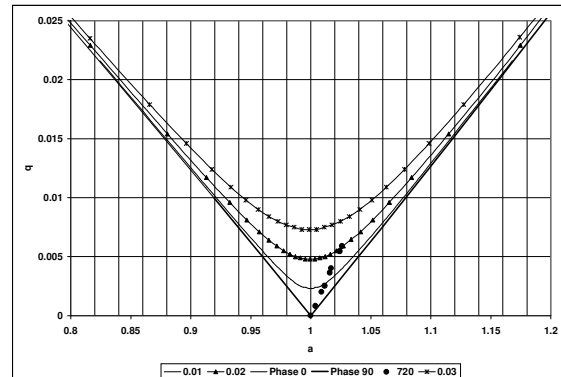


Figure 5a Strutt Diagram for 720' wavelength

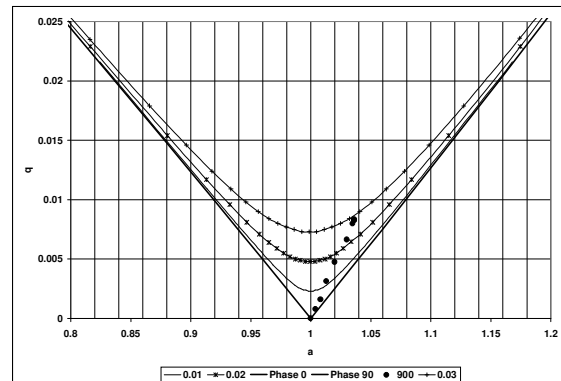


Figure 5b Strutt Diagram for 900' wavelength

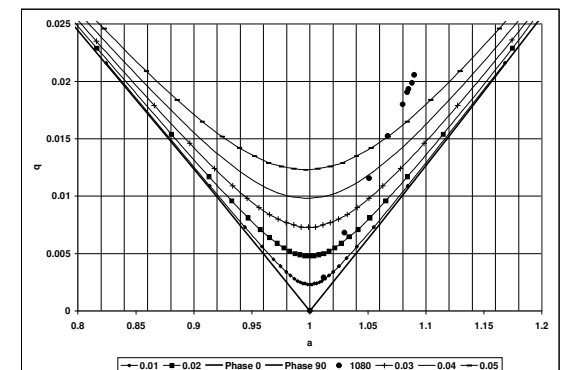


Figure 5c Strutt Diagram for 1080' wavelength

4. PARAMETRIC EXCITATION OF A SPAR BUOY

In this section, we consider the classic problem of coupled sway heave and roll motion of a SPAR buoy. This is a very important practical problem in offshore engineering since many

platform concepts are based upon this geometry. For example the so-called “classic SPAR” is a 100’ diameter and 600’ deep cylinder with a square moon-pool. Many other variations on this theme also exist. The buoy considered here is smaller but geometrically similar.

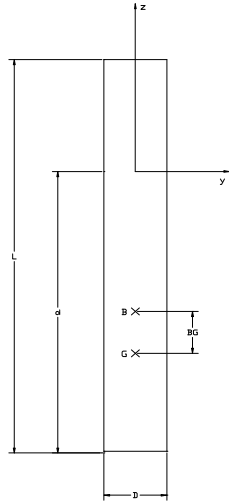


Figure 6 Circular cylinder buoy

Consider a buoy in regular incident waves in deep water. The buoy is a cylinder with circular cross section and the incident waves are propagating along the positive y-axis (see Figure 6). The wavelength λ is to be assumed larger than the diameter of the buoy enough so that the buoy does not generate any waves of significance. Linear potential theory can be assumed. In the figure, B means the centre of buoyancy and G the centre of gravity.

The velocity potential for the incident waves is written as

$$\phi = \frac{g\zeta_a}{\omega} e^{kz} \cos(\omega t - ky) \quad (13)$$

Without considering the effect of damping, the coupled equations of motion for sway, heave and roll can be written as[13]

$$(M + A_{22})\ddot{\eta}_2 + A_{24}\eta_4 = F_2 \cos \omega t \quad (14a)$$

$$(M + A_{33})\ddot{\eta}_3 + C_{33}\eta_3 = F_3 \cos \omega t \quad (14b)$$

$$A_{42}\ddot{\eta}_2 + (I_4 + A_{44})\ddot{\eta}_4 + C_{44}\eta_4 = F_4 \cos \omega t \quad (14c)$$

where, M is the mass of the buoy, I_4 is the moment of inertia about x-axis through the centre of gravity; A's are the added mass or moment of inertia coefficients, and C's hydrostatic restoring force coefficients, F's are the amplitudes of the wave exciting forces and moments. Using the potential flow theory, the heave and roll motions are given in closed form by Newman[14] as follows

$$\eta_3 = \zeta_a \cos \omega t \frac{e^{-kd}}{1 - kd} \quad (15)$$

$$\eta_4 = \zeta_a \cos \omega t \frac{1}{\omega_n^2 - \omega^2} \frac{F_4 - \frac{1}{2} \overline{BGF}_2}{(I_4 + A_{44}) + \frac{1}{2} M \overline{BG}^2} \quad (16)$$

The magnification curves or RAO of the heave and roll motion responses are shown in Figure 7 and Figure 8. In the calculation, the diameter of the buoy is $D = 2\text{m}$, the height is $L = 14\text{m}$, the draft is $d = 10\text{m}$, $\overline{BG} = 1.5\text{m}$, and the moment of inertia for roll is assumed $I_4 = 0.15ML^2$.

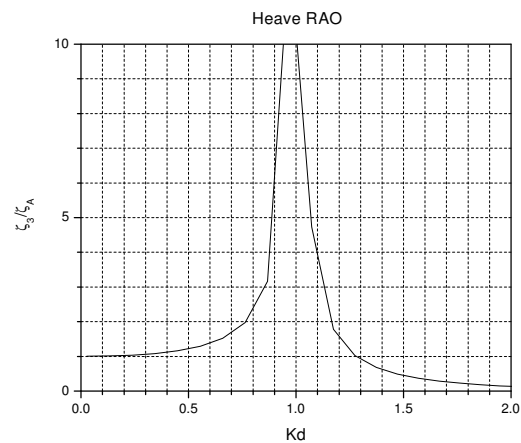


Figure 7 RAO of heave response of the buoy

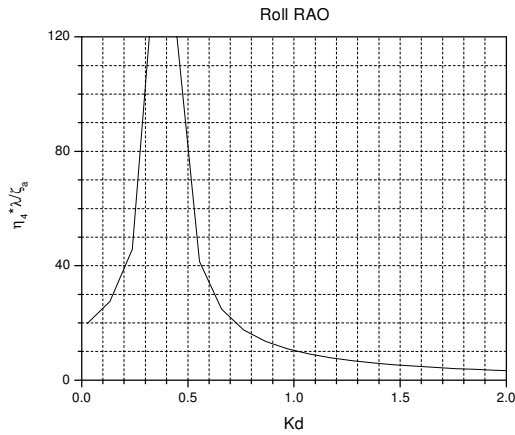


Figure 8 RAO of roll response of the buoy

A Mathieu type equation of motion with wave damping b_{44} included for roll motion is then derived and written as

$$\ddot{\eta}_4 + b_{44}\dot{\eta}_4 + \omega_n^2 \left[1 - \frac{1}{2} \frac{\eta_3(t)}{\overline{GM}_0} \right] \eta_4 = \frac{F_4 - \frac{1}{2} \overline{BG} F_2}{(I_4 + A_{44}) - \frac{1}{2} M \overline{BG}^2} \cos \omega t \quad (17)$$

where the natural frequency for the coupled sway and roll motion is given as

$$\omega_n = \left[\frac{\rho g \nabla \overline{GM}_0}{(I_4 + A_{44}) - \frac{1}{2} M \overline{BG}^2} \right]^{\frac{1}{2}} \quad (18)$$

and \overline{GM}_0 is the initial meta-centric height. In the above equation $\eta_3(t)$ is the displacement of the heave motion that has been solved previously.

Keep the configuration of the cylinder unchanged, if the wavelength of the incident wave is $\lambda = 40\text{m}$, the exciting frequency is twice of the natural frequency for roll as $\omega = 2\omega_n$. In this case, although the exciting frequency is away from the resonant frequencies of both heave and roll, the so called

internal or parametric excitation of time dependent restoring moment due to heave motion will cause roll instability. Figure 9 shows the unstable time history of the roll motion with the wave slope $2\zeta_a = 1/10$ by solving the homogeneous part of the equation of motion.

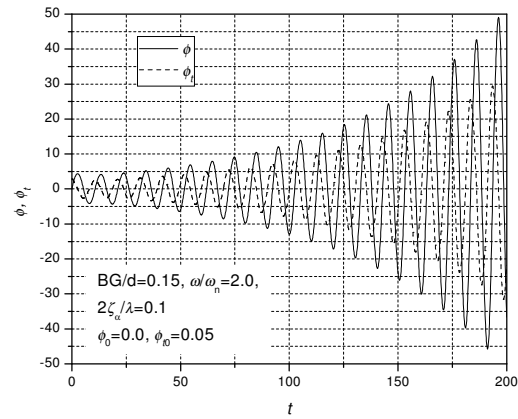


Figure 9 Roll time history of the buoy by parametric excitation

This simple example shows the strong possibility of the existence of Mathieu instability for the cylindrical type offshore platforms or production buoys in waves. Of course additional viscous damping and mooring forces should be considered in the analysis for the real platforms, it shows that even though the direct resonance have been avoided, large amplitude roll motion could still be induced due to the parametric excitation due to the coincidence of the design and wave environment.

Figure 10 shows that when the roll natural frequency is changed by moving the vertical position of the centre of gravity, the parametrically excited roll motion will decay even with higher wave excitation. Figure 11 shows that when the system is detuned and the damping is minimum the motion decay is very slow.

Figure 12 and 13 show the effect of initial conditions on the stable coupled motions. Numerical computation shows that the initial conditions play very important role on the amplitude of stable parametrically excited roll motion. Figure 13 shows that the amplitude of the parametric roll motion is doubled when the initial roll velocity is doubled.

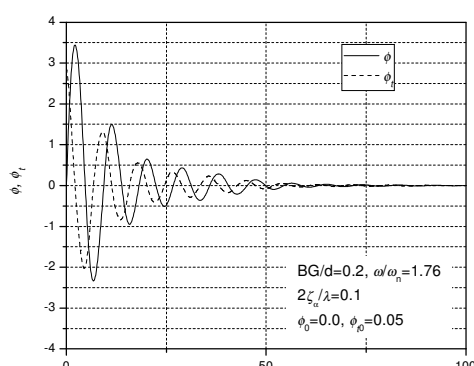


Figure 10 Roll time history of the buoy by parametric excitation

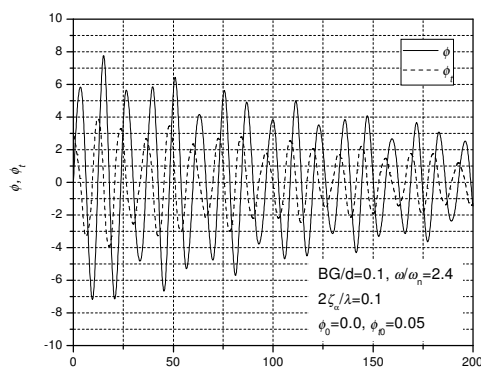


Figure 11 Roll time history of the buoy by parametric excitation

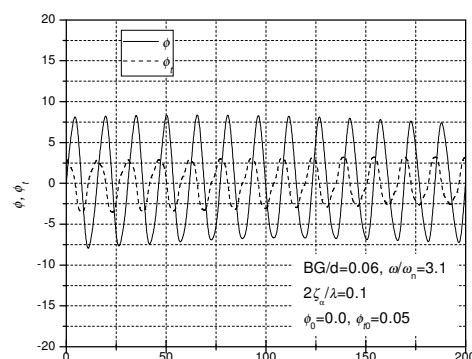


Figure 12 Roll time history of the buoy by parametric excitation

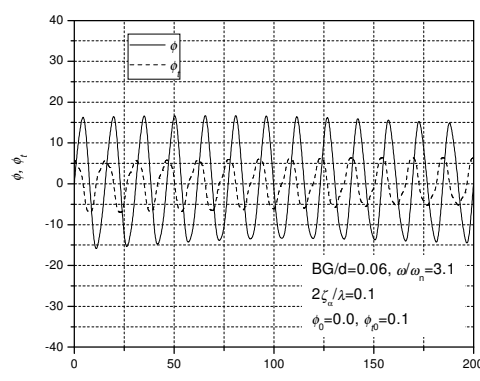


Figure 13 Roll time history of the buoy by parametric excitation

5. CONCLUSIONS

A parametric stability analysis of the US Navy Mobile Offshore Base has been performed using the Mathieu equation. For a range of regular wave amplitude the required linear radiated wave damping has been determined. It was found in all cases that the actual damping was more than adequate to assure stability. In a related work, Das and Falzarano[10] analysed additional wavelengths and analysed the additional 45' draft in more detail. Similar results were obtained.

In addition, we describe the results for coupled sway, heave and roll motion of a SPAR buoy.



In this case we determine the amount of damping required in order to eliminate this dangerous phenomenon. Although the results for the SPAR are quite interesting they represent a limited range of results that are possible and a more systematic bifurcation analysis should be undertaken in order to obtain a more complete picture of the system's behaviour[7].

It should also be noted that only the radiated wave damping was considered in this analysis and that the additional viscous damping could also be identified from model test[15] or predicted using viscous CFD[12].

6. ACKNOWLEDGEMENTS

The authors would like to acknowledge the past support of the US Navy Office of Naval Research's MOB Program/Project Office and program managers Gene Remers and Robert Taylor. The authors would also like to acknowledge the past support of the National Science Foundation Dynamic Systems and Control Program and program manager Dr. Alison Flatau.

7. REFERENCES

- [1] Paulling, J.R. and Rosenberg, R.M., "On Unstable Ship Motions Resulting from Nonlinear Coupling," *J. Ship Res.* 3, 36-46, 1959.
- [2] Esparza, I. and Falzarano, J.M., "Nonlinear Rolling Motion of a Statically Biased Ship under the Effect of External and Parametric Excitation," *ASME Vibrations Conference Symposium on Time-varying systems*, 1993.
- [3] Francescutto, A., "An experimental Investigation of Parametric Rolling in Head Waves," *OMAE 2000 Conference*, New Orleans, Feb. 14-17, 2000, also accepted into *OMAE Journal*.
- [4] Kerwin, J.E., "Notes on Rolling in Longitudinal Waves", *Int. Shipbuilding Progr.* 2, 1955.
- [5] Paulling, J.R., et al, "Head Seas Parametric Excitation of a Large Container Ship," April 2003, *SNAME Marine Technology*.
- [6] Blocki, W., "Ship safety in connection with parametric resonance of the roll," *Int. Shipbuilding Progr.* 27, 36-53, 1955.
- [7] Falzarano, J.M., Holappa, K. and Taz-Ul-Mulk, M., "A Generalized Analysis of Saturation Induced Rolling Motion," *Nonlinear Marine Dynamics*, *ASME WAM*, 1993.
- [8] Spyrou, K.J., "On the Parametric Rolling of Ships in Following Sea under Simultaneous Nonlinear Periodic Surging," *Phil. Trans. R. Soc. Lond. A*, 358, 1813-1834, 2000.
- [9] Falzarano, J.M., Uppu, K., Rodrigues, W.E., and Vassilev, R.H., "MOB SBU Transit Draft Dynamics and Stability Analytic Study," *Third International Workshop on Very Large Floating Structures (VLFS '99)* Honolulu, Sept., 1999, edited by Ertekin, R. and Kim, J..
- [10] Das, S. and Falzarano, J., "Transit Draft Roll Motion Stability Analysis Of The Mobile Offshore Base (MOB) Using Time Varying Coefficients," *Proceedings of the 11th International Offshore and Polar Engineering Conference*, Seattle, 2001.
- [11] Falzarano, J.M., Rodrigues, W.E., Vassilev, R.H., Das S. and Cheng, J., "MOB Transit Draft Dynamics and Stability", *OMAE 2000 Conference*, New Orleans, Feb. 14-17, 2000.

-
- [12] Korpus, R.A. and Falzarano, J.M., "Prediction of Viscous Ship Roll Damping using Unsteady Navier-Stokes Techniques," ASME OMAE Journal, 1997.
- [13] Faltinsen, O.M., *Sea Loads on Ships and Offshore Structures*, CAMBRIDGE University Press, 1990.
- [14] Newman, J.N., "The Motion of a Spar Buoy in Regular Waves," David Taylor Model Basin Report, No. 1499, May 1963.
- [15] Rodrigues, W. and Falzarano, J., "Transit Draft Heave Motion Analysis of the Mobile Offshore Base (MOB) using Reverse MI/SO Techniques," Proceedings of the 11th International Offshore and Polar Engineering Conference, Seattle, 2001.

